Compilers and computer architecture From strings to ASTs (2): context free grammars

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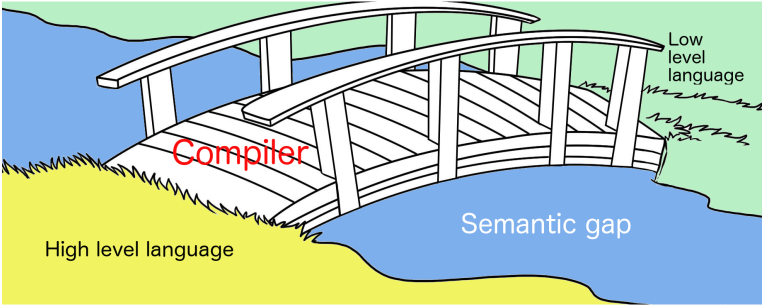
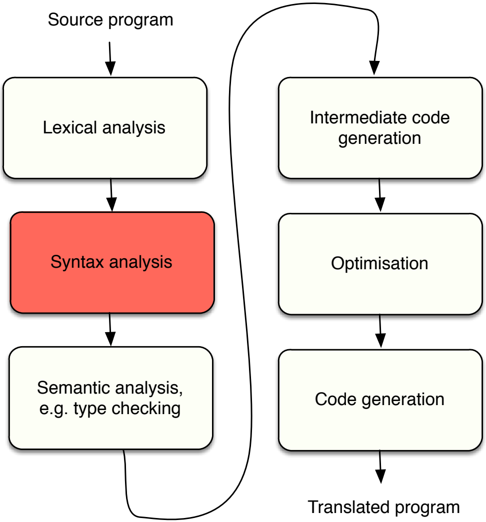
Recall the function of compilers



Recall we are discussing parsing



Introduction

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Remember, we want to take a program given as a string and:

- Check if it's syntactically correct, e.g. is every opened bracket later closed?
- Produce an AST to facilitate **efficient** code generation.

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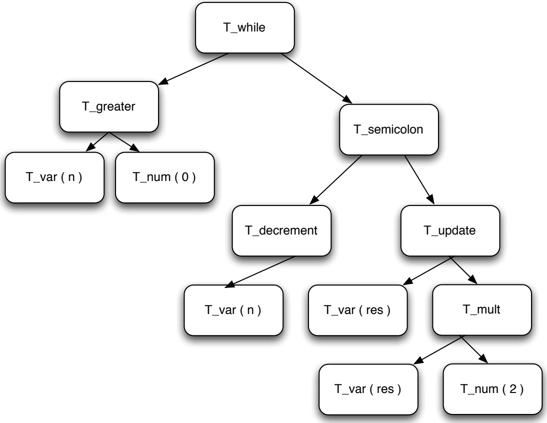
```
while( n > 0 ){
    n--;
    res *= 2; }
```



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We split that task into two phases, lexing and parsing. Lexing throws away some information (e.g. how many white-spaces) and prepares a token-list, which is used by the parser. The token-list simplifies the parser, because some detail is not important for syntactic correctness:

$$\texttt{if}\ x < 2 + 3\ \texttt{then}\ P\ \texttt{else}\ Q$$

is syntactically correct exactly when

$$\texttt{if}\ y < 111 + 222\ \texttt{then}\ P\ \texttt{else}\ Q$$

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So from the point of view of the next stage (parsing), all we need to know is that the input is

```
T_if T_var T_less T_int T_plus T_int T_then ...
```

Of course we cannot throw away the names of variables etc completely, as the later stages (type-checking and code generation) need them. They are just irrelevant for syntax checking. We keep them and our token-lists are like this

```
T_if T_var ( "x" ) T_less T_int ( 2 ) T_plus ...
```

Two tasks of syntax analysis

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Alphabet = $\{'(', ')'\}$.

Language = all **balanced** parentheses $()$, $()()$, $(())$, $((()(()())()(())))$, ..., note: the empty string is balanced.

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Summary: **FSAs can't count**, and likewise for REs (why?).

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Because programming languages contain many bracket-like constructs that can be nested, e.g.

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do ... while
if ( ... ) then { ... } else { ... }
3 + ( 3 - (x + 6) )
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But we must formalise the syntax of our language if we want to computer to process it. So we need a formalism that can 'count'.

