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Regular expression inference (REI) is a supervised machine learning and program synthesis problem that takes a cost metric for regular expressions, and positive and negative examples of strings as input. It outputs a regular expression that is *precise* (i.e., accepts all positive and rejects all negative examples), and *minimal* w.r.t. to the cost metric. We present a novel algorithm for REI over arbitrary alphabets that is enumerative and trades off time for space. Our main algorithmic idea is to implement the search space of regular expressions succinctly as a contiguous matrix of bitvectors. Collectively, the bitvectors represent, as characteristic sequences, all sub-languages of the infix-closure of the union of positive and negative examples. Mathematically, this is a semiring of (a variant of) formal power series. Infix-closure enables bottom-up compositional construction of larger from smaller regular expressions using the operations of our semiring. This minimises data movement and data-dependent branching, hence maximises data-parallelism. In addition, the infix-closure remains unchanged during the search, hence search can be staged: first pre-compute various expensive operations, and then run the compute intensive search process. We provide two C++ implementations, one for general purpose CPUs and one for Nvidia GPUs (using CUDA). We benchmark both on Google Colab Pro: the GPU implementation is on average over 1000x faster than the CPU implementation on the hardest benchmarks.

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1 INTRODUCTION

This paper answers the following quantitative research question in the affirmative:

Can well-known machine learning approaches, other than neural networks, benefit from GPU acceleration, in the sense of running at least 2 orders of magnitude faster than comparable CPU implementations?

What is the technical essence that allowed GPU acceleration of artificial neural networks (ANNs)? Simplifying a great deal, computer graphics algorithms often have the following characteristics:

- Highly parallel.
- Predictable data movement due to high spatial and temporal data locality.
- Little to no conditional execution based on non-local data.

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© 2023 Copyright held by the owner/author(s). 2475-1421/2023/6-ART160 https://doi.org/10.1145/3591274 Specialising processor architecture to such algorithms avoids the overhead of general purpose CPUs (e.g., branch prediction, complex cache hierarchies and out-of-order execution) [Dally et al. 2020; Hennessy and Patterson 2017] and leads to GPUs. Generalised matrix multiplication algorithms (GeMM) have similar advantages, which explains the popularity of GPUs in scientific computing. Training and inference of ANNs can also be reduced to GeMM, which we believe is why they can be accelerated on GPUs. As far as we are aware, few other well-known machine learning (ML) techniques are currently known to be accelerable on GPUs. Can we change this? In answer to this question, this paper considers precise and minimal regular expression inference (REI) from positive and negative examples of strings. REI, the most well-studied form of grammar inference [Wikipedia contributors 2022a], is one of the oldest approaches to ML [Angluin 1987; Gold 1967]. To the best of our knowledge, precise and minimal REI is an open problem for ANNs.

Regular expressions are a constrained mechanism for succinct, finite specification of finite and infinite languages. While all finite languages are definable, regular expressions can only specify simple infinite languages. Regular expressions are one of the most widely used and well-known formalisms in computer science, and we assume that the reader is familiar with them. Here is an example of a regular expression

$$10(0+1)^*$$

which specifies the language of all strings with characters from $\{0, 1\}$ that start with 10. We assume that regular expressions *r* have a *cost*, written cost(r). The exact nature of $cost(\cdot)$ will play a key role later, for now a naive understanding of cost, for example the length of *r* as a string, is sufficient. It is often bothersome to write down regular expressions explicitly, so instead we'd like to program them "by example": we give a set of positive and negative examples of strings, and get a suitable regular expression that accepts all positive examples and rejects all negative examples. For example

Positive : 10, 101, 100, 1010, 1011, 1000, 1001
Negative :
$$\epsilon$$
, 0, 1, 00, 11, 010 (1)

should, ideally, lead to the target expression $10(0 + 1)^*$. Clearly this is a special case of supervised learning, and, like all supervised learning, is subject to subtle problems, including that there isn't a unique way of generalising from a finite set of examples. The regular expression

$$10(0^* + 1^*)^* + 1000$$

and infinitely many others, correctly accept all positive examples and rejects all negative examples from (1). We want a canonical "natural" regular expression to be inferred from the example. But what does it mean to be canonical and natural? One might argue that the regular expression

$$10 + 101 + 100 + 1010 + 1011 + 1000 + 1001$$
(2)

is both, natural and canonical, and indeed minimal in some sense (it accepts exactly the positive examples and rejects every other string). At the same time it is unlikely to be what the author of the examples had in mind. Instead, it, in the language of modern ML, *overfits* on the examples. The ever-present option to overfit shows that REI is trivial without additional constraints such as minimality. With regular expressions, overfitting is easy to avoid, an insight we adapt from [Feser et al. 2015], since we can request a minimum cost regular expression that meets all examples. We reject (2) as it is much bigger than $10(0 + 1)^*$. Such minimisation can be seen as a form of regularisation, but there are multiple ways of measuring a regular expression's cost, so minimality is always relative to a chosen cost measurement. We come back to this later.

The core insight that lets us implement fast REI on a GPU is the following. First, instead of regular expressions, we search over regular languages, which are certain functions of type $r : \Sigma^* \to \{0, 1\}$, where Σ is the ambient alphabet (of arbitrary size), and r(w) = 1 iff w is in the language. Ignoring

cost, two regular expressions are equal with respect to a pair of sets of positive and negative examples, *P* and *N*, if their respective languages relate to the members of $P \cup N$ in the same way. Hence, during search, we can restrict our attention to functions $(P \cup N) \rightarrow \{0, 1\}$, but, in order to build up these functions in a compositional way that preserves cost-minimality, we instead search over a small generalisation:

$$ic(P \cup N) \to \{0, 1\}$$

Here ic(*S*) is the infix-closure of a set (*w* is an infix, aka substring, of string σ , if σ is of the form $\sigma_1 w \sigma_2$ for some strings σ_i). Since computer memory is totally ordered, we can implement each such function as a bitvector. Since *P* and *N* do not change during each REI run, all bitvectors that arise during search have the same length, and collectively form a binary matrix in memory. This representation allows us to implement REI mostly using matrix operations with little data-dependent branching, predictable data movement and enables a great deal of parallelism.

Contributions. In summary, our contributions are as follows:

- A data parallel algorithm for precise and minimal REI from positive and negative examples that is based around succinct representation of data structures and trades off memory for speed, to minimise data movement and data-dependent branching. Most algorithmic choices work for general grammar inference, and are not specific to regular expressions.
- Implementations of the algorithm on CPUs and GPUs.
- A parameterised benchmark suite with examples that we believe are useful for evaluating the performance of REI, beyond the present paper.
- Performance measurements of our implementations showing that both are faster than existing algorithms and that our GPU version is orders of magnitude faster than the CPU version. All measurements are available from [Valizadeh and Berger 2023].
- A mathematical foundation for our algorithm based on the well-established theory of formal power series.

2 MATHEMATICAL PRELIMINARIES

In order to establish terminology, we begin with a condensed review of standard mathematical concepts used later.

2.1 Background

Definition 2.1. We assume that Nat = {0, 1, 2, ...}. By \mathbb{B} we denote the set {0, 1} of Booleans. We use 0 for falsity and 1 for truth. We write $\mathfrak{P}(A)$ for the *powerset* of *A*. We write #*S* for the cardinality of the set *S*. The *characteristic function* of a set $S \in \mathfrak{P}(A)$ is the map $\mathbf{1}_S^A : A \to \mathbb{B}$ which maps $a \in A$ to 1 iff $a \in S$, and otherwise to 0. If *A* can be disambiguated from the context, we write $\mathbf{1}_S$ for $\mathbf{1}_S^A$. By the *average* of a list $x_1, ..., x_n$ of numbers we mean their arithmetic mean, i.e., $\frac{1}{n}(x_1 + \cdots + x_n)$.

Definition 2.2. A monoid is a tuple (M, \circ, ϵ) such that M is a set, $\circ : M^2 \to M$ is an associative function, and $\epsilon \in M$ such that: for all $m \circ \epsilon = m = \epsilon \circ m$. A set $S \subseteq M$ is *infix-closed* if whenever $a \circ b \circ c \in M$ then also $b \in M$. By ic(S), the *infix-closure* of a set S, we mean the smallest infix-closed superset of S. A semiring is a tuple $(S, +, \circ, 0, 1)$ such that: (S, +, 0) is a commutative monoid, $(S, \circ, 1)$ is a monoid, the distributive laws hold, i.e., $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ and $(a + b) \cdot c = (a \cdot c) + (b \cdot c)$, and, finally, $0 \cdot a = 0 = a \cdot 0$. We call \circ and + the product and sum of the semiring.

