Compilers and computer architecture: From strings to ASTs (1): finite state automata for lexing

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October 2019

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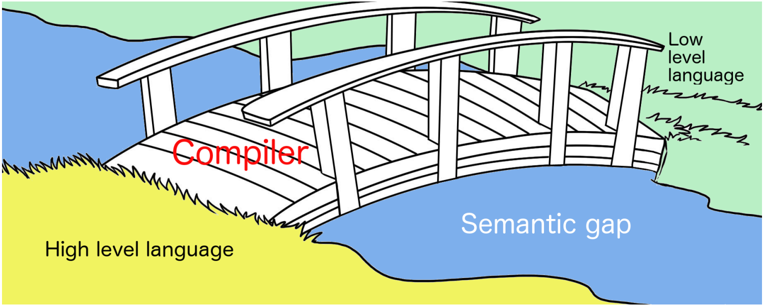
Recall the function of compilers



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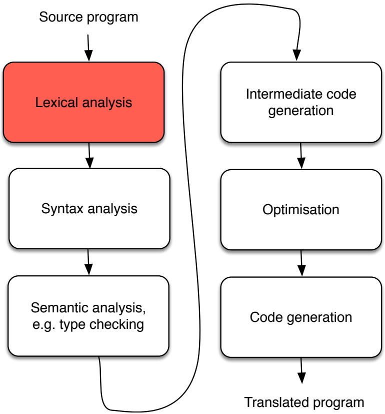

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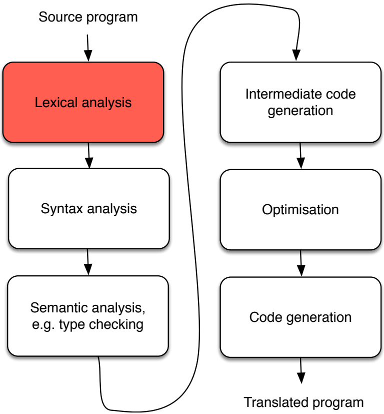

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We learned about regular expressions (REs). They enable us to specify simple language (finite and infinite).

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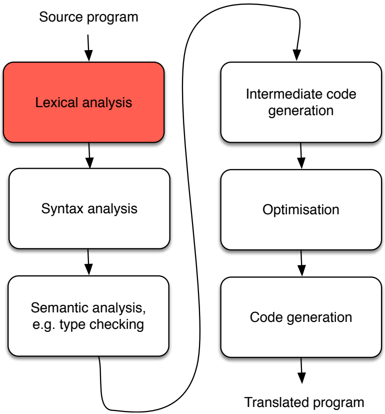

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We learned about regular expressions (REs). They enable us to specify simple language (finite and infinite).

The question we need to answer is: how to **decide**, given a string s and a regular expression R, if $s \in lang(R)$?

We will later see that this is the main step towards an **algorithm** for lexing (tokenisation).

Finite state automata

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FSAs are easiest explained in pictures. Here is one with the alphabet $\{0, 1\}$

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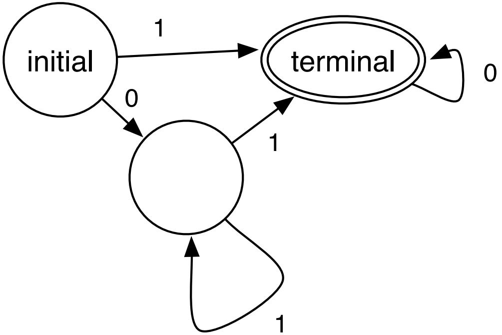

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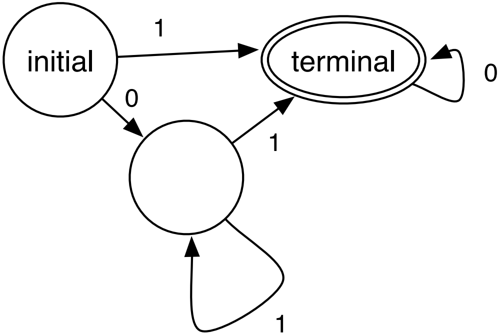

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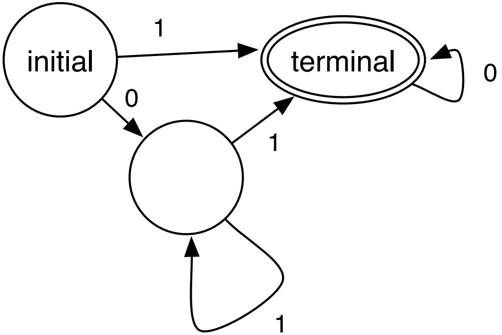

