

# Optimal Epidemic Information Dissemination in Uncertain Dynamic Environment

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**Abstract**—Optimization of stochastic epidemic information dissemination plays a significant role in enhancing the reliability of epidemic networks. This letter proposes a multi-stage decision-making optimization model for stochastic epidemic information dissemination based on dynamic programming, in which uncertainties in a dynamic environment are taken into account. We model the inherent bimodal dynamics of general epidemic mechanisms as a Markov chain, and a state transition equation is proposed based on this Markov chain. We further derive optimal policies and a theoretical closed-form expression for the maximal expected number of successfully delivered messages. The properties of the derived model are theoretically analyzed. Simulation results show an improvement in reliability, in terms of accumulative number of successfully delivered messages, of epidemic information dissemination in stochastic situations.

**Index Terms**—Epidemic mechanisms, information dissemination, dynamic programming, Markov chain.

## I. INTRODUCTION

EPIDEMIC algorithms emerge as an effective bio-inspired mechanism for information dissemination in large-scale distributed systems as they possess the characteristics of simplicity in implementation, robustness and high resilience to failures [1]. In an epidemic-based routing protocol, a communication terminal usually forwards a certain number of messages to a randomly selected subset or all of its neighboring nodes during a finite time period. This period is generally divided into successive time stages. The *fan-out* number in each epidemic stage, as a key control parameter, should be properly set to achieve high reliability in epidemic information dissemination.

So far, extensive variants of epidemic routing methods have been proposed, such as the strategic learning combining game theory gossip scheme [2], the optimal energy management strategies combining optimal control of activation and transmission [3], the random-efficient spreading with Markov chain [4], the sociality-aided adaptive recovery epidemic routing

scheme [5]. We refer the interested readers to [6] and [7] for a comprehensive understanding of epidemic mechanisms and routing protocols. Taken together, most previous studies base their modeling on deterministic ordinary differential equations (ODEs) to describe epidemic behaviors, thereby falling within the deterministic modeling paradigms in which modeling-related parameters are usually determined without consideration of uncertainties inherent in a targeted system, whereas in real scenarios of mobile communications such as DTNs and VANETs, uncertainties come from many coupled environmental factors. At this point, a stochastic modeling can be more appealing to deal with a decision-making problem in dynamic environments. Different to these works, taking into account uncertainties in epidemic dissemination, we derive our dynamic programming model by incorporating a probabilistic formulation into the epidemic dynamics motivated by epidemic bimodal behavior [1]. Therefore, our work actually falls within the probabilistic paradigm that allows us to cope with uncertain dynamic environments.

In this letter, we focus on modeling the optimal control of the fan-out of epidemic information dissemination in one node as a sequential multi-stage decision-making process. How to compute the optimal fan-out number of different messages from one node in each time stage so as to maximize the number of messages successfully delivered within the whole process is the core issue in our research. We introduce a state variable of two dimensions, one of which corresponds to the accumulative number of messages that are successfully forwarded to the destination nodes from one node, which could be varying randomly, the other being the predicted success in delivering the messages originated during each forwarding stage. A Markov chain is then constructed to model the stochastic dynamics of successive state transitions based on the inherent bimodal behavior of epidemic algorithms. With the Markov chain, we use the dynamic programming technique to develop the optimization model for stochastic epidemic information dissemination. Based on this, the optimal epidemic policies as well as the theoretical closed-form expression for optimal expected utility are derived. Further, simulations are conducted to prove the validity of the optimal epidemic policies and the closed-form expression.

## II. SYSTEM MODEL

Consider that epidemic information dissemination occurs during a finite time period. We discretize the time horizon of information dissemination of one node into finite time stages,

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i.e.,  $n \in \{0, 1, \dots, N\}$  where  $n$  is a time stage index. At the beginning of each stage  $n$ , a communication node always makes a decision on the number of messages that it would like to fan out, denoted by  $i_n$ . It is worth highlighting that our model focuses on the decision-making process of the individual node rather than the multi-hop epidemic process. Since increasing  $i_n$  would lead to communication link congestion, high occupancy of the node's storage buffer, high transmission power consumption, especially when the network is dense, the fan-out of the node should be limited by a certain threshold. Thus, we can limit  $i_n$  within  $[0, \bar{i}_n]$  where  $\bar{i}_n$  should be appropriately specified as the upper bound of  $i_n$ . In stage  $n$ , the accumulative number of successfully delivered messages originated from the information source can be represented by  $c_n$ . To quantify the benefit resulting from reliability and efficiency of epidemic information dissemination, we introduce a general nonnegative utility function,  $S(c_n)$ , as a performance measure, which maps  $c_n$  to an expected utility.  $S(c_n)$  can be prespecified according to a specific application scenario, which should be a monotonically increasing function of  $c_n$  due to the fact that more messages successfully delivered indicate better reliability and higher efficiency achieved by the epidemic mechanism. Then, the optimization objective of epidemic information dissemination is formulated as