Definition 2.3. An alphabet is a finite set Σ , and we often speak of *characters* when we mean the elements of Σ . A string of length $n \in N$ at over Σ is a map $\sigma : \{0, 1, ..., n - 1\} \rightarrow A$. We write $\|\sigma\|$ for *n*. A string over Σ is a string of length *n* for some *n*. We often omit stating the alphabet where

this is clear from the context. We often write σ_i instead of $\sigma(i)$. We write $\sigma \cdot \gamma$, or just $\sigma\gamma$ for the concatenation of two strings σ and γ . We write ϵ for the unique string of length 0. We write Σ^* for the set of all strings over Σ , and Σ^n the restriction of Σ^* to strings of length *n*. We write $\Sigma^{\leq n}$ for the set of strings of length not exceeding *n*.

Definition 2.4. A language over the alphabet Σ is a set of strings over Σ . We have the following well-known algebraic operations over Σ^* : constants \emptyset and ϵ (the language containing only the empty string), negation (aka complement), \overline{L} for $\Sigma^* \setminus L$, union (aka disjunction), $L \cup L'$, concatenation, $L_1 \cdot L_2$, or just L_1L_2 , for $\{\sigma \gamma \in \Sigma^* \mid \sigma \in L_1, \gamma \in L_2\}$, intersection or conjunction, $L_1 \cap L_2$. Finally there is the Kleene-star, where for all $n: L^0 = \{\epsilon\}, L^{n+1} = L^n \cdot L$ and then $L^* = \bigcup_{n \ge 0} L^n$. Those operations form various algebras, for example $(\mathfrak{P}(\Sigma^*), \cup, \cdot, \emptyset, \epsilon)$ is a semiring.

Definition 2.5. A partial order is a tuple (P, \sqsubseteq) where P is a set and \sqsubseteq , a subset of P^2 is a binary, reflexive, anti-symmetric and transitive relation on P. We write $p \sqsubset q$ to signify that $p \sqsubseteq q$ and $p \neq q$. We say the order is *total* if always $p \sqsubset q$, or $q \sqsubset p$ or p = q. If $Q \subseteq P$ then \sqsubseteq also orders Q by *restriction*. If a set Σ is ordered by \sqsubseteq , then this order can be lifted to Σ^* using the *shortlex* ordering: $\sigma \sqsubseteq \sigma'$ iff either $\|\sigma\| < \|\sigma'\|$, or for some i we have: $\sigma_i \sqsubseteq \sigma'_i$ and, at the same time, for all $j < i : \sigma_j = \sigma'_i$. Note that, by restriction, this orders every subset of Σ^* .

Note that anything stored in a computer's memory is always totally ordered, since the addresses of memory cells are integers.

Definition 2.6. The regular languages over an alphabet Σ , denoted Reg(Σ), are inductively given by the following constraints: the empty set is regular; for each $a \in \Sigma$, the language {a} is regular; if L_1 and L_2 are regular then so are $L_1 \cup L_2$ and $L_1 \cdot L_2$; if L is regular then L^* is regular.

Definition 2.7. The regular expressions over Σ , short RE(Σ), are given by the following grammar:

$$r ::= \emptyset \mid \epsilon \mid a \mid r \cdot r \mid r + r \mid r^*$$

Here *a* ranges over Σ . We call the *, ·, +, ..., \emptyset the *regular constructors (over* Σ) of regular expressions, where each regular constructor has the obvious *arity*, e.g., * has arity 1, while + has arity 2. We use various abbreviations, including *rr'* for concatenation $r \cdot r'$, and Σ^* for $(a_1 + \cdots + a_k)^*$ assuming that $\Sigma = \{a_1, ..., a_k\}$.

Definition 2.8. With each $r \in \mathsf{RE}(\Sigma)$ we associate the *denotation* of r, aka the *language of* r, abbreviated $\mathsf{Lang}(r)$, which is defined by the following clauses: $\mathsf{Lang}(\emptyset) = \emptyset$, $\mathsf{Lang}(\epsilon) = \{\epsilon\}$, $\mathsf{Lang}(a) = \{a\}$, $\mathsf{Lang}(r \cdot r') = \mathsf{Lang}(r) \cdot \mathsf{Lang}(r')$, $\mathsf{Lang}(r + r') = \mathsf{Lang}(r) \cup \mathsf{Lang}(r')$, $\mathsf{Lang}(r^*) = \mathsf{Lang}(r)^*$. This induces an equality on regular expressions: r is equivalent to r' iff $\mathsf{Lang}(r) = \mathsf{Lang}(r')$, for example $r + r \simeq r$, or $r^{**} \simeq r^*$. Note that each equivalence class has an infinite number of inhabitants. We write r? for the regular expression with the same language as $\epsilon + r$.

2.2 Key Structure: Formal Power Series

Formal power series (FPS) generalise characteristic functions $\mathbf{1}_{L}^{\Sigma} : \Sigma^* \to \mathbb{B}$ of formal languages L to functions $\Sigma^* \to S$ where S is a semiring. This is interesting for us, because well-behaved sets of such functions form semirings themselves, so the semiring structure on S can be lifted to FPS, see [Berstel and Reutenauer 1988; Droste and Kuich 2009; Golan 1999; Salomaa and Soittola 1978]. PARESY's core data structure is a generalisation of FPS.

Definition 2.9. Let Σ be an alphabet and S a semiring. A formal power series is a map

$$r: \Sigma^* \to S$$

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The *support* of *r*, written supp(*r*) is the set supp(*r*) = { $m \in \Sigma^* | r(m) \neq 0$ } A *polynomial* is a formal power series with finite support. We now make the following definitions. $\Sigma^* \langle S \rangle$ denotes the set of all formal power series $r : \Sigma^* \to S$. $\Sigma^* \langle S \rangle$ is the subset of $\Sigma^* \langle S \rangle$, but restricted to finite support. We define the following operations on $\Sigma^* \langle S \rangle$, and, by restriction, on $\Sigma^* \langle S \rangle$, for $\sigma \in \Sigma^*$.

$$\begin{array}{ll} - \ 0(\sigma) = 0 & - \ (r+s)(\sigma) = r(\sigma) + s(\sigma) \\ - \ 1(\sigma) = \begin{cases} 1 & \sigma = \epsilon & - \ (r \cdot s)(\sigma) = \oplus\{r(\sigma_1) \cdot s(\sigma_2) \mid \sigma_i \in \Sigma^*, \sigma_1 \cdot \sigma_2 = \sigma\} \\ 0 & \text{else} \end{cases}$$

Here \oplus is semiring addition lifted to finite sets. We note the formal similarity of $r \cdot s$ with convolutions. Under mild restrictions, which hold in our use-cases, we can define Kleene-star on $\Sigma^* \langle S \rangle$ and $\Sigma^* \langle \langle S \rangle \rangle$, details omitted for brevity.

While PARESY can be presented as code, we emphasise the connection with well-known mathematical structures, because they give us a shared language: semirings and polynomials are widely known and they are a minimal 'API' for grammar synthesis; ANNs are generalised matrix operations that also form semirings; and, finally, it puts in context what is REI specific (e.g., infix-closure), and what is not (almost everything else works for arbitrary formal languages).

3 THE PARESY ALGORITHM

This section presents PARESY, our *parallel regular expression synthesiser*. To save space we omit details, in particular about low-level optimisations. They can be found in the C++ implementations.

Specifications and cost homomorphisms. The input to the algorithm is a cost homomorphism, defined below, and a set of positive and negative examples (over an arbitrary alphabet).

Definition 3.1. Given an arbitrary alphabet Σ , a *specification (over* Σ) is a pair (P, N) where both P and N are finite subsets of Σ^* . We say a language $L \subseteq \Sigma^*$ satisfies (P, N), written $L \models (P, N)$, provided $P \subseteq L$ and $N \cap L = \emptyset$. We say a regular expression r satisfies (P, N), written $r \models (P, N)$, if Lang $(r) \models (P, N)$.

Definition 3.2. A cost function is a map $cost(\cdot) : RE(\Sigma) \rightarrow Nat$. It is a cost homomorphism if there are integer constants, $c_1, ..., c_5 > 0$, such that $cost(\emptyset) = cost(\epsilon) = cost(a) = c_1$ for all $a \in \Sigma$, $cost(r?) = cost(r) + c_2$, $cost(r^*) = cost(r) + c_3$, $cost(r \cdot r') = cost(r) + cost(r') + c_4$ and $cost(r+r') = cost(r) + cost(r') + c_5$. We call each c_i the cost of the corresponding regular constructor.

We write, e.g., cost(*) for c_3 and likewise for the other costs. From now on, whenever we present a 5-tuple of numbers, e.g., $(c_1, c_2, c_3, c_4, c_5)$, this is short for the cost homomorphism $(cost(a), cost(?), cost(*), cost(\cdot), cost(+))$ in this exact order, e.g., in (5, 2, 7, 2, 19), the cost of the Kleene-star is 7. Note that we allow $cost(r?) \neq cost(\epsilon) + cost(r)$, this is for convenient comparison with related work later. As long as all costs remain strictly positive, more complex cost homomorphisms can easily be accommodated, e.g., different costs for different alphabet characters.

