
How to catch a cricket ball

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Abstract. A cricket or baseball fielder can run so as to arrive at just the right place at just the right time to catch a ball. It is shown that if the fielder runs so that $d^2(\tan \alpha)/dt^2 = 0$, where α is the angle of elevation of gaze from fielder to ball, then the ball will generally be intercepted before it hits the ground. This is true whatever the aerodynamic drag experienced by the ball. The only exception is if the ball is not approaching the fielder before he starts to run.

1 Introduction

Our ability to run and catch a ball is a remarkable skill. From a dot in the distance rising in the air we know whether to move forward or backward, left or right, and then what speed to run at so as to arrive at just the right place at just the right time (McLeod and Dienes 1993). Indeed, the problem of how we do this was so compelling to Churchland (1988) that she described it as “every bit as mysterious ... as consciousness” (page 279).

What optic information specifies how we should move? Cues that would enable a fielder to move left or right have already been identified (Regan et al 1979). We shall explore algorithms that would enable a fielder to move forward or backward at the correct speed to intercept the ball.

First, we shall consider the case of a stationary fielder to determine what optic information could tell someone whether to initially move forward or backward. Next, we shall consider the case of a moving fielder to determine what strategy would bring the fielder to the right place at the right time to catch the ball. We shall show that the same optic variable which tells a stationary fielder which way to move can tell a moving fielder how fast to run. Finally, we shall show that the optic cue identified appears to be the one that people actually use.

2 The stationary fielder

What optic information could tell a stationary fielder to move forward or backward or to stay in the same place? Chapman (1968) showed that if the flight of the ball is parabolic then the acceleration of the tangent of the angle, α , of the fielder's line of sight to the ball with respect to the horizontal gives perfect information throughout the flight. The acceleration of the tangent [$d^2(\tan \alpha)/dt^2$] is equivalent to the acceleration of the vertical projection of the ball (see figure 1). The acceleration of the tangent is zero if (and only if) the fielder is at the place where the ball will fall; it is positive if the fielder is too far forward and negative if the fielder is too far back (figure 1). Thus, referring to figure 1, the distances between successive projections of the ball onto the vertical are equal only at point B, where the ball will land; the distances are successively greater at point C, which is too far forward, and successively smaller at point A, which is too far back. Further, it appears that people are sensitive to this information. Babbler and Dannemiller (1993; cf Todd 1981) showed subjects

a dot moving up a computer screen with positive or negative acceleration. If subjects were told to treat the dot as a ball, then they could use the vertical acceleration of the dot to say whether it would land in front of them or behind them.

Brancazio (1985) argued that Chapman's (1968) analysis was irrelevant to the real world problem because it ignored aerodynamic drag. He showed that if the fielder watches the trajectory of a ball whose flight is affected by drag from the place where it will fall the acceleration of the tangent of the angle of gaze will not be zero. He suggested that fielders could use the instantaneous acceleration of angle at the beginning of the flight $[d^2\alpha/dt^2]_{t=0}$. This does not give perfect information, but it is close to zero if the fielder is at the right location, and, except for small distances out, tends to be positive if the fielder is too far forward and negative if the fielder is too far back for a range of values for drag. Brancazio assumed that the best fielders could use this initial information to run directly to the right place and then wait for the ball.

Brancazio (1985) showed that both acceleration of tangent and acceleration of angle gave imperfect information for a stationary fielder watching the flight of a ball slowed by drag. But he did not systematically compare both cues to show which gave the *better* information. We calculated the behaviour of both cues over a range of possible trajectories. Appendix 1 describes how we modelled the ball trajectory. We assumed that the ball experienced just two forces: the force of gravity acting vertically downwards, and aerodynamic drag acting antiparallel to the direction of the motion of the ball. Drag was assumed to be proportional to the square of the velocity of the ball (Daish 1972), Q being the constant of proportionality. In the simulations reported below, the initial angle of the trajectory varied between 20° and 70° . Drag was set at $Q = 0.007 \text{ m}^{-1}$ and the initial velocity at 20 m s^{-1} . The conclusions that follow, however, are valid for a wide range of speeds (5 up to at least 35 m s^{-1}) and Q values (0 up to at least 0.1 m^{-1}), and for drag proportional to velocity rather than velocity squared (with L , the constant of proportionality for linear drag, equal to 0 up to at least 0.5 s^{-1}). The values for the acceleration of tangent and the acceleration of angle were calculated with the use of the formulae given in appendix 2.