$$\max \{S(c_N) | S(c_n), n = 0, 1, \dots, N - 1\}. \quad (1)$$

With the objective (1), a node would like to make a series of optimal decisions on  $i_n$  for  $\forall n \in \{0, 1, \dots, N\}$ . Thus, we formulate the problem as an  $N$ -stage dynamic decision-making process, in which a set of optimal  $i_n$ , denoted by  $\{i_n^*, n = 0, 1, \dots, N\}$ , needs to be computed and adopted. Since maximizing  $c_n$  is key to optimizing epidemic information dissemination, therefore, determining the optimal fan-out number in every time stage is a primary goal in our research.

As stated in Ref. [1], epidemic mechanisms are inherently *bimodal*, which means that one node would exhibit two types of behavior in a time stage. One is convergence to the endemic, which means that almost all other nodes in the network are infected and all messages can be successfully propagated epidemically from the original node to the destination nodes. The other case is disease-free equilibrium, which implies that only negligible nodes in the network are infected and no epidemic information is delivered successfully to other nodes. We introduce the concept of an ideal state in epidemic information dissemination in a time stage resulting from the first case of the bimodal behavior. We call this state "full messages delivery" hereafter. Considering uncertainties in a dynamic epidemic environment resulting from many complex factors including the evolving topology and different moving speed of nodes, we assume that full messages delivery is stochastic: the full messages delivery in each stage  $n$  can either be realized or not in a random way. We further define by a random variable  $M_n$  the possibility that full messages delivery can be realized in stage  $n$ , which follows a certain random distribution characterized by a cumulative distribution function (CDF)  $F_M(m)$  over  $[0, 1]$ . Correspondingly,  $M_n$  is observed by  $m_n \in [0, 1]$  varying along with  $n$ . We point out that since multiple factors in the network have influence on the bimodal

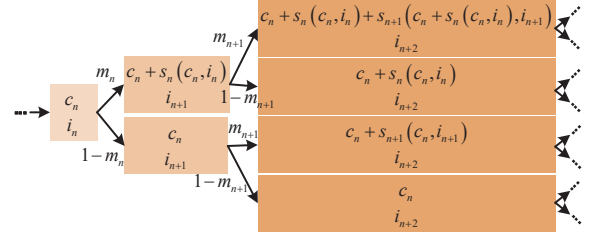


Figure 1. Markov chain diagram that defines the stochastic epidemic bimodal behavior. In a stochastic state  $(c_n, m_n)$ ,  $c_n$  is the accumulative number of successfully delivered messages at the beginning of  $n$ , while  $m_n$  is the probability of the full messages delivery in  $n$ .

dynamics, their comprehensive effect is aggregated into this single variable  $m_n$ .

With the mathematical notations given above, the stochastic epidemic bimodal behavior in an uncertain dynamic environment is consequently modeled as a binary Markov chain, as illustrated in Fig. 1, where we introduce a two-dimensional variable to capture the stochastic state in each time stage  $n \in \{0, 1, \dots, N\}$ ,  $(c_n, m_n)$ . The probability of transition between any two connected states in two successive time stages  $n$  and  $n + 1$  is then parameterized by  $m_n$ . Then we introduce  $s_n(c_n, i_n)$  to denote the full delivery response in stage  $n$ . Logically, a larger fan-out number or a larger accumulative number would imply more full delivery response. Therefore, we assume for  $n \in \{0, 1, \dots, N\}$ :

$$\frac{\partial s_n(c_n, i_n)}{\partial i_n} \geq 0, \quad \frac{\partial s_n(c_n, i_n)}{\partial c_n} \geq 0. \quad (2)$$

Therefore, based on the stochastic bimodal dynamics in Fig. 1, we derive the following state transition equation for  $n = 0, 1, \dots, N - 1$ :

$$c_{n+1} = \begin{cases} c_n + s_n(c_n, i_n), & \text{with a probability } M_n; \\ c_n, & \text{with a probability } 1 - M_n. \end{cases} \quad (3)$$