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CFGs are a generalisation of regular expression that is ideal for describing such recursive and nested structures.

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A **context-free grammar** is a tuple $(A, V, Init, R)$ where

- A is a finite set called **alphabet**.
- V is a finite, non-empty set of **variables**.
- $A \cap V = \emptyset$.
- $Init \in V$ is the **initial variable**.
- R is the finite set of **reductions**, where each reduction in R is of the form (l, r) such that
  - l is a variable, i.e. $l \in V$.
  - r is a string (possibly empty) over the **new** alphabet $A \cup V$.

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Note that the alphabet are often also called **terminal symbols**, reductions are also called **reduction steps** or **transitions** or **productions**, some people say **non-terminal symbol** for variable, and the initial variable is also called **start symbol**.

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Context free grammar

Example:

- $A = \{a, b\}$.
- $V = \{S\}$.
- The initial variable is S.
- R contains only three reductions:

$$S \to a\ S\ b$$
$$S \to S\ S$$
$$S \to \epsilon$$

Recall that ϵ is the empty string.

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To make this intuition precise, we need to say precisely what the language of a CFG is.

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How do we start this rewriting of variables? With the initial variable.

When does this rewriting of variables stop? When the string we arrive at by rewriting in a finite number of steps from the initial variable contains no more variables.

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Then: the **language** of a CFG is the set of all strings over the alphabet of the CFG that can be arrived at by rewriting from the initial variable.

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Let's do this with the CFG for balanced brackets (A, V, S, R) where

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$$
\begin{aligned}
S &\to (S) \\
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&\to ((S)S) \\
&\to (((S))S) \\
&\to (((S))SS) \\
&\to (((S))\epsilon S) = (((S))S) \\
&\to (((\epsilon))S) = ((())S) \\
&\to ((())\epsilon) = ((()))
\end{aligned}
$$

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Because only $S \rightarrow (\ S \ )$ introduces new brackets. But by construction it always introduces a closing bracket for each new open bracket.

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Such infinite reductions don't affect the language of the grammar. Only sequences of rewrites that end in a string free from variables count towards the language.

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If you like formal definitions ...

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Given a fixed CFG $\mathcal{G} = (A, V, S, R)$. For arbitrary strings $\sigma, \sigma' \in (V \cup A)^*$ we define the *one-step reduction relation* $\Rightarrow$ which relates strings from $(V \cup A)^*$ as follows.
$\sigma \Rightarrow \sigma'$ if and only if:

- $\sigma = \sigma_1 l \sigma_2$ where $l \in V$, and σ_1, σ_2 are strings from $(V \cup A)^*$.
- There is a reduction $l \longrightarrow \gamma$ in R.
- $\sigma' = \sigma_1 \gamma \sigma_2$.

The *language accepted by* $\mathcal{G}$, written $\text{lang}(\mathcal{G})$ is given as follows.

$$\text{lang}(\mathcal{G}) \stackrel{def}{=} \{\gamma_n \mid S \to \gamma_1 \to \cdots \to \gamma_n, \text{ where } \gamma_n \in A^*\}$$

The sequence $S \to \gamma_1 \to \cdots \to \gamma_n$ is called **derivation.**

Note: only strings free from variables can be in $\text{lang}(\mathcal{G})$.

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Consider the following CFG where `while`, `if`, `;` etc are elements of the alphabet, and M is a variable.

$$
\begin{array}{lcl}
M & \rightarrow & \texttt{while}\ M\ \texttt{do}\ M \\
M & \rightarrow & \texttt{if}\ M\ \texttt{then}\ M \\
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We do this until we reach a string without variables.

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Variables are written in ⟨angle-brackets⟩, the alphabet isn't.

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Instead of multiple reductions from the same variable, like

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we write

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Finally, many write $::=$ instead of $\to$.

Simple arithmetic expressions

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- $7 * 4 + 222$

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- $7 * 4 + 222$
- $7 * (4 + 222)$ ...

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What's this?

(The syntax of) regular expressions can be described by a CFG (but not by an RE)!

REs vs CFGs

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In principle yes, but lexing based on REs (and FSAs) is simpler and faster!