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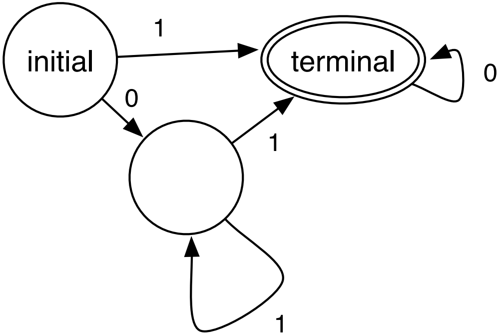

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If we cannot find a path that terminates at the end of the input, and the automaton is NOT in an accepting state, the input string as a whole is rejected and is NOT in the language of the automaton.

FSA, formal definition

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A **finite state automaton** (FSA) is a tuple $\mathcal{A} = (A, S, i, F, R)$ such that the following is true.

- A is a finite set, called the **alphabet** of the automaton.
- S is a non-empty finite set of **states**.
- $i \in S$ is the **initial state**.
- $F \subseteq S$ is the set of **terminal**, or **accepting states** of the automaton. **Note:** F can be empty. (What happens then?)
- R is the **transition relation**, i.e. it is a relation on states, characters and states. More formally, R is a subset of $S \times A \times S$. We often write $s \xrightarrow{\alpha} t$ instead of $(s, \alpha, t) \in R$

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We say $\mathcal{A}$ is **deterministic** if whenever $s \xrightarrow{\alpha} t$ and $s \xrightarrow{\alpha} t'$ then $t = t'$. Otherwise $\mathcal{A}$ is **non-deterministic**.

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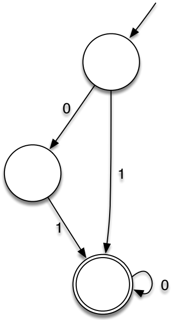

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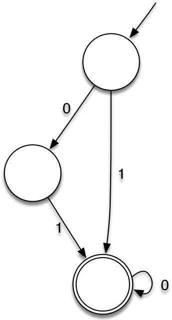

The finite state automaton on the left is deterministic, that on the right non-deterministic. Each has one accepting state, indicated by double circles. Initial states are often drawn with an incoming arrow without source.

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A string $(\alpha_1, ..., \alpha_n)$ is **accepted** by the automaton if and only if there is a path

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The **language** of an automaton $\mathcal{A}$ is the set of all accepted strings. We write lang($\mathcal{A}$) for this language.

FSA examples

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In class.

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- For each non-deterministic FSA F over alphabet A, there is an deterministic FSA F' over A such that $\text{lang}(F') = \text{lang}(F)$, and vice versa.

Deterministic vs non-deterministic FSA: why bother?

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This is a familiar story: we look at something from two angles (1) convenient for humans vs (2) convenient for the machine.

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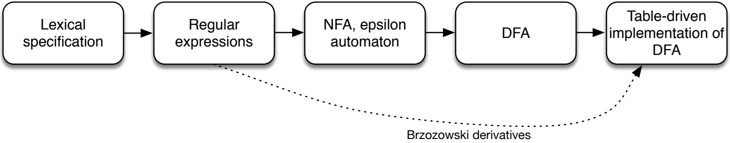

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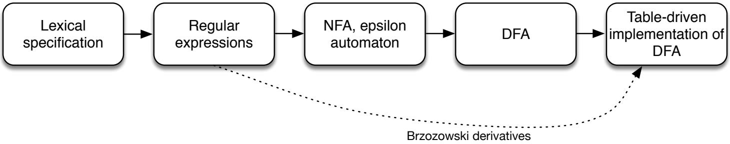

We are using ϵ-automata which can be seen as a special case of NFAs. ϵ-automata make the conversion from REs to Java implementations easier.

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Formally, an **ϵ-automaton with alphabet** A is a (usually non-deterministic) FSA with alphabet $A \cup \{\epsilon\}$.

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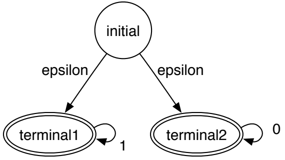

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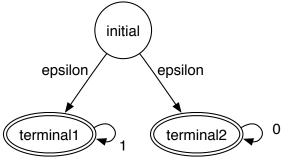

The language $0^*|1^*$ as a regular expression.