Now, we use  $R_n(c_n, m_n)$  to denote the maximal expected accumulative number of messages that can be successfully delivered in the remaining  $N - n$  stages when the node is currently in the state  $(c_n, m_n)$ . Then, the utility function  $S(c_n)$  in stage  $n$  can be expressed as

$$S(c_n) = \begin{cases} R_{n+1}(c_n + s_n(c_n, i_n), t), & \text{with a probability } M_n; \\ R_{n+1}(c_n, t), & \text{with a probability } 1 - M_n. \end{cases} \quad (4)$$

Based on dynamic programming [8] and considering randomness represented by probability distribution  $F_M(m)$ , we propose the following Bellman equation as  $R_n$  denotes the maximal expected value of  $S_n$ :

$$\begin{cases} R_n(c_n, m_n) = \max_{0 \leq i_n \leq \bar{i}} \mathbb{E} [S(c_n)] \\ = \sup_{0 \leq i_n \leq \bar{i}} \int_0^1 \left\{ m_n R_{n+1}(c_n + s_n(c_n, i_n), t) \right. \\ \left. + (1 - m_n) R_{n+1}(c_n, t) \right\} dF_M(t); \\ R_N(c_N, m_N) = S(c_N). \end{cases} \quad (5)$$

To transform (1) into a dynamic programming formulation. Hence, by backward induction using the Bellman equation (5),

we are allowed to derive the optimal value of  $R_n(c_n, m_n)$  from the optimal decisions in the remaining  $N - n$  stages, i.e., from  $\{i_k^*, k = N, N - 1, \dots, n + 1\}$ . With prior knowledge on  $s(c_n, i_n)$  and  $F_M(n)$  under specific application scenarios, we will be able to derive the maximal expected utility  $S(c_N)$  and the optimal policy  $i_n^*$  in each decision making stage.

### III. MODEL ANALYSIS

In this section, we analyze the theoretical properties of the dynamic programming model proposed above.

**Lemma 1:**  $R_n(e, t)$  is an increasing function of  $e$  for any  $n \in \{0, 1, \dots, N\}$  and  $t \in [0, 1]$ .

*Proof:* For  $n = N$ , we have  $R_N(c_N, t) = S(c_N)$ . According to the definition of  $S(c_N)$ , we can see that  $R_N(c_N, t)$  is an increasing function of the variable  $c_N$  and independent of  $t$ , which indicates that the lemma holds in this case. As for  $n = N - 1, N - 2, \dots, 1$ , due to induction and the fact that  $m_n R_{n+1}(c_n + s_n(c_n, i_n), t) + (1 - m_n) R_{n+1}(c_n, t)$  is a convex combination of  $R_n(\cdot)$ ,  $R_n(c_n, m_n)$  is also an increasing function of  $c_n$  for  $m_n \in [0, 1]$ . ■

**Lemma 2:** Suppose  $\partial R_{n+1}(e, t)/\partial e$  exists. Given a specific stage  $n$  and a fixed accumulative number of successfully delivered messages  $c_n$ ,  $i_n$  is an implicitly nondecreasing function,  $i_n(m_n)$ , of  $m_n$ .

*Proof:* From **Lemma 1**, it sees that  $\partial R_{n+1}(e, t)/\partial e \geq 0$  for any  $e$  and  $t \in [0, 1]$ . It follows this result and  $\partial s_n(c_n, i_n)/\partial i_n \geq 0$  that

$$\begin{aligned} & \frac{\partial}{\partial i_n} [R_{n+1}(c_n + s_n(c_n, i_n), t) - R_{n+1}(c_n, t)] \\ &= \frac{\partial R_{n+1}(c_n + s_n(c_n, i_n), t)}{\partial (c_n + s_n(c_n, i_n))} \frac{\partial s_n(c_n, i_n)}{\partial i_n} \\ &= \frac{\partial R_{n+1}(e, t)}{\partial e} \frac{\partial s_n(c_n, i_n)}{\partial i_n} \geq 0. \end{aligned} \quad (6)$$

The inequality (6) further indicates that

$$\frac{\partial}{\partial i_n} R_{n+1}(c_n + s_n(c_n, i_n), t) - \frac{\partial}{\partial i_n} R_{n+1}(c_n, t) \geq 0. \quad (7)$$

Therefore we can have

$$\frac{\partial^2}{\partial i_n \partial m_n} \left\{ m_n R_{n+1}(c_n + s_n(c_n, i_n), t) + (1 - m_n) R_{n+1}(c_n, t) \right\} \geq 0. \quad (8)$$