Core intuitions. REI is a search problem over $RE(\Sigma)$, the syntax of regular expressions. We have to decide how to represent the search space $RE(\Sigma)$ in an implementation. The natural answer, using $RE(\Sigma)$ itself, is wasteful for several reasons.

- Redundancy: each regular language is denoted by infinitely many regular expressions. For example 00 + 1 and 1 + 00 denote the same language.
- Not succinct: each regular expression is a tree (i.e., requiring additional pointers).
- Slow contains-check: the search will carry out many *contains-checks*, to determine if a candidate expression accepts or rejects a given string. Depending on implementation details, this amounts to expensive 'walking' of the tree representing the candidate.

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In order to avoid those inefficiencies, we represent regular expressions, simplifying a bit, by their languages, i.e., the search space is (a subset of) $Lang(\Sigma)$. In memory, we could represent each language *L* by its *characteristic function*

$$\mathbf{1}_L: \Sigma^* \to \mathbb{B}$$

which is formal power series in the sense of Def. 2.9. Mathematically, a function is an unordered set of pairs. Since computer memory is a totally ordered sequence of bits, we get a total order on Σ^* (e.g., using the shortlex order to lift a chosen total order on Σ). Hence we can represent $\mathbf{1}_L$ as a list of 0s and 1s in memory. We call this list *characteristic sequence* (CS). This turns every language into a bitvector, albeit infinitely long. Fortunately, we only need to implement a finite segment of these characteristic functions: the algorithm returns an r with $r \models (P, N)$. As we represent regular expressions by L, that means we need to check $L \models (P, N)$, i.e., only words in $P \cup N$, a finite set. Hence we can represent languages as finite functions

$$\mathbf{1}_{L}:(P\cup N)\to\mathbb{B}$$

which amounts to a bitvector of length $\#(P \cup N)$. In the rest of this text we will not carefully distinguish between a language *L*, and its representations as function $\mathbf{1}_L : (P \cup N) \to \mathbb{B}$, bitvector or CS.

We need not just synthesise a regular expression meeting the target specification, but a minimal one. In order to do this we lift the ambient cost function to regular languages.

Definition 3.3. Given $cost(\cdot)$, we set cost(L) to cost(r) for a minimal r with Lang(r) = L.

We now present a key insight that does not seem to appear in the extensive literature on formal power series and semirings, the mathematics behind our implementation.

LEMMA 3.4. Assume $cost(\cdot)$ is cost homomorphism, then for all $L, L_1, L_2 \in Reg(\Sigma): cost(L^*) \leq cost(L) + cost(*), cost(L?) \leq cost(L) + cost(?), cost(L_1 \cdot L_2) \leq cost(L_1) + cost(L_2) + cost(\cdot), cost(L_1 + L_2) \leq cost(L_1) + cost(L_2) + cost(+).$

This lemma enables compositional, bottom-up construction of regular languages with increasing cost: in order to construct a regular language with cost c, we choose an outermost regular constructor, subtract its cost, and recurse. E.g., for + we split the remaining cost c - cost(+) into suitable pairs c_l and c_r and find all languages L_l of cost c_l and L_r of cost c_r . Then $L_l + L_r$ has target cost not exceeding c. Computing the sum of two languages is just bitwise-or over the corresponding two bitvectors. Then we check if $L_l + L_r$ meets the specification. If not we continue to search, if yes, we reverse engineer a corresponding regular expression (see below) and return it. Kleene-star and concatenation are somewhat more complex and discussed next.

First space-time trade-off: infix-closure. There is a problem with using characteristic sequences $(P \cup N) \rightarrow \mathbb{B}$ for concatenation and Kleene-star. Recall from Def. 2.9 that the product of formal power series, understood as mappings from Σ^* into some semiring, is given as

$$(r \cdot s)(\sigma) = \bigoplus \{ r(\sigma_1) \cdot s(\sigma_2) \mid \sigma_i \in \Sigma^*, \sigma_1 \cdot \sigma_2 = \sigma \}$$

We emphasise the underlined part: if we define, for $r, s : (P \cup N) \rightarrow \mathbb{B}$

$$(r \cdot s)(\sigma) = \bigoplus \{ r(\sigma_1) \cdot s(\sigma_2) \mid \sigma_1 \cdot \sigma_2 = \sigma \}$$

the question arises: what do σ_1 , σ_2 range over? The answer cannot be: over $P \cup N$. Consider the specification ({01}, \emptyset). PARESY works bottom-up, starting from the lowest cost CS, as we will see below. The only way to construct (the CS corresponding to) $0 \cdot 1$ is as concatenation of (the CS corresponding to) 0 and (the CS corresponding to) 1, both of which are lower cost than $0 \cdot 1$. But

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the set of functions on $(P \cup N) \to \mathbb{B}$ corresponding to the alphabet characters 0, 1 is empty. One might say: $P \cup N$ is incomplete for bottom-up synthesis! We are looking for a smallest but finite superset of $P \cup N$ that is closed under regular operations. This motivates the next definition.

Definition 3.5. Let S be a semiring, and $I \subseteq \Sigma^*$ be finite and infix-closed. An *infix power series* (IPS) is a map

 $r: I \rightarrow S$

We denote the set of all IPS by $I\langle S \rangle^{ic}$. We define the following operations on $I\langle S \rangle^{ic}$, for $\sigma \in I$.

$$-0(\sigma) = 0. \qquad -(r+s)(\sigma) = r(\sigma) + s(\sigma).$$

$$-1(\sigma) = \begin{cases} 1 \quad \sigma = \epsilon \\ 0 \quad \text{else} \end{cases}, \qquad -r^*(\sigma) = \bigoplus_{n=0}^{\infty} r^n(\sigma).$$

$$-(r \cdot s)(\sigma) = \bigoplus \{r(\sigma_1) \cdot s(\sigma_2) \mid \sigma_1, \sigma_2 \in I, \sigma_1 \cdot \sigma_2 = \sigma\}$$

Here r^0 is the characteristic function of the language $\{\epsilon\}$ and $r^{n+1} = r^n \cdot r$. It is straightforward to show that $\bigoplus_{n=0}^{\infty}$ is well-defined because $ic(P \cup N)$ is finite, see [Droste and Kuich 2009] for the more complex general case. Boolean operations like negation or conjunction are similarly easy to define, and omitted for brevity.

Noting that $(\mathbb{B}, \land, \lor, 0, 1)$ forms a semiring, this gives us PARESY's search space: $ic(P \cup N) \langle \mathbb{B} \rangle^{ic}$. In other words functions $r : ic(P \cup N) \rightarrow \mathbb{B}$, which, with the assumed total order on $ic(P \cup N)$, give us bitvectors. The *i*-th element of each bitvector corresponding to *r* stores the value r(w), where *w* is the *i*-th element of $ic(P \cup N)$.

Example 3.6. Consider the specification $P = \{1, 011, 1011, 11011\}$ and $N = \{\epsilon, 10, 101, 0011\}$. Then ic($P \cup N$) is

Assume $ic(P \cup N)$ is ordered as above, and consider the regular expression $r = (0?1)^*1$. The intersection of Lang(r) with $ic(P \cup N)$ is {11011, 1011, 011, 11, 1}. This can be represented as CS, relative to $ic(P \cup N)$:



Here binary strings in green are in *P*, red means *N*, and grey indicates strings that arise from infix-closure. The *i*th bit is 1 (dark blue square) exactly when the language contains the *i*th word of $ic(P \cup N)$. Putting multiple CSs contiguously in memory yields a matrix.

Recovering a regular expression from a regular language. The algorithm sketched above computes formal languages, but not regular expressions. If we build up minimal-cost regular languages until we find the first *L* that is compatible with (P, N), we cannot efficiently, from *L* alone, produce a minimal regular expression *r* such that Lang(r) = L. We solve this problem by associating with each bitvector *bv* enough information to be able to reverse-engineer a suitable regular expression. Simplifying at bit, it means we track the outermost regular constructor used to construct *bv* and pointers to the component bitvectors. We store bitvectors produced using the same outermost regular constructor consecutively, and track, in a lightweight manner, the information about block beginning / end in a small external table. This allows us, recursively, to construct a regular expression corresponding to the *bv*, on demand. Details are fiddly and can be found in the implementation.

Second space-time trade-off: bitvector length. How long should bitvectors representing languages be? If $n = \#ic(P \cup N)$, then *n* bits suffice, but instead we choose the smallest power-of-2, not below *n*. We make this space-time trade-off because the instruction sets of all modern processors are designed for operations working on power-of-2 sized data, typically 8, 16, 32, 64 bits, sometimes 128 bits. All other bit-widths must be expressed in terms of those, and hence are much slower.