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- $w' \in \text{lang}(\mathcal{A})$ as a normal (i.e. walking any edge consumes the first character of the input string).

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So we have

$$\text{lang}_\epsilon(\mathcal{A}) = \{w \mid w' \in \text{lang}(\mathcal{A}), w \text{ is } w' \text{ with } \epsilon \text{ removed}\}$$

ϵ-automata are enough for non-deterministic FSA

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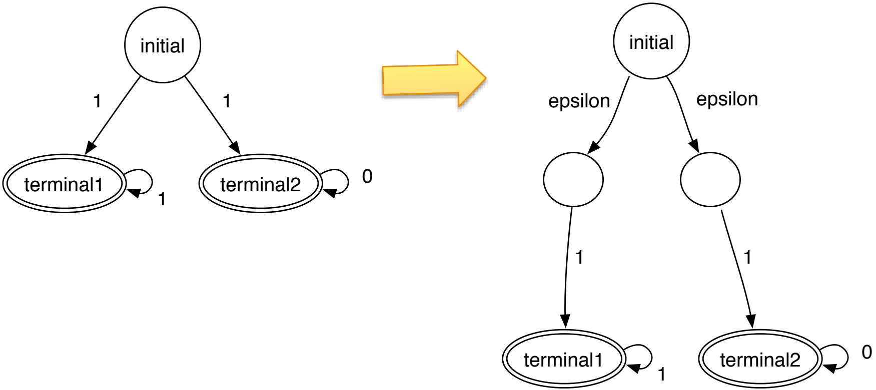
Non-determinism can always be translated to ϵ-automata that are deterministic except for ϵ-transitions.



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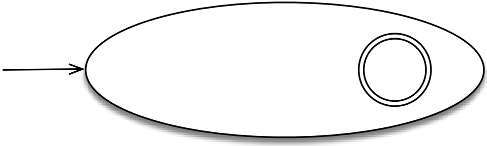
We don't need to details of each FSA in the translation, we will only be manipulating the initial and final state. All our translations have just one final state. We use the following notation to represent the FSAs arising in our translations.



Translation of $\emptyset$

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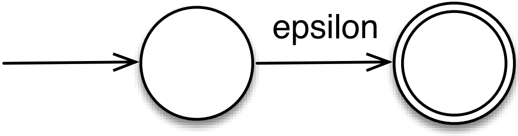

Translation of $'c'$

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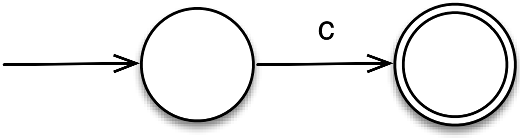

Translation of (A)

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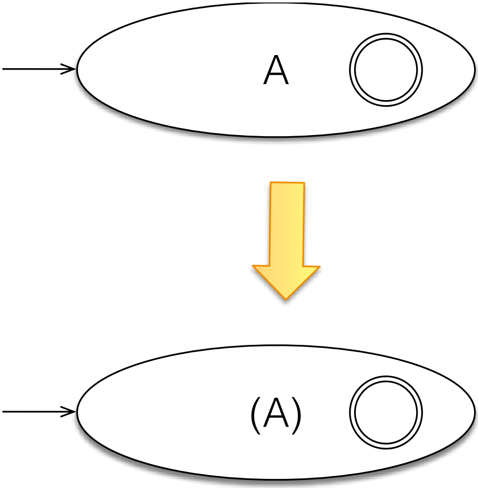

Translation of $A|B$

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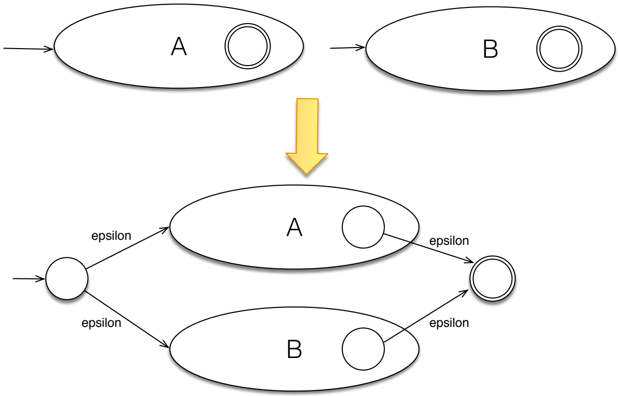
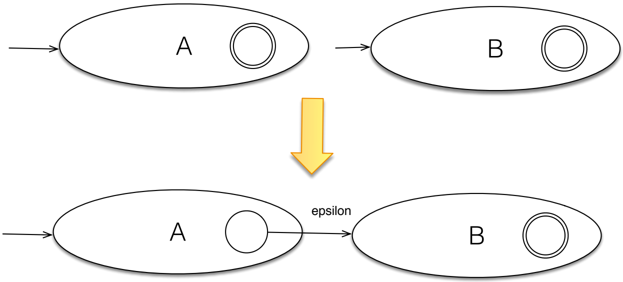