According to *Topkis' Characterization Theorem* in the field of economics and game theory [9],  $m_n R_{n+1}(c_n + s_n(c_n, i_n), t) + (1 - m_n) R_{n+1}(c_n, t)$  has the supermodularity, implying that an increase in  $m_n$  will increase the marginal payoff, i.e., the partial derivative  $\partial S_n(m_n, i_n)/\partial i_n$  with respect to  $i_n$ . That is, a larger  $m_n$  will make an incentive for the transmitter to raise  $i_n$  in order to maximize  $S_n(m_n, i_n)$ . Hence, we arrive at **Lemma 2**. ■

### IV. A CASE STUDY

To show how to apply the proposed model (5), we consider a specific epidemic scenario where we assume that the full delivery response  $s_n$  equals  $i_n$ , i.e.,  $s_n(c_n, i_n) = i_n$ , and

the probability of full messages delivery  $M_n$  is uniformly distributed within  $[0, 1]$ , i.e.,  $M_n \sim U[0, 1]$  and

$$F_M(m) = \begin{cases} 0, & m < 0; \\ m, & m \in [0, 1]; \\ 1, & m \geq 1. \end{cases} \quad (9)$$

Furthermore, we introduce a nonnegative parameter,  $\gamma \geq 0$ , to capture the individual decision-making preference of a node: the value of  $\gamma = 1$  refers to the case that the node is neutral in resource consumption incurred by forwarding information; in contrast,  $\gamma \in [0, 1)$  indicates that the node, acting in its self-interest, prefers to conserving its communication resources, while  $\gamma > 1$  implies that the node is more likely to consume more resources for information dissemination in the uncertain dynamic environment. A higher  $\gamma$  indicates more effort the mobile node would like to make in order to maximize the epidemic reliability and can increase  $i_n$  for any stage  $n$ . Next, we are ready to derive **Theorem 1** to characterize the optimal epidemic policy and the corresponding optimal value function. **Theorem 1** can perform as a significant guidance to solve  $R_n^*(c_n, m_n)$  step by step.

**Theorem 1:** Suppose  $M_n \sim U[0, 1]$  for  $n = 0, 1, \dots, N$ ,  $S(c_N) = c_N$ , and  $\gamma \geq 0$ . The optimal information dissemination policy in any time stage  $n$ ,  $i_n^*$ , can be expressed as

$$i_n^* = \gamma m_n \bar{i}_n, \quad (10)$$

and the optimal expected value given by (5) is formulated as

$$R_n^*(c_n, m_n) = m_n i_n^* + \frac{1}{2} \sum_{k=n+1}^{N-1} i_k^* + c_n. \quad (11)$$

*Proof:* Given  $M_n \sim U[0, 1]$  for  $n = 0, 1, \dots, N$ , we can easily see  $dF_M(t) = dt$  for  $t \in [0, 1]$ . According to the definitions of  $m_n$  and  $\gamma$ , we can further modify the upper bound of  $i_n$  as  $\gamma m_n \bar{i}_n$ , i.e.,  $i_n \in [0, \gamma m_n \bar{i}_n]$ .

From Equation (2), it is obvious that  $\partial c_{n+1}/\partial i_n \geq 0$ . At this point,  $c_{n+1}$  is a nondecreasing function of  $i_n$ . According to **Lemma 1** and (5), the increase of  $c_n$  will improve  $R_n(c_n, m_n)$ . In other words, any node should adopt the maximum potential policy of  $i_n$ ,  $i_n^* = \gamma m_n \bar{i}_n$ , as its optimal policy for epidemic information dissemination in any time stage  $n$  in order to maximize its expected utility  $R_n(c_n, m_n)$  as much as possible. Such a form of the optimal policy is also consistent with **Lemma 2**. Then, by applying the optimal policy of  $i_n^*$  to the solution of optimal expected value of accumulative number of delivered messages, we can conduct a backward mathematical induction to derive  $R_n^*(c_n, m_n)$  and prove **Theorem 1**. ■

We point out in other application scenarios,  $s_n(c_n, i_n)$  and  $F_M(m)$  would have different forms, which may be more complex. Once their formulation can be well defined according to the targeted scenario of interest, we can use the model above to determine the optimal policies and maximal expected utility.