Third space-time trade-off: caching. If we implement our algorithm naively, lower cost languages will be recomputed repeatedly when computing higher-cost languages. We prevent this with *dynamic programming* in the sense of [Fisler et al. 2022]: we construct all needed languages bottom-up, from lower to higher cost, and keep the constructed languages in memory for later re-use in a matrix that we describe below. This caching is our third space-time trade-off. It is one of the main reasons for the performance of our algorithm, but, because the number of regular languages increases exponentially with increasing cost, makes available memory the scalability limit. We will see in Sec. 4 that on a modern GPU the algorithm can solve virtually all synthesis tasks in at most a few seconds, provided they fit in memory.

Matrix representation: language cache. During each PARESY run, P and N are fixed, and so is the size of $ic(P \cup N)$. Hence each potential language, i.e., all bitvectors that arise during the search, have the same length. We store all next to each other in memory, ordered by increasing cost. This amounts to a matrix called *language cache*, the core data structure of PARESY. Ordering the language cache by increasing cost, and noting that each individual bitvector is itself a one-dimensional matrix, that means the language cache is a matrix of matrices of matrices, where the *c*-th entry contains exactly the languages of cost *c*. The complex, yet regular structure of the language cache allows us to implement REI mostly using matrix operations with little data-dependent branching, predictable data movement and enables a great deal of parallelism. For (P, N) from Example 3.6, the language cache could contain something like the blue squares in the figure below.



We annotate every row with a regular expression accepting the language of the row. Assuming (1, 1, 1, 1, 1) as cost function, the regular expression is minimal for (P, N). The costs on the right show the language cache naturally decomposes into clusters of equal cost.

Uniqueness checking. The compositional construction of languages is not injective: target languages might be constructed more than once. For example a union of languages $\{001\} + \{\epsilon\}$ results in the same language as $\{001, \epsilon\} + \{\epsilon\}$. In order to avoid the performance penalty from this duplication we remove them as soon as possible. Unlike other operations of our algorithm, uniqueness is a

global property: as soon as a new CS is constructed, we compare it to all previously constructed CS. We add it to the matrix only if it is genuinely new. Several things are noteworthy about our approach to uniqueness checking:

- The performance of uniqueness checking is crucial to performance.
- It works on all formal languages. It is not tied to regular languages.
- It is subtly different from the *pruning* techniques proposed in e.g., [Lee et al. 2016a], in the sense that they prune paths in their search space *before* constructing regular expressions. We remove languages *after* we construct them. Pruning before construction is an interesting direction for further work.
- Computing global properties like uniqueness efficiently on GPUs is challenging. Our uniqueness checker is a modified form of the HashSet class from WarpCore [Jünger 2022; Jünger et al. 2020], a CUDA library for high-performance hashing of 32 and 64 bit integers. We can use WarpCore because we represent as (sequences of) unsigned integers (powers-of-2, see above).

Staging: guide table. We have seen how easy it is to compute the union of two formal languages by bitwise-or. Fast computation of concatenation or the Kleene-star is harder because of the convolutional nature of concatenation, which the Kleene-star iterates. Recall that the product in $I\langle S \rangle^{ic}$ is defined abstractly as follows:

$$(r \cdot s)(w) = \bigoplus \{ r(\sigma_1) \cdot s(\sigma_2) \mid \sigma_1, \sigma_2 \in I, \sigma_1 \cdot \sigma_2 = w \}$$

In our case of characteristic sequences $ic(P \cup N) \rightarrow \mathbb{B}$, the check $\sigma_1 \cdot \sigma_2 = w$ is somewhat expensive, and, if our algorithm was implemented naively, would have to be re-run every time we construct a new characteristic sequence from old using concatenation or Kleene-star. Fortunately *P*, *N* remain constant, and we pre-compute all ways in which a word *w* can be split. This amounts to a function

$$gt: ic(P \cup N) \to \mathfrak{P}(ic(P \cup N)^2)$$

that returns $gt(w) = \{(\sigma_1, \sigma_2) \mid \sigma_1 \cdot \sigma_2 = w\}$. The concrete implementation is the *guide table*, an array of arrays of pairs of offsets into the language cache, and we omit details for brevity.

OnTheFly mode. Space-time trade-offs make the algorithm memory intensive. This is the main scaling limitation. The OnTheFly mode alleviates the problem somewhat, without compromising on minimality and precision. The insight enabling OnTheFly is that, depending on cost function, computing a regular language of target cost might make reference only to lower cost CSs that are still cached, e.g., if the cost of all regular constructors is > 55, then the algorithm needs only CSs of target cost minus 55. That means, even when we have run out of language cache space, the algorithm can continue for a while, creating new CSs from old CSs still in the language cache, but these new CSs are neither cached, not checked for uniqueness. Avoiding uniqueness checks makes OnTheFly much faster. Our algorithm automatically switches to OnTheFly mode when the language cache is full. Eventually, OnTheFly mode needs access to CSs that are no longer cached, then synthesis stops without having found a suitable regular expression. Our implementation and measurements in Sec. 4 refer to this as out-of-memory error.

GPU language cache implementation. The explanations so far used mathematical concepts, because we wanted to separate low-level implementation details from high-level abstract structural ideas. However, the block matrix structure of the language cache, essentially a matrix of matrices of matrices, is itself interesting, and, especially on GPUs where locality of memory access matters, important for performance.

The figure below sketches how a new cost-level is built put. Grey parts are temporary data, while blue is permanent data. L and R denote auxiliary data, allowing the conversion of a CS to a corresponding regular expression. In (a) we see how a new cost level is built up from data already in the language cache. The newly constructed CS are first held in temporary storage. Only CS in temporary memory that passes the uniqueness check is then copied into the language cache in (b) filling the new cost level. The use of intermediate temporary storage enables parallelism on GPUs.



The overall structure of the algorithm is simple:

- Allocate the language cache as contiguous array of bytes (which need not be initialised, as the
 rest of the algorithm guarantees that the first read always happens after the first assignment).
 The internal structure of the language cache as contiguous array emerges during search.
- During searching, the language cache is filled in a single sweep from left to right, in a writeonce manner. No element of the language cache is ever removed or even changed, once in the language cache.
- The cost of CSs stored in the language cache is never decreasing.
- A newly created CS is not directly stored in the language cache.
- Search terminates only if the algorithm finds a solution to the specification, or OnTheFly has exhausted the language cache's supply of CSs.

The pseudocode below adds more detail. For simplicity, pseudocode models the language cache as an list of fixed size entries, indexed by cost: accessing the *i*-th language cache entry returns a list of CSs with cost *c*, we ignore the aforementioned auxiliary information.

Algorithm 1 Main function of synthesis algorithm

```
1: Input Positive and negative examples (P, N), cost, maxCost
 2: Output A minimal RE w.r.t. cost and maxCost and consistent with (P, N), otherwise "not_found"
 3:
 4: if \mathcal{P} == \{\} then return \emptyset
 5: if \mathcal{P} == \{""\} then return \epsilon
 6: languageCache = [list of CSs of alphabet]
                                                                                                       IanguageCache is global variable
 7: for c \leftarrow cost(\epsilon) + 1 to maxCost do
        questions = buildQuestionMark( c - cost(?))
 8:
 9:
        stars = buildStar(c - cost(*))
10:
        concats = buildConcat(c - cost(\cdot))
11:
        unions = buildUnion(c - cost(+))
12:
        languageCache[c] = questions ++ stars ++ concats ++ unions
                                                                                                                      ► ++ is concatenation
13: return "not found"
                                                                ▶ Procedures in loop will return solution directly to caller of main, if found
```

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Alg. 1 shows the overall structure of PARESY. Lines 4 and 5 handle trivial specifications. Line 6 fills the initial language cache with the initial CSs, corresponding to characters of the alphabet Σ , at index *cost*(*a*)). The loop then sweeps over all allowed costs, with increasing cost. For each cost *c*, it considers all CSs of cost *c*, first with question mark as outermost constructor, then with Kleene-star, then with concatenation, and finally union. Each returns either a regular expression that solves (*P*, *N*) and the algorithm terminates (we don't model the trivial details), or else a list of all *new* CSs. Line 12 then concatenates all those new CSs, and makes them the language cache entry for cost *c*. As mentioned above, we omit details about having a single contiguous cache, this is straightforward, but fiddly. (We access the language cache through a layer of indirection that translates cost into memory offsets, using a data-structure called *startPoints* that is dynamically updated. This is made easy by the write-once nature of the language cache.)