Translation of AB

Translation of *AB*



Translation of A^*

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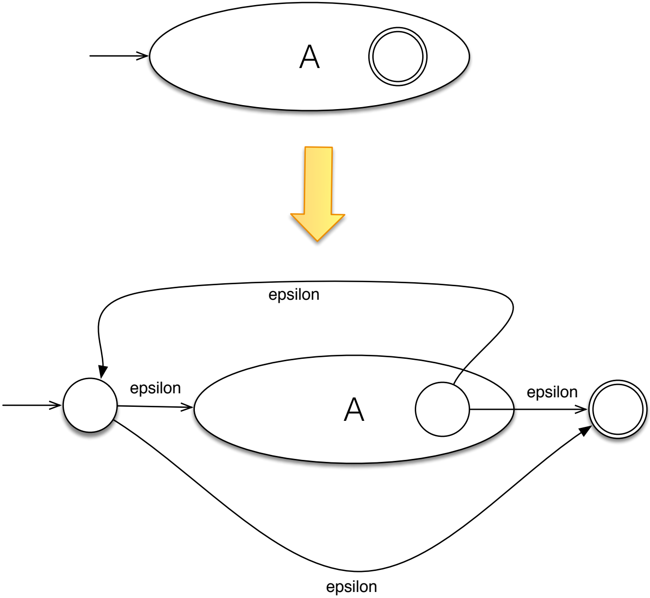

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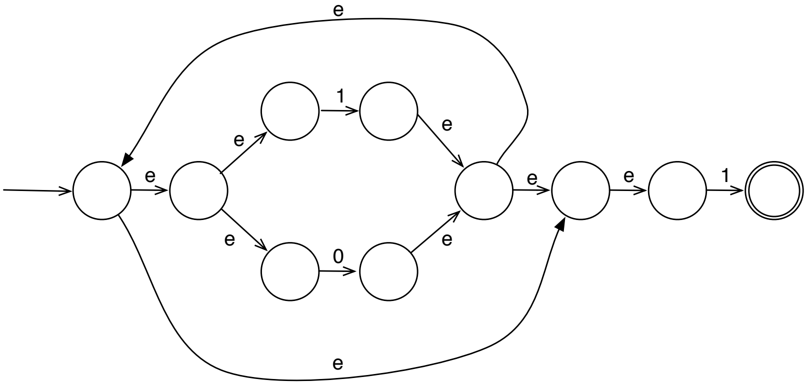

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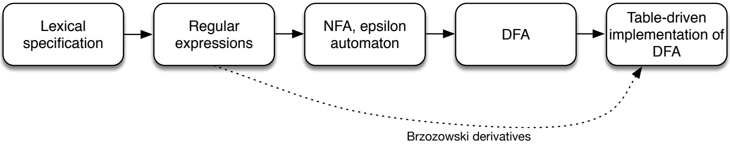

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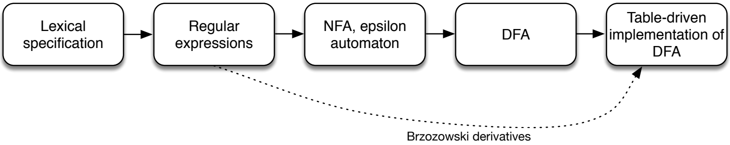

Now we want to translate our NFAs (ϵ-automata) to DFAs, because we can implement DFAs in e.g. Java (computers can't handle non-determinism).