Figure 2a shows how the acceleration of the tangent of the angle of gaze varies with the time of flight, t , for a fielder watching a ball projected towards him with an initial projection angle of 20° . The curves show what would be seen by a fielder

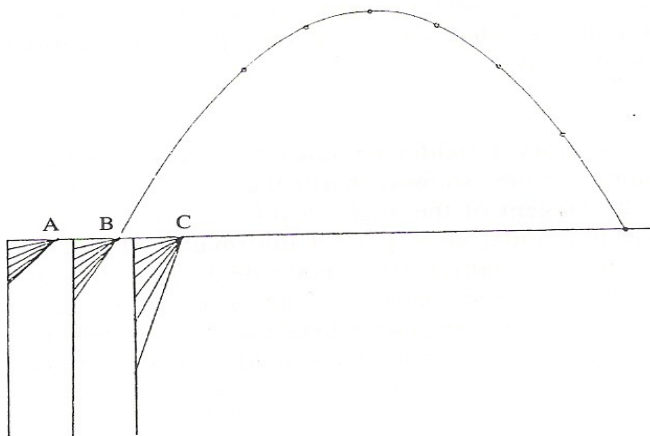


Figure 1. The vertical projection of six successive points lying on a parabolic path for fielders standing at positions A, B, or C. Only when the fielder is standing at the right location for interception does the vertical projection of the points move at constant velocity.

standing at different distances from the point where the ball will land. If the fielder is at the right position, it is close to zero; otherwise the acceleration of tangent is systematically positive or negative depending on whether the fielder is too far forward or back. As the angle of the trajectory is increased, the same qualitative results are obtained. Figure 2b shows the plot of the acceleration of the tangent of the angle of gaze against time of flight for a 70° trajectory.

It can be seen that the acceleration of the tangent of the angle of elevation of gaze is positive for someone in front of the place where the ball will land, negative for someone standing too far back, and close to zero for someone viewing the ball from the place where it will land. Thus, the fielder could use the sign of the acceleration of the tangent to tell him whether he should move forwards or backwards to catch the ball.

However, the picture is very different with the acceleration of the angle. Although this gives qualitatively similar results with low trajectories, it ceases to provide useful information with steep trajectories. Figure 3 shows the result with a projection angle of 70° . Except for $t = 0$ s, and shortly before the ball flies over the fielder's head, the acceleration of angle is negative whether the fielder is too far forward or too far back. Indeed, Babbler and Dannemiller's (1993) measurement of subjects' sensitivity to vertical acceleration indicates that the positive accelerations of the magnitude and duration shown at the beginning of this flight would be completely undetectable by fielders.⁽¹⁾