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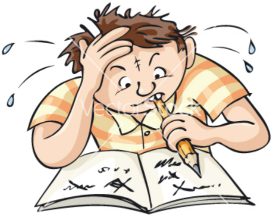

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CFGs allow us to specify the grammar for programming languages. But that's not all we want. We also want:

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The key idea to solving both problems in one go is the **parse tree**.

CFG vs AST

CFG vs AST

Here is a grammar that you were asked to write an AST for in the tutorials.

```
P   ::=   x := e | if0 e then P else P
     |    whileGt0 e do P | P; P

e   ::=   e + e | e - e | e * e | (e) | e % e
     |     x | 0 | 1 | 2 | ...
```

CFG vs AST

Here's a plausible definition of ASTs for the language: Syntax.java

CFG vs AST

Do you notice something?

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Looks very similar. The CFG is (almost?) a description of data type for the AST.

This is no coincidence, and we will use this similarity to construct the AST as we parse, in that we will see the parsing process as a tree. How?

Derivations and parse trees

Derivations and parse trees

Recall that a derivation in a CFG (A, V, I, R) is a sequence

$$I \to t_1 \to t_2 \to \cdots \to t_n$$

where t_n is free from variables, and each step is goverend by a reduction from R.

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I.e. nodes in the tree are elements of $A \cup V$. Let's look at an example.

Example parse tree

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Recall our CFG for arithmetic expressions: Let's do another example. Grammar:

$$E \quad \rightarrow \quad E + E \mid E * E \mid (E) \mid 0 \mid 1 \mid \ldots$$

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$$E \quad \rightarrow \quad E + E \mid E * E \mid (E) \mid 0 \mid 1 \mid \ldots$$

Let's say we have the string "4 * 3 + 17". Let's parse this string and build the corresponding parse tree.

Example parse tree

E

Example parse tree

$$E \quad \rightarrow \quad E + E$$

Example parse tree

$$
\begin{aligned}
E &\rightarrow E + E \\
  &\rightarrow E * E + E
\end{aligned}
$$

Example parse tree

$$
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E &\rightarrow E + E \\
&\rightarrow E * E + E \\
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Example parse tree

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\end{aligned}
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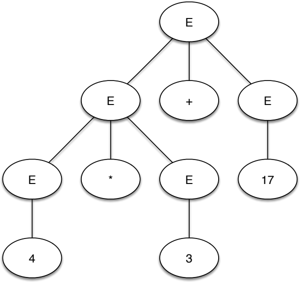

Example parse tree

$$
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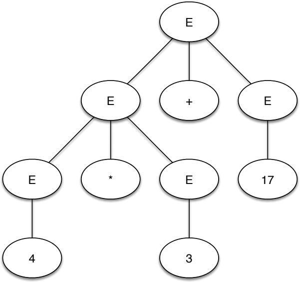

Let's do this in detail on the board.

Derivations and parse trees

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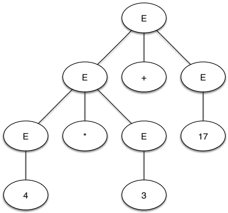
The following is important about parse trees.

- Terminal symbols are at the leaves of the tree.
- Variables symbols are at the non-leave nodes.
- An **in-order** traversal of the tree returns the input string.
- The parse tree reveals bracketing structure explicitly.

Left- vs rightmost derivation

Left- vs rightmost derivation

BUT ... usually there are many derivations for a chosen string, giving the same parse tree. For example:



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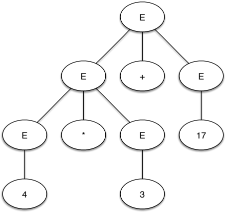

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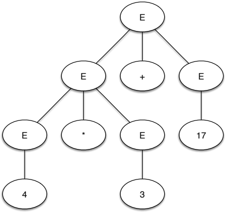

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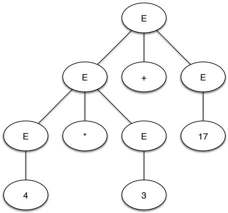

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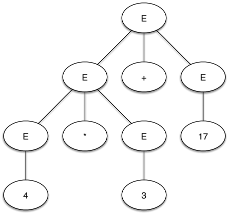