Alg. 2 is pseudocode for constructing CSs with concatenation as outermost constructor. It relies on the guide table, described earlier, which pre-computes all the ways each string w can be split into strings from ic($P \cup N$). In code we access the guide table with the index of the target word in ic($P \cup N$), rather than the word itself. Line 5 splits the available cost *c* into all pairs (*L*, *R*) of costs that sum up to c. The next two lines then retrieve all CSs of costs L and R from the language cache. On GPUs this is done in parallel. For each pair (lCS, rCS) Line 10 loops over all words in $ic(P \cup N)$. Here w is the index of the word in $ic(P \cup N)$, which we assume to be totally ordered. Line 9 initialises the local variable *newCS* which is the CS we are constructing. It is initialised to the empty language. Line 8 initialises *i*, the 'pointer' into *newCS* at the position of *w*. This lets us set the bit at the right place to *newCS*. In Line 11 we start searching through all guide table entries for w, the (index of the) current word we are interested in. Let's say the guide table entry contains (l, r). If *lCS* contains the *l*-th word and *rCS* contains the *r*-th word (that is checked in Line 12), then that means w is in the CS under construction, so Line 13 sets the relevant bit i. Once we have finished with the current w, we update our 'pointer' *i* (by logical left shift) in Line 14, and process the next word. Once *newCS* is constructed, we check if it was already constructed previously. If yes, we try with new choices for lCS and rCS. If not, we check if it solves the specification (P, N) and if it does, the program converts it into a minimal cost RE, and terminates. Otherwise we add *newCS* to the language cache. For brevity we omit the pseudocode for *buildStar* which just iterates concatenation a finite number of times, and *buildUnion* and *buildQuestionMark* which are straightforward.

Algorithm 2 Pseudocode for concatenation (buildConcat procedure in Alg. 1)

1: 2:	Input cost c, globals used: languageCache, P, N Output A list of new CSs generated by concatenation
3:	
4:	$outList \leftarrow []$
5:	for all L, R such that $L + R = c do$
6:	for all $lCS \in languageCache(L)$ do
7:	for all $rCS \in languageCache(R)$ do
8:	$i \leftarrow 1$
9:	$newCS \leftarrow 0$
10:	for $w \leftarrow 0$ to $\#ic(P \cup N) - 1$ do
11:	for all pair $(l, r) \in \operatorname{gt}[w]$ do
12:	if $(lCS \& l) \neq 0$ and $(rCS \& r) \neq 0$ then
13:	$newCS \leftarrow newCS \mid i$
14:	$i \leftarrow i \ll 1$
15:	$isUnique \leftarrow hashSet.insert(newCS)$
16:	if isUnique then
17:	if $newCS \models (P, N)$ then
18:	print <i>newCS</i> and terminate program
19:	outList.insert(newCS)
20:	return outList



The image below visualises how the guide table speeds up concatenation.

Checking if $w \in ic(P \cup N)$ is accepted by $r_1 \cdot r_2$ means checking, for all possible splits $w_1 \cdot w_2 = w$, if w_i is accepted by r_i . This amounts to folding (in the sense of functional programming) over all splits. As pairs w_1, w_2 are infixes of w, fast access to infixes is crucial, and the reason why our CS have domain $ic(P \cup N)$ rather $P \cup N$. If, given language cache entries *lCS* and *rCS*, we want to compute $lCS \cdot rCS$, the CS representing concatenation of lCS and rCS, we have to compute this fold for each word *w*-more precisely for each bit *k* in the new bitvector $lCS \cdot rCS$. The loop starting on Line 10 does this. Each row in the guide table corresponds to, and is created as soon as (P, N) is available, from words w. Each row in the guide table is a *contiguous* list, hence bitvector, of pairs (i, j) corresponding to a split $w = w_i \cdot w_j$. Here *i* is the position of w_i in CSs, and *j* the position of w_i . (This indexing uses a power-of-two representation aka one-hot [Wikipedia 2023], details omitted.) The image highlights the word w = "110" at index 10 in the language cache, and the guide table row for "110". The guide table entry (6,1) corresponds to splitting "110" into "11" and "0". Here 6 is the index of "11" in each CS (read from *lCS*) while 1 is the index of "0" (read from rCS). Line 12 computes if this particular split generates w, and Line 13 computes the disjunction with other possible splits (we fold over all splits, as fast exits are data-dependent branching and problematic on GPUs).

Ideally, we'd like to implement the algorithm only once, and run it on both, CPU and GPU. But we found CUDA programming in a GPU friendly manner leads to unnatural CPU code, that would probably perform badly on a CPU. So we implemented the algorithm twice, once for CPUs and once for Nvidia GPUs. Note that many modern CPUs offer GPU-like features, e.g., Intel's streaming extensions [Wikipedia contributors 2022c] or Arm's Scalable Vector Extension [Stephens et al. 2017]. We explicitly avoided using those, since they are blurring the lines between CPUs and GPUs, making the comparison less informative.

Relationship between CPU and GPU implementation. One purpose of the present work is to understand the speed-ups over CPUs that can be gained by a GPU-friendly implementation of REI. Alas, there was no suitable existing reference implementation, so we had to produce one ourselves.

4 EVALUATION OF ALGORITHM PERFORMANCE

Contemporary ML is an empirical field, and new algorithmic approaches ought to be evaluated on reproducible benchmarks. To keep measurement and contributions focused, our evaluation centres on the speed of our algorithm on a GPU. We'd like to compare our work with existing comparable precise and minimal REI, on widely agreed upon benchmarks. This proved difficult: all existing implementations of REI we consider compromise on precision or minimality (often

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both). Existing benchmarks are unsuitable because they are either much too easy for PARESY or they use large alphabets and long strings that lead to out-of-memory errors in our implementation. (Other approaches that benchmark with large alphabets and long strings compromise on precision, so solve a much easier problem.) In short: there is no comparable CPU implementation, and no suitable benchmark suite. We solve both problems by implementing our algorithm on a CPU and a GPU, and developing suitable benchmarks. All measurements and related artefacts necessary for reproducing our measurement are available from [Valizadeh and Berger 2023]. For brevity, the paper discusses only the most interesting observations.

4.1 Hardware and Software Used for Benchmarking

Benchmarks in Sections 4.3 and 4.3 run on GOOGLE COLAB PRO [Google 2022]. We use Colab Pro because it is a widely used industry standard for running ML workloads. Another reason is that we did not have access to modern GPUs outside the cloud. **Colab CPU parameters:** Intel Xeon ("cpu family 6, model 79"), 2.20 GHz, RAM: 25 GB, running Ubuntu. We use the g++ compiler, version 7.5.0, with the -03 optimisation setting. We use std::unordered_set [C++ Standards Committee 2022] to implement uniqueness checking. From now on we will refer to this as Colab-CPU. **Colab GPU parameters:** Nvidia A100-SXM4-40GB, Driver: Nvidia-SMI460.32.03, RAM: for comparison, we restricted the program's memory usage to the 25 GB available on the Colab-CPU, CUDA Version: 11.2. We use the nvcc Nvidia CUDA compiler driver, with CUDA compilation tools version 11.2.152. We use the WarpCore library Version 1.0.0-alpha.1 [Jünger 2022] to implement uniqueness checking. From now on we will refer to this as Colab-CPU **Darameters:** with a 2.5 GHz Quad-Core Intel Core i7, with 16 MB RAM. We compile CPU C++ code using Apple clang version 11.0.3. We compile ALPHAREGEX using version 4.12.0 of the Ocaml system. We compile with the native-code compiler (ocamlopt). From now on we will refer to this as Laptop-CPU.

4.2 Threats to Validity

Benchmarking is fraught with methodological difficulties that we are intimately aware of, see [Barrett et al. 2017; Dehghani et al. 2021; Gregg 2018; Hooker 1995] for a discussion. A comparison between CPU and GPU is intrinsically apples-to-oranges, and there are numerous ways in which our measurements could be improved.

- It is unclear to what extent GPU, CPU on Google Colab Pro are virtualised. This may affect the reproducibility of measurements.
- For the Colab-CPU we could neither determine the exact version of the processor nor the version of Ubuntu.
- Our benchmarks are largely random strings, and those are likely quite different from strings that we expect to see in practically relevant REI. We conjecture that random strings over an alphabet Σ tend to be more difficult for grammar inference than more structured, human written examples.
- Benchmarking against AlphaReGex compares a C++ with Ocaml, which could be seen as disadvantaging the AlphaReGex. On the other hand, AlphaReGex does not always return a minimal regular expression, so solves a simpler problem.
- We were hampered by a "measurement threshold" of around 0.2 seconds, a minimal time the Colab-GPU would take on *any* task, including toy programs that do nothing at all on the GPU. We believe that this might be GPU latency [Wilper et al. 2020], possibly compounded by the Colab framework. This stood in the way of evaluating the performance of PARESY on small benchmarks.