### V. NUMERICAL RESULTS

To evaluate our proposed multi-stage dynamic programming model in an uncertain dynamic environment, we have set up

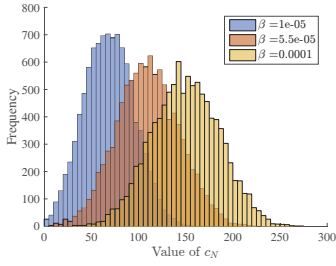
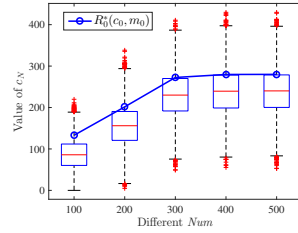
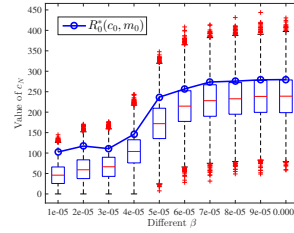


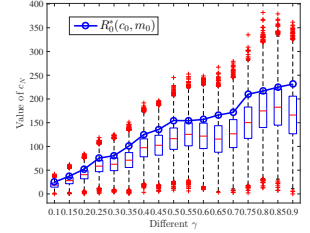
Figure 2. Distribution pattern of the accumulative number of successfully delivered messages at the end of the final stage in different environments with different intercontact rates.



(a) Case I



(b) Case II



(c) Case III

Figure 3. Comparison of the accumulative number of successfully delivered messages obtained by calculating the closed-form optimal value function  $R_0^*(c_0, m_0)$  along with the optimal policy  $i_n^* = \gamma m_n \bar{i}_n$  in each stage  $n$  and by Monte Carlo simulations under different parameter settings.

a simulation scenario, where  $Num$  mobile nodes are assumed to follow a random waypoint mobility model and the rate of pairwise contact between any two mobile nodes is denoted by  $\beta$ . Two nodes, uniformly and randomly selected from these nodes, are treated as an information source and a destination. We divide the total simulation duration of 3600s into  $N = 6$  successive stages each with a time interval of  $\Delta T = 600$ s. According to [6], the probability that a message is successfully forwarded to the destination in the basic epidemic fashion (i.e. without any immunization or adaptive mechanisms) during a given time stage can be analytically approximated by  $Prob(\Delta T) = 1 - Num / [Num - 1 + \exp(\beta Num \Delta T)]$ . Considering randomness in each stage  $n$ , we further simulate  $m_n$  using  $m_n = Prob(\Delta T) + (1 - Prob(\Delta T))\eta_n$  in our experiment where  $\eta_n$  is assumed a random variable uniformly distributed over  $[0, 1]$ .

To show the influence of randomness in the epidemic environment consisting of  $Num = 200$  mobile nodes each with an individual forwarding preference  $\gamma = 0.5$  on the final accumulative number of successfully delivered messages,  $c_N$ , we have conducted Monte Carlo simulations where we set  $\beta \in \{0.1 \times 10^{-4}, 0.55 \times 10^{-4}, 1 \times 10^{-4}\}$  and  $\bar{i}_n = 100$  for each  $n$ . Allowing for stochasticity, we set the policy for basic epidemic information dissemination in stage  $n$ ,  $i_n$ , as  $i_n = \gamma m_n \bar{i}_n \eta_n$ . Interestingly, it can be observed from Fig. 2 that the distribution pattern of stochastic  $c_N$  under different settings of  $\beta$  is bell-shaped, suggesting that the sum of multiple random variables  $\{I_1, \dots, I_n, \dots, I_6\}$  in six successive time stages (where each  $I_n$  is used to denote the random number of successfully delivered messages in stage  $n$ ),  $\sum_{n=1}^6 I_n$ , would have an approximately normal distribution under the dynamic environment. This is consistent with the central limit theorem.

Furthermore, we provide a diverse array of figures in Fig. 3 that shows the influence of different parameters and compares the results offered by computing the theoretical model with those obtained by the basic epidemic simulations. In Fig. 3, we have also performed Monte Carlo simulations with  $10^5$  replications per parameter point, and the results are illustrated with a series of boxplots where the expected performance is marked by a red median bar. Notably, it can also be found that our  $R_0^*(c_0, m_0)$ , determined by using the optimal policy (10) and the closed-form optimal utility (11) at each parameter

point in any case, is always higher than the value of  $c_N$  at the third quartile of the corresponding box. The observation indicates that our proposed optimal policy based on multi-stage dynamic programming can achieve more than about 25% of where the mean reliability of the basic epidemic mechanism in the uncertain dynamic environment simulated under different environmental conditions characterized by  $Num$ ,  $\beta$  and  $\gamma$ .

## VI. CONCLUSION

In this letter, we have studied the issue of epidemic information dissemination in a dynamic uncertain environment and considered stochastic epidemic bimodal dynamics. The theoretical properties of the proposed model were investigated, and a closed-form optimal expected utility function as well as the underlying optimal policy for epidemic information dissemination was derived. Numerical results have demonstrated that our model can bring an improvement in terms of reliability in epidemic information dissemination when compared to the basic epidemic mechanism.

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