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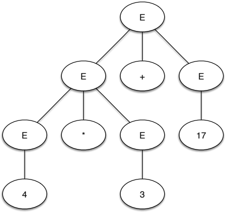

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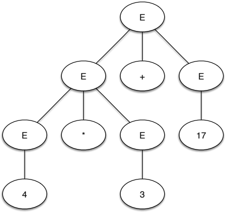

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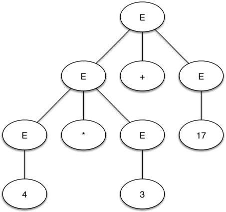

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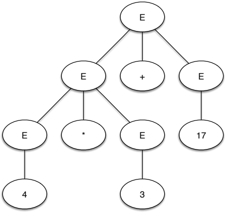

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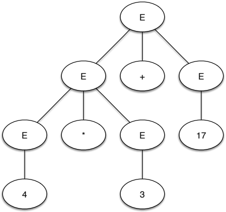

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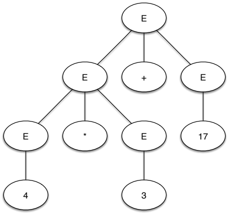

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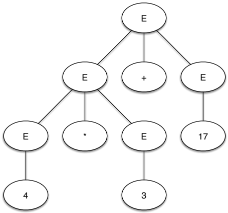

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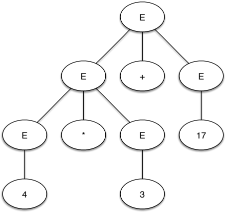

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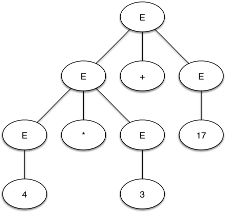

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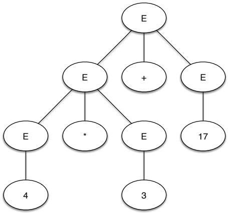

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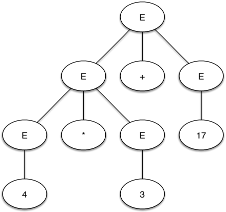

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In parsing we usually use either left- or rightmost derivations to construct a parse tree.

Left- vs rightmost derivation

Question: do left- and rightmost derivations lead to the same parse tree?

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Answer: For a context-free grammar: YES. It really doesn't matter in what order variables are rewritten. Why?

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Question: do left- and rightmost derivations lead to the same parse tree?

Answer: For a context-free grammar: YES. It really doesn't matter in what order variables are rewritten. Why? Because the rest of the string is unaffected by rewriting a variable, so we can modify the order.

Which rule to choose?

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Alas there is a second degree of freedom:

Which rule to choose?

Alas there is a second degree of freedom: which rule to choose?

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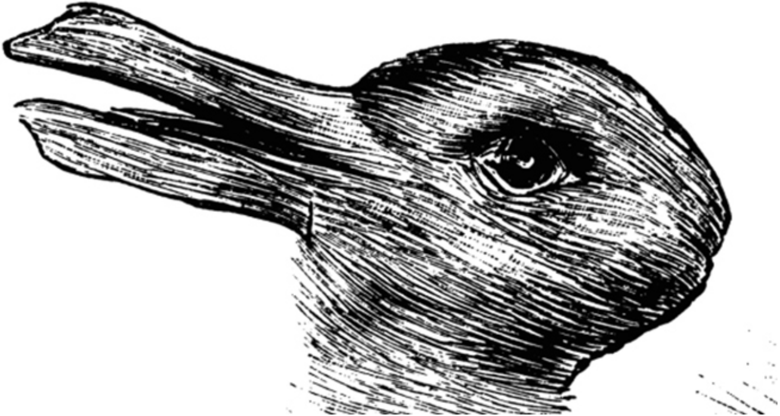
In constrast: it can make a big difference which **rule** we apply when rewriting a variable

This leads to an important subject: **ambiguity.**

Ambiguity



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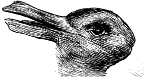

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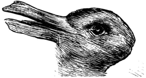

In the grammar

$$E \quad \rightarrow \quad E + E \mid E * E \mid (E) \mid 0 \mid 1 \mid ...$$

the string 4 * 3 + 17 has **two** distinct parse trees!