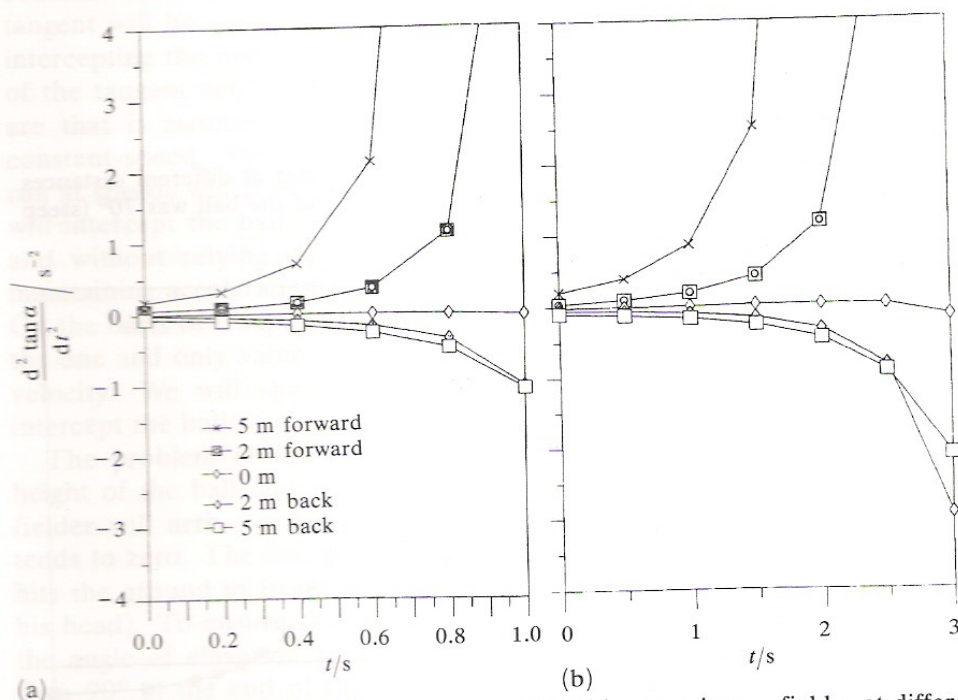


Figure 2. The acceleration of tangent experienced by a stationary fielder at different distances from the final landing point of the ball. The initial projection angle of the ball was (a) 20° (shallow trajectory) and (b) 70° (steep trajectory).

⁽¹⁾ Babbler and Dannemillar (1993) showed that subjects' sensitivity to angular accelerations could be measured by the velocity change ratio: (final angular velocity - initial angular velocity) / (average angular velocity). This ratio needed to be at least 0.2 before subjects could detect that there was an acceleration. Now consider the trajectory shown in figures 3 and 4. Even when the fielder is 16 m too far forward, the angular acceleration is positive for only 80 ms. Over this time period, the velocity change ratio is 0.06. According to the results of Babbler and Dannemillar, the fielder could not detect this; but, by the same calculations, he could detect the subsequent negative acceleration after only 250 ms.

Thus, in contrast to the acceleration of tangent, the sign of the acceleration of angle does not provide the fielder with information about whether he should move forward or backward.

Another way of showing the problem with acceleration of angle for steep trajectories is a plot of this cue against fielder distance from the correct location at the

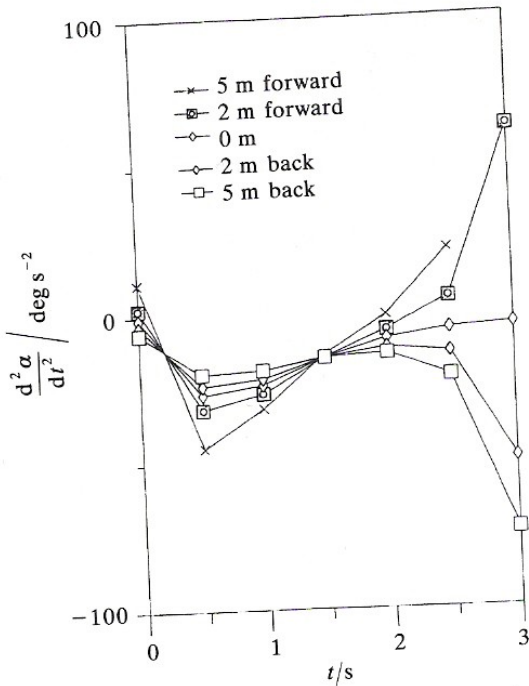


Figure 3. The acceleration of angle experienced by a stationary fielder at different distances from the final landing point of the ball. The initial projection angle of the ball was 70° (steep trajectory).