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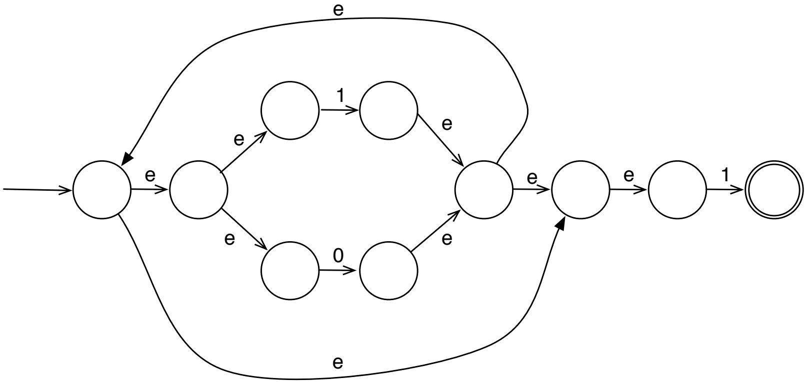

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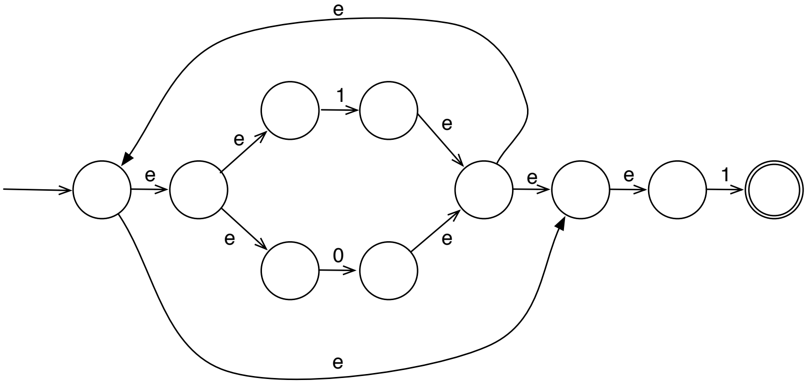

The **ϵ-closure** of a set of states S in an automaton is the set of all states reachable from a state in S by 0 or more ϵ-transitions.

From NFAs (ϵ-automata) to DFAs: ϵ-closure.

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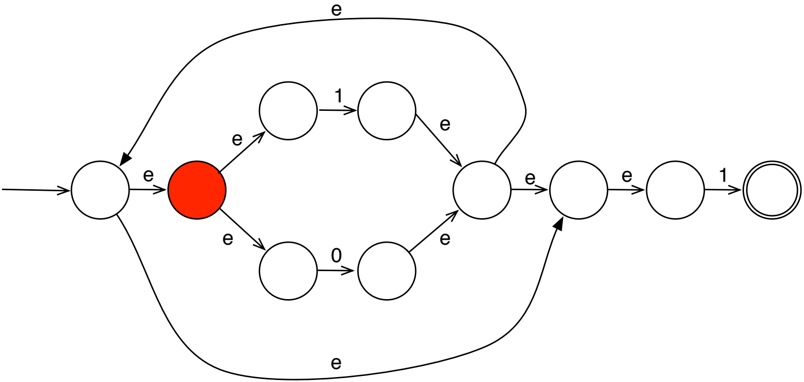

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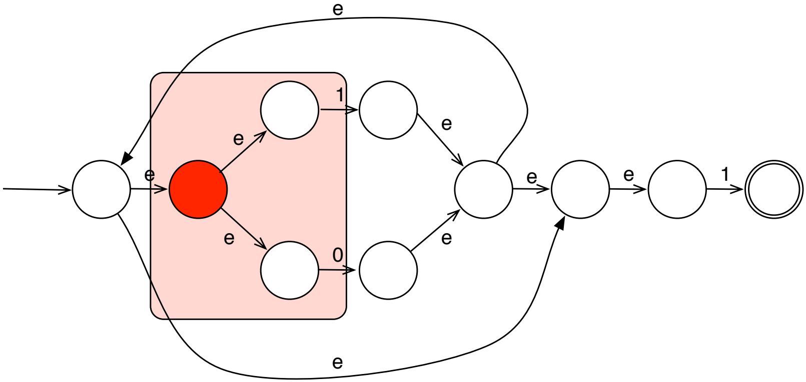

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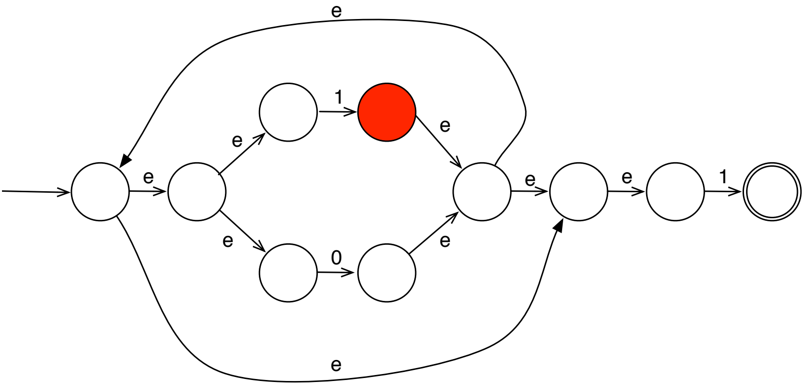