We believe the speedups we find are unlikely only the effect of measurement bias. We encourage others to replicate our experiments and improve our measurement methodology.

4.3 Benchmark Construction

A good benchmark suite should be tunable by a small number of explainable parameters that allows users to achieve hardness levels, from trivial to beyond the edge-of-infeasibility, and any point in-between. The benchmarks should be suitably random to reduce biasing measurements, yet remain fully reproducible. We are interested in both, space and time complexity, since our algorithm make a space-time trade-off, and quickly solves virtually every problem instance that fits in the available memory. That means we need to be able to fine-tune benchmarks to target memory availability. PARESY's memory usage is governed by the size of $ic(P \cup N)$, which, in turn depends on two related factors: the length of the longest strings in $P \cup N$, and what might be called the heterogeneity of infixes: e.g., $ic({aaa, aa}) = {aaa, aa, a, e}$ is smaller than $ic({abc, de}) = {abc, ab, bc, de, a, b, c, d, e, e}$, despite both being computed from two strings of identical lengths. This suggest a reproducible way of constructing benchmarks with the following natural parameters.

- Alphabet Σ ,
- *le* is the maximal length of example strings,
- -p and n, the numbers of positive and negative examples, respectively.

With those parameters, we define two complementary benchmark generation schemes. Both create instances (P, N) by sampling uniformly from two different spaces of random strings.

- $\text{ Type 1: } \{ (P, N) \in \Sigma^{\leq le} \times \Sigma^{\leq le} \mid \forall w \in P \cup N. \#P = p, \#N = n, P \cap N = \emptyset \}$
- Type 2: {(($P_0, ..., P_{le}$), ($N_0, ..., N_{le}$)) ∈ Y × Y | Σ_i#P_i = p, Σ_i#N_i = n, ∀i.P_i ∩ N_i = ∅}

Here Y is $\mathfrak{P}(\Sigma^0) \times \cdots \times \mathfrak{P}(\Sigma^{le})$, and the benchmark corresponding to $((P_0, ..., P_{le}), (N_0, ..., N_{le}))$ is $(\bigcup_i P_i, \bigcup_i N_i)$. Type 1 and Type 2 have different flavour: since there are exponentially more long strings than short, Type 1 specifications are dominated by long strings. In order also to be able to study the effects of short strings in specifications, Type 2, gives each length the same chance of occurring in positive or negative examples. Hence short strings, like ϵ , are likely to be in most Type 2 specifications. Our main benchmarks, used below, are generated using the following parameter. The alphabet is $\{0, 1\}$. The remaining parameters have been chosen to be as hard as possible on the GPU while avoiding running out of memory. Type 1 benchmarks: *p* and *n* both range over 8 to 12, and *le* over 0 to 7. Type 2 benchmarks: *p* and *n* both range over 7 to 14, and *le* over 0 to 10. While PARESY can deal with arbitrary alphabets (with the expected increase in search space size), we restrict our attention to $\{0, 1\}$ because our main point of comparison, ALPHAREGEX, can only handle binary alphabets.

Measurement (1): impact of cost functions. How big is the impact of cost function on run time? Intuitively, it should be strong, since any specific cost function induces a search order, and the solution to a given synthesis problem can come earlier or later in the search process depending on that order. Figure 1 hones in on, and quantifies the effect of search order on synthesis performance. The measurements were done with the Colab-GPU. We run and time 5160 benchmarks (200 examples from TYPE 1, and 230 from TYPE 2, all with 12 different cost functions), on the Colab-GPU 3 times, and take the average of those three runs for each benchmark. We plot those examples that don't timeout within 5 seconds at least for one of the cost functions). We do *not* run the measurements on a CPU because that would be too time-consuming (order of weeks). Since the algorithm is the same on CPU and GPU, and Table 1 indicates that, indeed, search order affects CPU and GPU similarly, we conjecture that we would see similar effects with CPUs. Here is a summary of observations.



Fig. 1. Plotting 3325 benchmarks with 12 different cost functions on the Colab-GPU. The x-axis are the benchmark names, sorted by increasing duration of each named benchmark using the (1, 1, 1, 1, 1) cost function. Benchmarks that run more than 5 seconds (only 3.62%), or run out of memory, are omitted.

- Measurements cluster on the bottom left of the figure, meaning that slow benchmarks are the exception rather than the rule: 60% of benchmarks run in under 1 second and 73% under 2 seconds. We discuss outliers briefly at the end of this section.
- The (1, 1, 1, 1, 1) cost function shows a clean and steep increase in synthesis time, which we do not see for other cost functions. This is partly a consequence of the fact that the x-axis is benchmarks, ordered by increasing time using (1 1 1 1 1) (details of sorting in case of tie are in [Valizadeh and Berger 2023]).
- The (1, 1, 10, 1, 1) cost function which makes the Kleene-star expensive, is often fast. This might be surprising since the Kleene-star is the one mechanism regular expressions have to 'generalise', to exploit patterns. Our benchmarks use random strings, so it's unlikely that we get many opportunities that make the Kleene-star useful. Therefore cost functions that avoid the cost of the Kleene-star are likely to be faster. It would be interesting to use this cost function on benchmarks with a lot of repetition.
- The (1, 1, 1, 1, 10) cost function that makes union expensive is usually the slowest one. We can infer from long running-times that the algorithm does not run out of memory. This is only possible if most newly generated CSs fail their uniqueness checks.

Measurement (2): CPU vs GPU versions of our algorithm. Table 1 compares PARESY on a CPU and a GPU. Our choice of benchmark parameters resulted in benchmarks that can fit into Colab-GPU, but take on average about 1 hour each on the Colab-CPU. For each pair of (type, cost-function), we chose from the previous benchmark the longest-running benchmark that neither ran out-of-memory nor timed out. We run and time these 24 benchmarks 3 times, and report the average of those three runs for each benchmark. Here is a summary of core observations.

 Our GPU is three orders of magnitude faster than our CPU version of the same algorithm, and the speed-up does not depend on the chosen cost function.

Input					CPU		GPU		
Туре	No	# P	# N	Cost Function	Sec	Sec	Speed-up	# REs	
1	50	10	12	(1, 1, 1, 1, 1)	5080.7850	4.9512	1026x	26,774,099,142	
1	51	12	9	(10, 1, 1, 1, 1)	4699.8137	4.4966	1045x	23,824,118,297	
1	73	10	11	(1, 10, 1, 1, 1)	5805.2168	3.7144	1562x	22,703,639,676	
1	20	9	9	(1, 1, 10, 1, 1)	2893.4835	2.8935	1000x	13,567,472,188	
1	73	10	11	(1, 1, 1, 10, 1)	2901.9297	2.9504	983x	11,706,686,339	
1	31	8	9	(1, 1, 1, 1, 10)	5856.6925	3.9973	1465x	14,210,157,835	
1	57	12	10	(10, 10, 10, 10, 1)	2804.6793	3.4322	817x	14,163,906,090	
1	50	10	12	(10, 10, 10, 1, 10)	4519.9456	4.9096	920x	23,349,552,935	
1	57	12	10	(10, 10, 1, 10, 10)	4301.8548	4.5243	950x	20,257,045,497	
1	97	12	12	(10, 1, 10, 10, 10)	5608.7286	4.7782	1173x	19,680,542,658	
1	61	12	10	(1, 10, 10, 10, 10)	2915.0938	3.0532	954x	14,322,039,866	
1	88	12	9	(20, 20, 20, 5, 30)	6899.0045	4.6904	1470x	25,193,577,825	
2	88	14	8	(1, 1, 1, 1, 1)	3783.9772	4.2462	891x	23,697,549,545	
2	150	14	12	(10, 1, 1, 1, 1)	4228.2773	4.4120	958x	23,125,803,623	
2	158	12	14	(1, 10, 1, 1, 1)	2975.9956	2.4887	1195x	11,432,891,412	
2	136	11	14	(1, 1, 10, 1, 1)	3374.8873	3.6080	935x	18,241,755,827	
2	107	12	12	(1, 1, 1, 10, 1)	2432.4320	4.4120	551x	24,954,272,802	
2	32	10	7	(1, 1, 1, 1, 10)	7400.8135	4.6482	1592x	16,729,795,052	
2	136	11	14	(10, 10, 10, 10, 1)	2907.9182	3.9689	732x	17,476,988,322	
2	200	13	8	(10, 10, 10, 1, 10)	9687.7952	4.5366	2135x	6,037,014,423	
2	107	12	12	(10, 10, 1, 10, 10)	3383.1937	4.5071	750x	20,697,274,025	
2	81	8	14	(10, 1, 10, 10, 10)	3497.9013	4.6699	749x	21,869,903,022	
2	88	14	8	(1, 10, 10, 10, 10)	3405.5536	4.1602	818x	21,889,508,744	
2	158	12	14	(20, 20, 20, 5, 30)	5804.8112	4.9228	1179x	23,163,079,580	
		A	verag	e	4465.4493	4.1238	1077x	19.127.861.447	

Table 1. Comparison of PARESY on hardest examples, using the Colab-CPU and Colab-GPU.