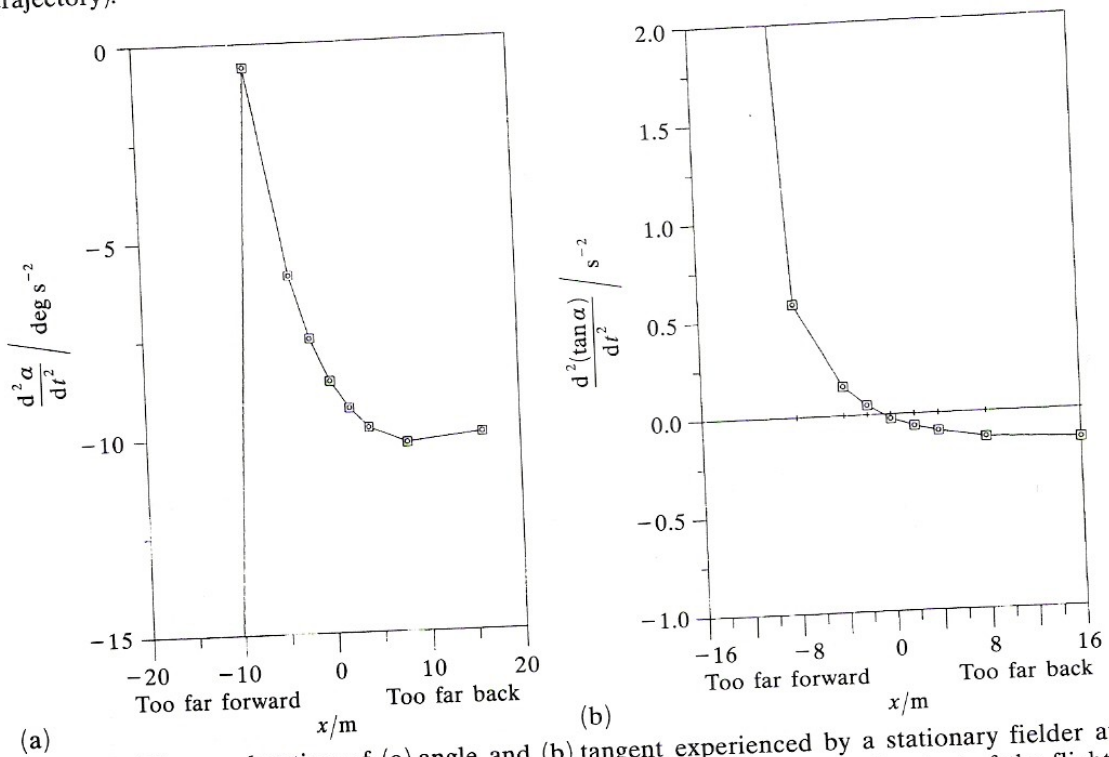


Figure 4. The acceleration of (a) angle and (b) tangent experienced by a stationary fielder at different distances from the final landing point of the ball 100 ms after the start of the flight. The initial projection angle of the ball was 70° .

beginning of the flight (figure 4a). At 100 ms into the flight, the acceleration of angle is negative regardless of whether the fielder is too far forward or back. Figure 4b shows the same plot for the acceleration of tangent, which is well behaved. If the fielder is too far forward it is positive; if he is too far back it is negative. Note that the slope changes with distance: there is an asymmetry between being too far forward or back. If fielders have a certain threshold for detecting acceleration of tangent, the information would become available later if the fielder is too far back than it would if he is too far forward.

In summary, the acceleration of tangent, as suggested by Chapman (1968), but not the acceleration of angle, as suggested by Brancazio (1985), is a well behaved cue for the stationary fielder. Chapman had previously shown that the acceleration of tangent could be used in a vacuum, but that does not mean that the cue is ecologically useful. We have shown that the cue does, in fact, have ecological validity and can be used over a wide range of real world values of drag, speed, and projection angle; the figures illustrate the use of representative values. The fact that the acceleration of tangent is not always zero for a fielder standing in the correct place is not important; the actual zero point in fact lies very close to the correct location.