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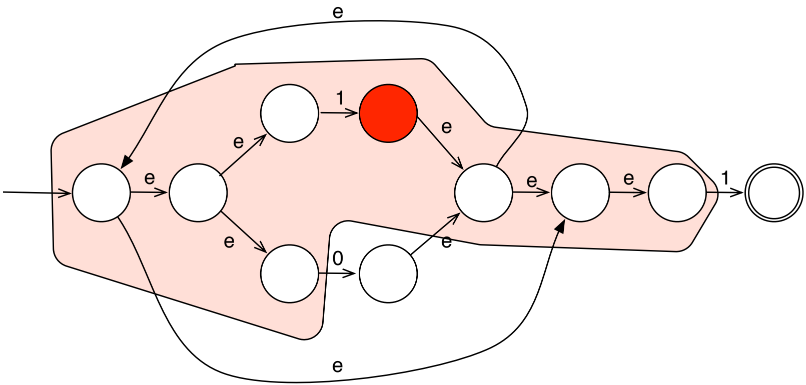

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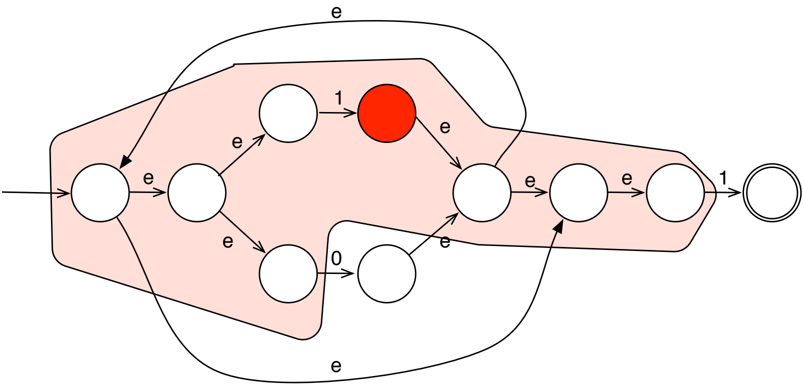

Now we construct a deterministic FSA using ϵ-closure such that

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- The new final states are all non-empty sets $X \subseteq S$ such that $X \cap F \neq \emptyset$. (Why non-empty?)
- We have a new transition from X to Y with the label a exactly when Y = ϵ-closure of $a(X)$.

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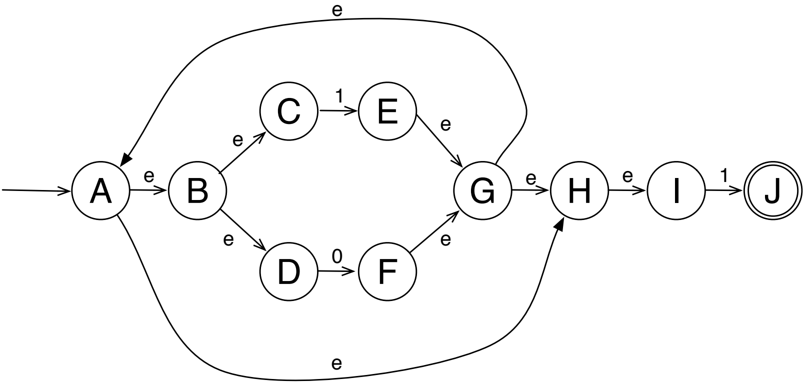
The example