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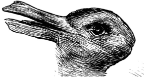

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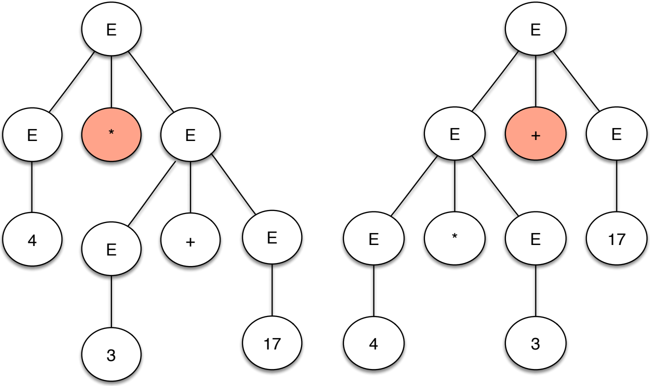

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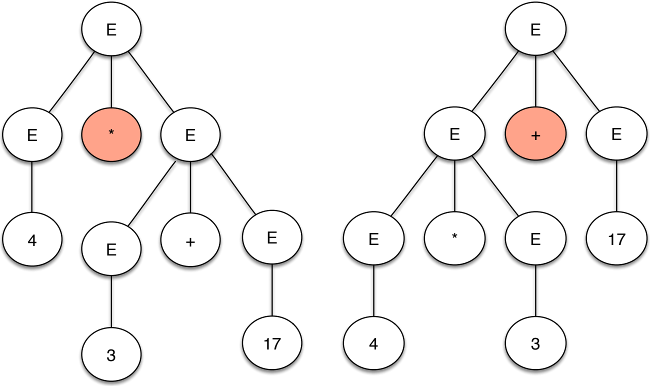
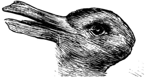


A CFG with this property is called **ambiguous**.

Ambiguity



More precisely: a context-free grammar is **ambiguous** if there is a string in the language of the grammar that has more than one parse tree.

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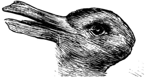

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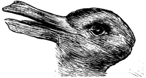
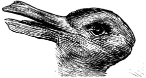
Note that this has **nothing** to do with left- vs right-derivation. Each of the ambiguous parse trees has a left- and a right-derivation.

Ambiguity



We also have ambiguity in natural language, e.g.

Ambiguity



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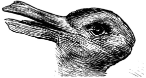

Ambiguity is programming languages is bad, because it leaves the meaning of a program unclear, e.g. the compiler should generate different code for $1 + 2 * 3$ when it's uderstood as $(1 + 2) * 3$ than for $1 + (2 * 3)$.

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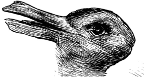

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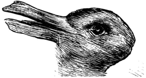
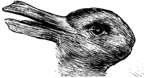


Ambiguity is programming languages is bad, because it leaves the meaning of a program unclear, e.g. the compiler should generate different code for $1 + 2 * 3$ when it's uderstood as $(1 + 2) * 3$ than for $1 + (2 * 3)$.

Can we automatically check whether a grammar is ambigouous?

Bad news: ambiguity of grammars is undecidable, i.e. no algorithm can exist that takes as input a CFG and returns "Ambiguous" or "Not ambiguous" correctly for all CFGs.

Ambiguity



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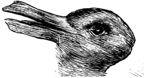

There are several ways to deal with ambiguity.

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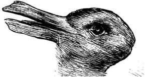

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- **Parser returns all possible parse trees**, leaving choice to later compiler phases. Example: combinator parsers often do this, Earley parser. Downside: kicks can down the road ... need to disambiguate later (i.e. doesn't really solve the problem), and does too much work if some of the parse trees are later discarded.

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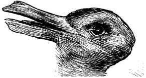

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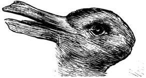

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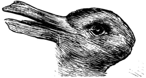
- **Use non-ambiguous grammar**. Easier said than done ...
- **Rewriting the grammar to remove ambiguity**. For example by enforcing precedence that * binds more tightly than +. We look at this now.