3 The moving fielder

For a parabolic trajectory Chapman (1968) showed that if the fielder runs at a constant velocity towards the point where the ball will drop, the acceleration of tangent will be zero if and only if the fielder runs at a velocity which results in him intercepting the ball. In other words, if the fielder ran so as to keep the acceleration of the tangent zero he would reach the ball. Two limitations of Chapman's analysis are that it assumed parabolic flight, and that it required the fielder to run at a constant speed. We will give a general analysis to show that if the fielder is able to run at speeds which keeps the acceleration of the tangent of the angle of gaze zero, he will intercept the ball, except in one identifiable case, regardless of the level of drag, and without relying on the fielder running at constant velocity. In fact, typically, maintaining acceleration of tangent at zero does not lead to constant running velocity. On the basis of Chapman's analysis it might be supposed that the fielder needs to find the one and only value of $d(\tan \alpha)/dt$ that, if kept constant, leads to constant running velocity. We will show that maintaining any value of $d(\tan \alpha)/dt$ is sufficient to intercept the ball.

The problem of intercepting a ball can be expressed easily. If y is the vertical height of the ball and x is the horizontal distance between ball and fielder, then the fielder will arrive at the right place provided he ensures that x tends to zero as y tends to zero. The fielder will fail to catch the ball if y reaches zero before x (the ball hits the ground in front of the fielder) or x reaches zero before y (the ball goes over his head). To ensure that x and y tend to zero together the fielder must ensure that the angle of elevation, α , of the ball with respect to the fielder is positive and less than 90° at the end of the flight. This does not, of course, require that the flight is parabolic, as in Chapman's analytical solution. It will be true of any flight, no matter what the value of drag experienced by the object.

3.1 *Constant α could work*

One simple strategy which would ensure that α remains between 0° and 90° throughout the flight would be to start moving at some arbitrary time during the flight when the angle of elevation was α , and run at a speed which kept α constant. There is an attractive simplicity to this strategy, but there are severe practical problems in implementing it for many flights. For example, simple calculations show that, in vacuo, the fielder needs to maintain an acceleration of $g \cot \alpha$, and this can be prohibitively large

(eg for $\alpha = 40^\circ$ the fielder must maintain an acceleration of 11 m s^{-2}). Further, when the ball reaches maximum height, the fielder should be running away from the ball at the same horizontal speed as that of the ball, regardless of whether he is too far forward or too far back! For many flights, if the fielder is too far back, or only slightly too far forward, and the strategy is initiated early in the trajectory, then the fielder must run even further back at a large velocity. In contrast to this prediction, we know that subjects do initiate some strategy early on in the flight that involves running in the appropriate direction (McLeod and Dienes 1993).

3.2 Constant $d\alpha/dt$ does not work

An alternative to maintaining a constant angle would be to allow the angle to change during the flight but in a way which prevented it reaching 0° or 90° . Ball flights all start with α increasing. If the ball will go over the fielder's head, the value of $d\alpha/dt$ increases during the flight until $\alpha > 90^\circ$. If it is going to fall short, α reaches a maximum value and then $d\alpha/dt$ becomes negative until α reaches zero. One way to ensure that the ball never fell short would be to start running while $d\alpha/dt$ was positive and to run at a speed that ensured that it remained positive. The problem with this strategy is that α might exceed 90° before $y = 0$; in other words, it will not prevent the ball going over the fielder's head. For example, for a cricket ball starting off with a speed of 20 m s^{-1} , projected at an angle of 60° , and with $Q = 0.007 \text{ m}^{-1}$, if the fielder initiates a strategy of keeping $d\alpha/dt$ constant after 0.5 s of flight when he is 5 m too far forward, the ball will fly over the fielder's head after 2.2 s of flight with over 1 s of flight still left.