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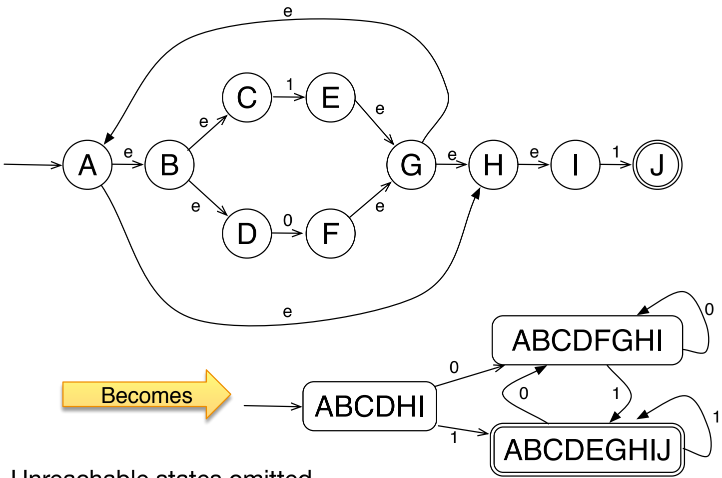
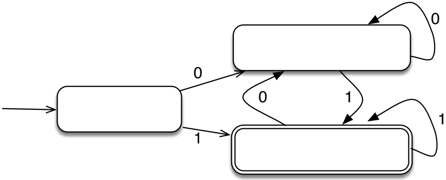


Unreachable states omitted

From NFAs (ϵ-automata) to DFAs



Check that the language of the new FSA is $(1|0)^*1$ as required.

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Fortunately in many cases, most of them are inactive and can be ignored. However in pathological cases, all states are needed.

From NFAs (ϵ-automata) to DFAs

This example shows that the translation in the naive form presented here is not particularly efficient: of the 1023 states it introduces, only 3 are needed (active). It is possible to improve the translation, so the inactive states disappear.

Implementation of DFAs and NFAs

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Remember the lexer construction pipeline?

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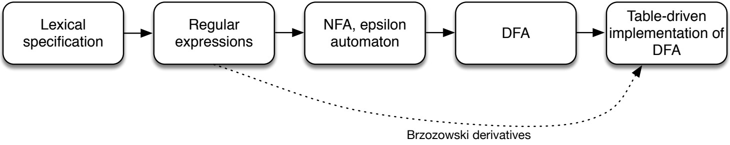

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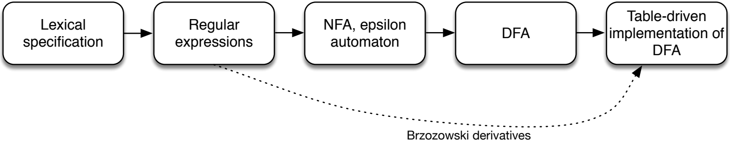

Now we want to translate DFAs into real programs e.g. Java.

Implementation of DFAs

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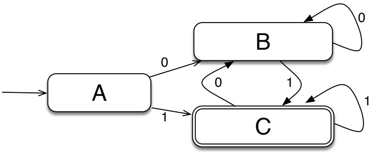

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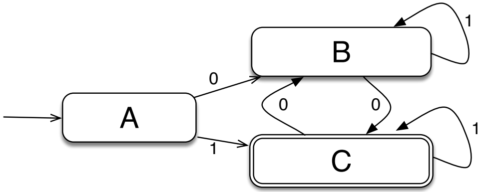

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A	B	C
B	C	B
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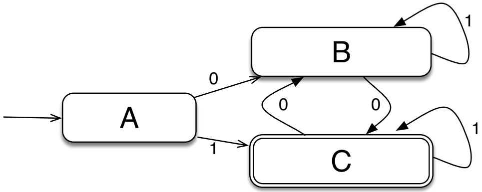

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In code: p/t/o

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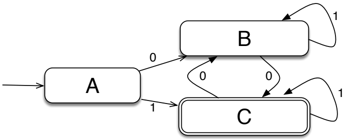

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```
def scan ( input : Array [ Char ] )
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   var s = A // current state
   val acceptingState = C
   while ( i < input.length ) {
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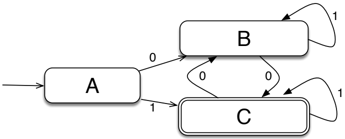

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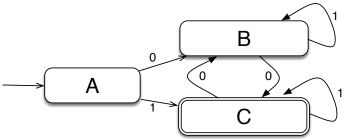

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Question: what if one of the state lacks outgoing transitions on some labels? Answer: add artificial error states, and from the error state a transition back to itself for every character.

Implementation of DFAs

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This idea, using a 2-dimensional table to implement an FSA is fundamental. Most (all?) real-world implementations of REs, FSAs etc use variants of it. It is worth understanding well.