- Almost half of TYPE 1 examples in Table 1, contain ϵ . This is surprising, because TYPE 1 strongly favours long strings. Indeed, we created TYPE 2 only so that we can also run benchmarks with specifications contain short strings, including, in particular ϵ . So ϵ seems to make inference disproportional harder. We conjecture that this is an effect of using regular expressions, rather than an artefact of our algorithm.

Measurement (3): comparison with ALPHAREGEX. We compare our algorithm against ALPHAREGEX, the state-of-the-art REI system. We use the Laptop-CPU because we could not get Ocaml, required for ALPHAREGEX, to run on the Colab-CPU. We do not run our own benchmarks because, after informal experiments, we felt ALPHAREGEX would take too long. Moreover, most of our benchmarks contain ϵ , which ALPHAREGEX does not handle. Instead we use the benchmark from [Lee et al. 2016a, 2017] (slightly adapted): because ALPHAREGEX solves those quickly. We run these benchmarks only use the CPU version of PARESY since almost all are solved on the Colab-GPU by PARESY below the measurement threshold. We adapt some benchmarks because ALPHAREGEX uses the "wild card" heuristic, which PARESY does not support: we replace 'X' by 0 + 1, the meaning intended by ALPHAREGEX. In order to mitigate the methodological problems arising from comparing C++ and Ocaml, we report the regular expressions checked by both for compliance with the ambient specification, as those depend only on the algorithm. Here is a summary of observations.

Table 2. Running PARESY and ALPHARECEX on the laptop-CPU. Benchmarks labelled with \dagger replace the "wild card" x with the intended 0 + 1, since PARESY does not support their "wild card" heuristic. Cost **bold and underlined** is not minimal. On the Colab-GPU, PARESY does not accept no6 and no9, because they require 128 and 256 bits for their respective ic($P \cup N$), which WarpCore does not currently support; no9 needs > 256 bits which even our CPU version does not currently support. All but three examples that we can run on the Colab-GPU, finish in approx. 0.2 seconds, i.e., below the measurement threshold.

	Runi	ning Time	s (sec)	Co	st(RE)	# REs		
No	αR	Paresy	Speed-up	αR	Paresy	αR	Paresy	Increase
no1 [†]	0.0092	0.0054	2 x	85	85	27	77	2.85x
no2 [†]	0.0402	0.0042	10x	<u>155</u>	110	704	470	0.67x
no3 [†]	52.6748	24.1921	$2\mathbf{x}$	280	280	282,815	32,329,412	114.31x
no4 [†]	0.0277	0.0051	5x	155	140	520	3,821	7.35x
no5 [†]	50.5782	3.144	16x	265	265	292,115	9,960,260	34.10x
no6 [†]	231.4295	4.1491	56x	240	240	659,702	4,731,056	7.17x
no7	7.211	0.5716	13x	230	225	92,702	581,659	6.27x
no8	0.0673	0.0096	7 x	160	160	1,318	7,776	5.90x
no9†	>20000	N/A	N/A	N/A	N/A	N/A	N/A	N/A
no10	1.4267	0.0057	250x	155	155	21,457	6,772	0.32x
no11	0.01	0.0041	$2\mathbf{x}$	85	85	92	77	0.84x
no12	0.1614	0.0137	12x	185	185	3,721	23,675	6.36x
no13	1.105	0.1961	6x	220	220	17,957	483,161	26.91x
no14 [†]	>20000	116.5046	N/A	N/A	310	N/A	261,293,189	N/A
no15 [†]	25.9066	0.2952	88x	240	240	187,484	1,721,174	9.18x
no16 [†]	3.8588	0.0594	65x	205	205	32,039	225,377	7.03x
no17	2.1345	0.326	7 x	230	225	31,476	659,386	20.95x
no18	0.1004	0.0074	14x	155	155	1,710	8,010	4.68x
no19 [†]	0.0207	0.0057	4x	130	130	164	1,867	11.38x
no20 [†]	25.6899	0.0192	1338x	160	160	97,510	21,135	0.22x
no21	0.8064	0.1993	$4\mathbf{x}$	225	225	18,887	481,762	25.51x
no22 [†]	68.6433	1.1519	60x	265	265	495,783	6,202,349	12.51x
no23	1.5421	0.028	55x	<u>210</u>	180	22,411	32,068	1.43x
no24	10.7292	0.0491	219x	200	200	127,893	109,433	0.86x
no25	19.182	3.7642	5x	265	240	260,104	3,205,741	12.32x

- ALPHAREGEX does not always return minimal-cost regular expressions in nearly 25% of their own benchmarks. This is surprising, given the abstract of [Lee et al. 2016a]. We believed this is a direct consequence of their heuristics¹.
- ALPHAREGEX's pruning heuristics often work well, and can sometimes decrease the number of regular expressions checked by an order or two of magnitude. Surprisingly, and despite their pruning heuristics, in about 20% of benchmarks ALPHAREGEX checks more regular expressions than PARESY. In all benchmarks, PARESY is faster despite generating and checking many more regular expressions.

¹In passing we note that sometimes the lower-cost regular expression we synthesise does not meet the corresponding English language description in [Lee et al. 2016a]. For example, in benchmark "no25", the description in English is "at most one pair of consecutive 1s". We synthesise $0 + ((1 + 00)(0 + 1))^*$, which is lower cost than the solution ALPHAREGEX finds, and meets their all positive and negative examples, but accepts strings like 1111.

- ALPHAREGEX is always slower than the CPU version of PARESY, in an extreme case by more than three orders of magnitude.
- The benchmark running out-of-memory with PARESY ("no9") can be executed by ALPHAREGEX, albeit not within 20000 sec².

A note on outliers. We have repeatedly stated that PARESY solves virtually all benchmarks (that fit into memory at all) in a few seconds. However, there is a small number of outliers that take much longer. The table below quantifies outliers w.r.t. the full benchmark suite.

Duration (sec)	<2	<3	<4	<5	<10	<25	<50	<100	<200	<400	<800
% of benchmarks	89.48	94.06	95.71	96.38	98.14	98.84	99.28	99.59	99.83	99.91	100.00

Performance evaluation. PARESY is exponential in (asymptotic) space and time complexity. PARESY terminates no later than with the maximally overfitted regular expression $w_1 + ... + w_i$ for (P, N), assuming $P = \{w_1, ..., w_i\}$, see (2) from the introduction. Let X be the number of regular expressions with cost not exceeding that of $w_1 + ... + w_i$, then X is an upper bound on the number of generated CSs. Regarding space complexity, we store only unique CSs. Let Y be the number of unique CSs in the language cache, so $Y \leq X$. Each CS uses approx. k bits where k is size of $ic(P \cup N)$. We use additional memory for uniqueness checking and reconstructing concrete regular expressions from CSs: overall approx. $3 \cdot k$ bits for each CS. This bound works regardless of alphabet size. Hence the worst-case size of the language cache is $3 \cdot k \cdot Y \leq 3 \cdot k \cdot X$ bits. Getting tighter average and worst-case bounds on the number of unique CSs for a given specification is an interesting open problem. We leave a detailed investigation of the performance overheads of cache misses, data-depended branching and hardware synchronisation overhead (e.g., insertion of unique CSs into the shared language cache) as future work.

Summary of evaluation. We believe, that, despite the difficulties with measurement methodology we noted, the speed-ups we are seeing from running PARESY, especially on GPUs, are not just an effect of measurement bias. We believe that the main reason for the performance improvement we are seeing, is that our algorithm is GPU-friendly.

5 CONCLUSION

The present work combines three main themes: ML, GPU programming and algorithms & data structures. Each is vast and we could not possibly do justice to those fields here. Instead, we highlight some key works that have influenced our thinking.

5.1 Related Work

Program synthesis techniques. FlashFill [Gulwani 2011] reinvigorated program synthesis a decade ago. While FlashFill does not do REI but synthesises string transformers, there is conceptual overlap with PARESY, in that both return minimal solutions w.r.t. a cost function $(size(\cdot) in FlashFill)$, and that both make crucial use of infixes. The main differences are: (i) FlashFill represents infixes of strings with a syntactic construct e.g., SubStr(s, 2, 5), resp. a DAG, and explicitly represents startand end-positions of infixes as numbers. In contrast, infixes are implicitly represented as bits in our CS and accessed by position. (ii) PARESY runs on GPUs, while, to the best of our knowledge, FlashFill is implemented only for CPUs. We don't believe the FlashFill algorithm can be implemented efficiently on GPUs without substantial re-engineering. (iii) Unlike our use of semirings which

²AlphaRegex solves it quickly using the "wild card" heuristic.