3.3 Constant $d(\tan\alpha)/dt$ does work

Consider the plot of tangent against angle. As angle increases from 0° so does the tangent. At 90° tangent goes to infinity. If instead of keeping $d\alpha/dt$ constant the runner ran so as to keep $d(\tan\alpha)/dt$ constant, the problem would be solved. If the tangent were increased in a controlled way [ie with a finite positive $d(\tan\alpha)/dt$ for the duration of the flight] starting with any angle between 0° and 90° , then the angle must be greater than 0° and less than 90° at the end of the flight. That is, provided the fielder could run fast enough to ensure that $d(\tan\alpha)/dt$ remained finite and positive he would catch the ball. The logic of the argument holds whether the ball is initially approaching the fielder or moving away; whether the trajectory occurs in vacuo or with drag: if a positive constant $d(\tan\alpha)/dt$ has been maintained, the angle at the end of the flight must be between 0° and 90° , and this must mean interception.

At the beginning of any flight, $d(\tan\alpha)/dt$ is positive. So if the fielder starts to run soon after the ball appears and runs at a speed that keeps the rate of change of the tangent constant (or, equivalently, the acceleration of the tangent zero) he would always arrive at the right place at the right time to catch the ball. The only limitation would be on whether he could run fast enough to ensure that $d(\tan\alpha)/dt$ was constant.

If the fielder waits until the ball has started to fall before he runs, $d(\tan\alpha)/dt$ may be negative. What will happen if he uses the strategy of keeping $d(\tan\alpha)/dt$ constant when it is negative? The condition for catching the ball in this case is that the time it takes for the tangent to go to zero is greater than the remaining time of the flight, T . That is, we require

$$-\tan\alpha \left/ \frac{d(\tan\alpha)}{dt} \right. > T. \quad (1)$$

Appendix 3 shows that inequality (1) will always be satisfied for a trajectory with any amount of drag if the fielder initiates the strategy of keeping $d(\tan\alpha)/dt$ constant when the ball is approaching the fielder. If the ball is not approaching the fielder the strategy will not work. Note that by 'approaching' we mean that there is a component

of the ball velocity towards the fielder. As described below, the fielder could be initially viewing the flight path from an angle, and then the strategy would work, as long as there is a component of the ball velocity towards the fielder.

If $d(\tan \alpha)/dt$ were initially negative, we have suggested that one strategy would be to run so as to maintain this negative value. Another strategy would be to run towards the ball until $d(\tan \alpha)/dt$ acquired a positive value, and then run so as to maintain the positive value. This strategy would always work, regardless of whether the ball was approaching the fielder or not.

In conclusion, the acceleration of tangent, but not the acceleration of angle, is a good cue for moving as well as for stationary fielders. In fact, if the fielder runs so as to keep the acceleration of the tangent zero, but not the acceleration of angle, starting at any time in the flight of the ball, he is guaranteed to intercept the ball regardless of the level of drag, except in the case where the ball is not approaching the fielder.

The strategy of keeping the acceleration of tangent constant will work in the normal case when the fielder has to move sideways as well as forwards or backwards to catch the ball: so long as the fielder always orients towards the ball the logic of the previous arguments applies. That is, when the horizontal component of the ball's velocity in the vertical plane containing the ball and the catcher is in the direction of the catcher, the strategy will always work. The fielder could also apply some mixture of strategies; for example, keeping a constant left-right bearing with respect to the ball and otherwise moving so as to keep $d(\tan \alpha)/dt$ constant.

4 What people really do

McLeod and Dienes (1993) videoed people catching balls fired directly towards them with a bowling machine. They were filmed against a structured background from which their position was measured at 120 ms intervals. McLeod and Dienes showed that fielders do not run to the place where the ball will fall and wait for it. Rather, they run at speeds that take them through the point where the ball will fall at the exact time that it arrives. If the flight duration is lengthened by increasing the angle of projection, or if the fielder starts closer to the right location, then the fielder runs at an overall slower speed. That is, fielders do not know where to go, only how to get there. That is exactly what the theory that fielders run at a speed that keeps $d(\tan \alpha)/dt$ constant predicts.