Implementation of DFAs

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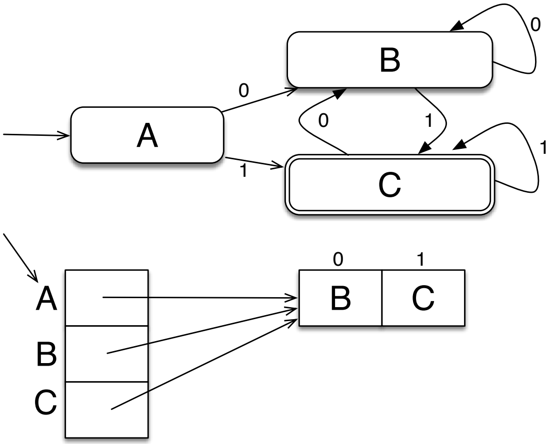
Many rows in the array are identical (all in the example below, first and third row in the previous example). That is often the case in the implementation of lexers. We can save space by sharing rows (or columns):



Outputting a token list

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We have reached our intermediate goal: going from REs to algorithms that **decide** the language of the RE, i.e. respond with TRUE/FALSE for each input string. But in lexing we want a token list (or an error message). Fortunately, this is only a small variant of the decision problem.

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```
Hello ( 123 then ...
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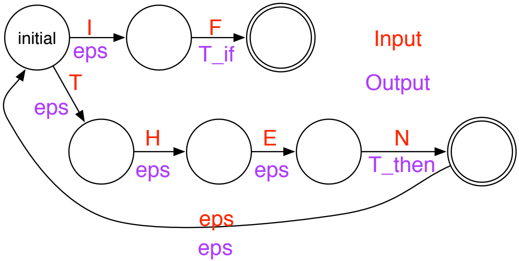
should yield a token list:

```
T_Ident ( "Hello" ),
T_Left Bracket,
T_Num ( 123 ),
T_Then,
...
```

Mealy automata

Mealy automata

We use **Mealy automata**, which is a variant of FSAs which have not only an input action, but also an output action. A picture says more than a 1000 words.



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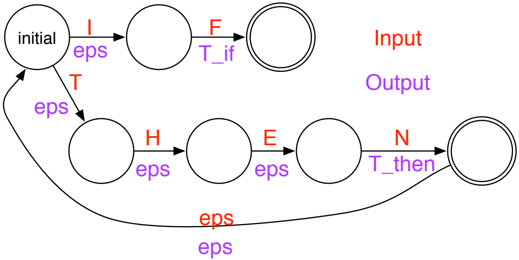

With a Mealy automaton, when we have a path

$$i \xrightarrow[u_1]{w_1} s_1 \xrightarrow[u_2]{w_2} s_2 \cdots s_{n-1} \xrightarrow[u_n]{w_n} s_n$$

whenever we accept (and consume) the input string $w_1 ... w_n$ we create an output $u_1, ..., u_n$. There are many variants, e.g. **Moore automata**.

Mealy automata: implementation

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We can implement Mealy automata by agumenting the 2-dimensional table with appropriate outputs that we accumulate as we consume the input string.

Lexer generators

Lexer generators

Lexers can be written by hand, but much easier to let the computer do that work. **Lexer generators** take as input an ordered list of REs (ordering gives priority, see below) together with **actions** (think Mealy automaton) associated with each RE, and returns a working lexer. Actions allow you to associate Java code with regular expressions. Examples: **Flex**, **JFlex**.

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- An expert can probably produce faster lexers than a generator.

Implementing lexers using regular expression libraries

Modern programming languages often have elaborate regular expression libraries. They can be used for implementing lexers too. But you have to ensure things like “longest-match” and “keywords-first” heuristics.

Key disadvantage: regular expressions tend to be **slow**, so not suitable for industrial strength compilers. But OK for toy compilers.

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  `R = Keywords | Identifier | Number | ...`
- Construct an FSA (Mealy automaton) for *R*. Let a lexer generator do this work.

Error handling in lexers

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Use a RE that matches **any** character in the alphabet. Give this RE the lowest priority. Because it matches any character it will also always be a shortest possible match.

It catches anything that is not allowed by all previous REs. The output associated with this RE can be used for error messages.

Conclusion

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The point of lexing is to have a ROUGH classification of the input program that enables the next stage (parsing) to determine of the program is syntactically well-formed, and to construct the AST. Regular expressions and FSAs are convenient tools for implementation of lexers.

The material in the textbooks

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- Dragon Book: Chapter 2.6, Chapter 3.
- Appel, Palsberg: Chapter 2.
- “Engineering a compiler”: Chapter 2: especially sections 2.1 to 2.5.