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immediately also generalise to context-free, and, indeed, all formal languages, FlashFill's data structures are not related to abstract mathematics, making it difficult to see how they generalise. (iv) FlashFill's cost function is not configurable. (v) FlashFill is used as an incremental synthesis tool in Microsoft Excel. PARESY is currently not incremental.

Our CSs are a variation on the theme of *observational equivalences*, a standard technique to mitigate the cost imposed on synthesis by the redundancies of syntax: intuitively, programs are equivalent if they have the same behaviour in all contexts, e.g., r^* and $\epsilon + r^*r$. An ideal synthesis mechanism searches over canonical representatives of programs quotiented by this equivalence, alas observational equivalence is not computable in general. When synthesising programs from examples, it is natural to consider programs equivalent if they relate to the examples in the same manner, FlashFill [Gulwani 2011] and TRANSIT [Udupa et al. 2013] do this. PARESY does something subtly different: we do not identify regular expressions w.r.t. to words in the examples, but instead over-approximate and identify them if they have the same CS over the infix-closure of the examples. This over-approximates: this is a heuristic for quickly discarding obviously unsuitable candidates, e.g., the "fingerprints" in superoptimisation [Bansal and Aiken 2006]. Similar techniques are used in e.g., equality saturation [Nandi et al. 2021].

Regular expression synthesis from examples. ALPHAREGEX [Lee et al. 2016a,b] works from positive and negative examples, and a configurable (albeit only by editing and recompiling source code) cost homomorphism. ALPHAREGEX uses top-down, exhaustive search over regular expressions extended with a concept of 'hole'. The clean mathematical semantics of extended regular expressions enables elegant pruning heuristics. All synthesised regular expressions are precise. [Lee et al. 2016a] claims that synthesised regular expressions are minimal ("the method automatically synthesizes the simplest possible regular expression that ..."), but we found several counterexamples (see Table 2). ALPHAREGEX has two main restrictions: binary alphabet only, and examples must not contain the empty string (PARESY has neither restriction). FLASHREGEX [Li et al. 2020] presents an interesting twist on regular expression synthesis: regular expressions are often used in security sensitive applications, and, if chosen naively, can enable denial-of-service attacks, called ReDOS [Wikipedia contributors 2022b]. FLASHREGEX optimises for generating regular expressions from positive and negative examples that are not susceptible to ReDOS. Since this work is optimising in a different direction from ours, the two approaches are not directly comparable. However, an interesting research problem is to investigate if PARESY can be extended so it guarantees lack of ReDOS vulnerability in addition to, or instead of minimality.

A widespread use of regular expressions is for information extraction: extract(r, w) returns all substrings w' of w such that $w' \in Lang(r)$. Examples include extracting URLs from a web-page. Much research has been done on inference of regular expressions for information extraction, often in the context of XML, or linguistics. We mention only [Bartoli et al. 2014, 2016; Bex et al. 2010; Li et al. 2008], where the reader can find more references. The learning problem in those papers is the same as ours: from given positive and negative examples, construct a suitable regular expression. The resulting regular expression is not guaranteed to be minimal and it also not, in general, precise. The papers use genetic programming as search mechanism.

Often input to REI changes only gradually and incrementalising this problem is an appealing proposition. Several papers consider this problem. We mention two: [Pan et al. 2019] design a heuristic algorithm called RFIXER that repairs a regular expression with given positive and negative examples. When the regular expression is incorrect on the examples, RFixer automatically synthesises the syntactically smallest repair of the original regular expression that is correct on the given examples. This can be seen as a incremental regular expression synthesis. [Wang et al.

2016] considers the widespread use of regular expressions for data filtering, for example to 'zoom in' on relevant data in a spreadsheet. This can be done with regular expressions: "show me only the strings that start with something matching regular expression r". Since construction of suitable regular expressions is considered hard, it should be learned from examples. Like us [Wang et al. 2016] consider positive and negative examples, but they consider only the star-free subset of regular expressions, making the search space much easier. We can already search in the star-free fragment, by setting cost(*) high enough, however, our algorithm is not incremental. In either case, incrementality typically requires sophisticated support data structures to work well. We leave the question of incrementalising our algorithm as important future work.

We are reluctant to compare with deep-learning based regular expression synthesis, because existing implementations like [Li et al. 2021; Locascio et al. 2016; Park et al. 2019; Zhong et al. 2018] all work from natural language specification and make neither minimality nor precision guarantees. Indeed, given the intrinsic ambiguity of natural language, what would such guarantees even mean? In order to fill this notable lacuna, the present authors are preparing a comparison between PARESY and REI on generative pre-trained transformers [Vaswani et al. 2017].

Acceleration of regular expression contains-checking. Regular expressions are widely used, and performance-critical for many applications. So it is not surprising that there is work on accelerating regular expressions with GPUs or even dedicated hardware. It is crucial to understand that this existing work is accelerating a subtly different problem, which we call the *contains-check* for regular expressions (also known under different terms, including, but not limited to matching, pattern matching, evaluation, language containment). The table below summarises the difference.

	REI	Contains-check
Input	Sets of strings	String
Output	Regular expression	True/False

It is not clear that accelerating REI is automatically also advantageous for contains-checking, or vice-versa. Be that as it may, we briefly survey existing work on the acceleration of regular expression contains-checking.

Much has been written about implementing fast regular expression contains-checking on CPUs. We mention only [Qiu et al. 2021], which presents two ideas that might be interesting on GPUs, too: the problem with contains-checks, from the point of view of parallelism, is that the contains-check appears to be sequential. In order to parallelise contains-checking of string w, we could break w into parts, e.g., to check if r_1r_2 contains w, we could break w into $w = w_1w_2$ and then check in parallel if r_1 contains w_1 and r_2 contains w_2 . The problem is that's it's not easy to know where to split w. [Qiu et al. 2021] suggests doing this speculatively, and perform a 'rollback' in case of miss-speculation. Contains-check acceleration on GPUs is a relatively new field, not surprising given that GPUs are relatively recent and hard to program. The first work to do so was [Cascarano et al. 2010]. The paper exploits the parallelism GPUs offer through a non-deterministic automata representation of regular languages. Later, [Liu et al. 2020; Zu et al. 2012] improves [Cascarano et al. 2010] with more carefully designed data structures.

5.2 Future Work

It is natural to consider generalisation of the PARESY algorithm to other classes of grammars, whether more expressive (e.g., context-free or context-sensitive), or less (e.g., restricted star-height). Many of our core algorithmic choices, in particular the choice of bitvectors to represent languages, and the use of uniqueness as generic pruning technique, work for any grammar.

It is also natural to ask if PARESY can be made less memory intensive by compromising on minimality or precision. In other words, can local search benefit from ideas for GPU acceleration of global search? In closing, we sketch how to implement *REI with error*, a simple local search technique that requires changing only a few lines of code. Intuitively it is clear that REI becomes easier if we drop the requirement that the result be precise. Let's introduce an *allowed error* parameter, a number between 0 and 1, that quantifies the *allowed error*. Solving (P, N) then requires finding r with $r \models (P', N')$ such that $P' \subseteq P$ and $N' \subseteq N$, and the allowed error is an upper bound on the fraction of $P \cup N$ that is not in $P' \cup N'$. Consider the specification $P = \{00, 1101, 0001, 0111, 001, 1, 10, 1100, 111, 1010\}$ and $N = \{\epsilon, 0, 0000, 0011, 010, 011, 100, 1000, 1001, 111, 1110\}$, the top row of Table 1. The table below shows the dependency of synthesis cost (quantified by number of regular expressions checked for compliance with the specification) on allowed error. We used the (1, 1, 1, 1, 1) cost function.

Allowed Error	# REs	RE	Cost(RE)
0 %	26,774,099,142	10?+0?(00+10*10?(0+1))1?	28
5 %	319,649,322	$((0+1)0+(0+11)^*1)(100?)?$	22
10 %	18,698,767	$(10+0^*1)(10?(0+1))?$	18
15 %	794,598	(0+1)0+(0+11)*1	14
20 %	116,912	$(0+11)^*(1+00)$	12
25 %	2,073	(0+11)*1	8
30 %	2,073	(0+11)*1	8
35 %	1,124	1+(0+1)0	7
40 %	50	10?	4
45 %	3	1	1
50 %	1	Ø	1

What we see might be an exponential dependency between allowed error and synthesis cost.

Finally, we challenge the deep learning community to solve minimal and precise REI with ANNs, and then compare speed, memory usage and power consumption with PARESY.

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CODE AND DATA AVAILABILITY

Our evaluated artefacts are not available in immutable archival format. All benchmarks, source code and data are available from [Valizadeh and Berger 2023].

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