If fielders were keeping angle constant, then a plot of angle against time would be a horizontal line. If fielders were keeping the rate of change of angle constant, then a plot of angle against time should be any straight line. Figure 5a shows how, for several different flights, elevation angle varies over the course of a catch.⁽²⁾ Clearly, fielders are not keeping the angle constant. Note that the increase of angle with time is negatively accelerated: multiple regression showed that the quadratic component was highly significant for each flight, $ps < 0.0001$. This is not consistent with subjects maintaining a constant rate of change of angle, but would be consistent with, for example, subjects maintaining a constant rate of change of tangent. Figure 5b shows how the tangent behaves over the course of a catch for the flights shown in figure 5a. Note that the plots are virtually straight lines. The rate of change of tangent is kept constant at its starting value for the duration of the catch. The increase in variance explained by the assumption of a linear relationship in the plot of tangent against time as compared to the plot of angle against time can be determined

⁽²⁾ The fielder's position at each time was averaged for five successful catches that happened to have the same trajectory and starting position; thus, each curve in figures 5a and 5b is the average of five catches. The results for individual catches are analysed in McLeod and Dienes (submitted) and were not qualitatively different.

by William's test of difference between nonindependent correlations (Howell 1982, page 243). For each of the four flights, the increase in variance explained was significant, $ps < 0.0001$; that is, the correlation between tangent and time was significantly greater than the correlation between angle and time for each flight.⁽³⁾ The assumption that fielders keep the rate of change of tangent constant fits the data better than the assumption that the fielders keep the rate of change of angle constant.

The data are only for cases when $d(\tan\alpha)/dt$ was initially positive. In the case of an initial negative $d(\tan\alpha)/dt$ we do not know if fielders attempt to maintain the negative value, or run so as to produce a positive value.

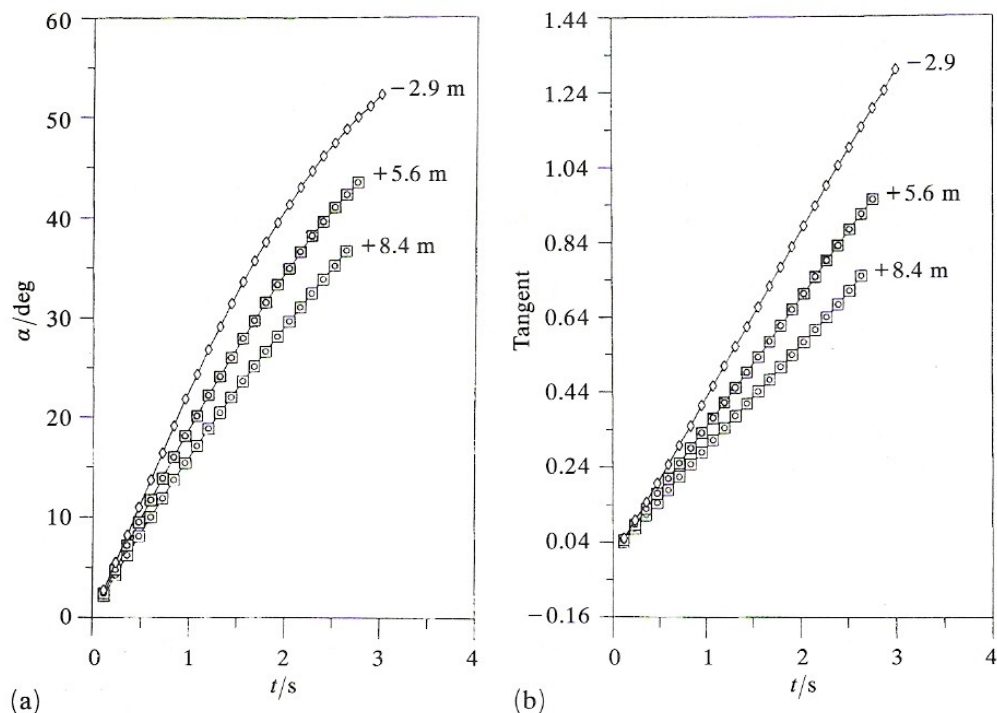


Figure 5. (a) The elevation angle actually experienced by a fielder during different runs. Each curve represents a different starting distance from the point of interception. The fielder ran either 2.9 m backwards, 5.6 m forwards, or 8.4 m forwards to catch the ball. In all the cases, the initial separation between the fielder and the ball was 45 m, the ball was projected at an angle of 45° , and the initial ball velocities varied between 20 and 25 m s^{-1} so that the ball landed in front of or behind the initial position of the fielder. (b) The tangent actually experienced by the fielders running the three distances shown in (a).