Ambiguity: grammar rewriting



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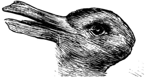

The problem with

$$E \quad \to \quad E + E \mid E * E \mid (E) \mid 0 \mid 1 \mid \ldots$$

is that addition and multiplication have the same status. But in our everyday understanding, we think of $a * b + c$ as meaning $(a * b) + c$. Moreover, we evaluate $a + b + c$ as $(a + b) + c$. But there's nothing in the naive grammar that ensures this.

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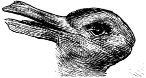

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Let's bake these preferences into the grammar.

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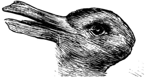

Let's rewrite

$$E \rightarrow E + E \mid E * E \mid (E) \mid 0 \mid 1 \mid \ldots$$

to

$$\begin{aligned} E &\rightarrow F + E \mid F \\ F &\rightarrow N * F \mid N \mid (E) * F \mid (E) \\ N &\rightarrow 0 \mid 1 \mid \ldots \end{aligned}$$

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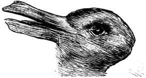

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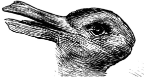
Examples in class.

If-Then ambiguity



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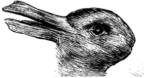
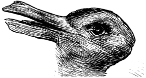


Here is a problem that often arises when specifying programming languages.

$$
\begin{array}{lcl}
M & \rightarrow & \texttt{if}\ M\ \texttt{then}\ M \\
  & \mid & \texttt{if}\ M\ \texttt{then}\ M\ \texttt{else}\ M \\
  & \mid & \ldots
\end{array}
$$

If-Then ambiguity



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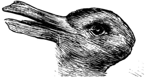

Now we can find two distinct parse trees for

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if B then if B′ then P else Q
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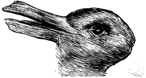

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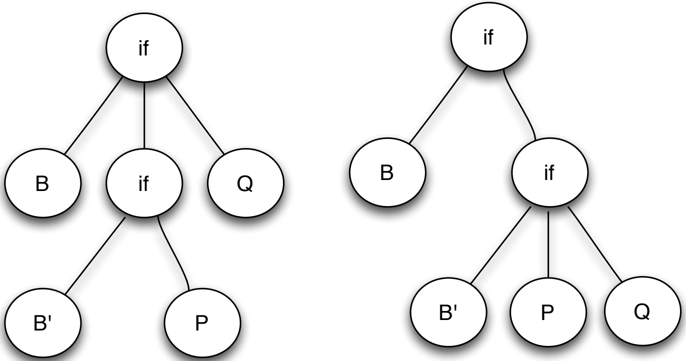

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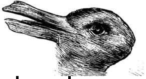

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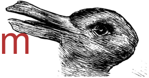
Let's do this for the if-then ambiguity by saying:

`else` always closes the nearest **unclosed** `if`, so `if-then-else` has priority over `if-then`.

M	$\rightarrow$	ITE	only if-then-else
	$\mid$	$BOTH$	both if-then and if-then-else
	$\mid$	...	
ITE	$\rightarrow$	`if` M `then` ITE `else` ITE	
	$\mid$	...	other reductions
$BOTH$	$\rightarrow$	`if` M `then` M	
	$\mid$	`if` M `then` ITE `else` $BOTH$	no other reductions

If-Then ambiguity, aka the dangling-else problem



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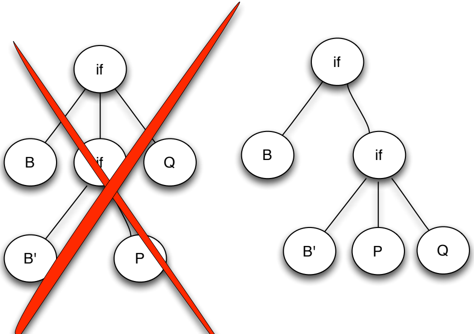
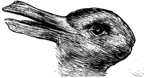


Ambiguity: general algorithm?



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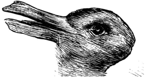

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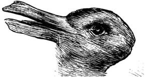

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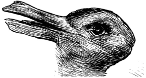

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Fortunately, such languages are esoteric and not relevant for programming languages. For languages relevant in programming, it is generally straightforward to produce an unambiguous CFG.

I will not ask you in the exam to convert an ambiguous CFG into an unambiguous CFG. You should just know what ambiguity means in parsing and why it is a problem.