5 Conclusion

We have shown that the acceleration of tangent of elevation angle is a useful cue for informing a stationary fielder whether to move forwards or back, and a useful cue for allowing a moving fielder to intercept the ball at just the right time. It appears that fielders do use this cue because they run to catch balls at speeds that keeps the rate of change of tangent constant, arriving at the right place at just the right time (McLeod and Dienes 1993). One problem that this paper has not addressed is how such a strategy could be physiologically implemented. The strategy of keeping the rate of change of tangent constant is guaranteed to work partly because the tangent approaches infinity as angle approaches 90° ; but it is clearly difficult for neural activity directly to code infinitely large numbers. Fortunately, the fielder does not

⁽³⁾ Two of the curves in figure 5b did not have a significant quadratic component ($ps > 0.05$), but one of the curves in fact had a significant positive quadratic component; in the latter case, the quadratic effect accounted for less than 0.1% of the variance.

need to code an infinite tangent; all that is required to stop the ball going over the head is that the rate of change of angle approaches zero as angle approaches 90° . The fielder could do this by capitalizing on the following relationship

$$\frac{d\alpha}{dt} = \cos^2 \alpha \frac{d(\tan \alpha)}{dt}. \quad (2)$$

If $d(\tan \alpha)/dt$ is set equal to a constant, k , then the fielder needs to run so that

$$\frac{d\alpha}{dt} - k \cos^2 \alpha = 0. \quad (3)$$

The left hand side of equation (3) involves calculating only finite values, but maintaining this relationship will ensure that the rate of change of tangent is constant. If the left hand side of equation (3) becomes greater than zero, that means that $d(\tan \alpha)/dt$ has increased, so the fielder should accelerate away from the ball. Similarly, if the left hand side of equation (3) becomes less than zero, the fielder should accelerate towards the ball. However, the problem of how, in detail, fielders could come to learn to keep the acceleration of tangent zero, remains a problem for further research.

Another problem is how α is specified. Brancazio (1985) suggested that fielders use the vestibular system. The otolith organs can indeed detect rotation of the head (Benson 1982), and rotation of the head could be used to measure α . However, the otolith organs are also sensitive to linear horizontal acceleration, so the effect of the fielder's motion would have to be compensated for. The fielder might also use the rotation of the eyeball relative to the head and the motion of the image of the ball across the retina during a saccade. Another possibility is that the fielder estimates the distance away of the ball and its height from the ground to calculate α . If the latter is the case, then interfering with depth perception should interfere with the fielder's ability to implement the strategy. So far, we have found that distorting depth perception does not impair interception (McLeod and Dienes, submitted).

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Appendix 1. Equations of a cricket ball

Once a ball leaves the bat it experiences two forces: the force of gravity acting downwards and aerodynamic drag acting antiparallel to the direction of its motion. For a spherical object like a cricket ball, drag is proportional to the square of its velocity (Daish 1972); in other cases, drag can be proportional to velocity or to other powers of velocity (De Mestre 1990). For the simulations reported here, drag was assumed proportional to the square of velocity. But to test the generality of the findings, the simulations were also conducted with drag proportional to velocity. These two methods of calculating trajectories are now described in turn.

For drag proportional to the square of velocity

$$\frac{d^2x}{dt^2} = -Qv^2 \cos \theta, \quad (\text{A1.1})$$

$$\frac{d^2y}{dt^2} = -g - Qv^2 \sin \theta, \quad (\text{A1.2})$$

where x is the horizontal distance between the (stationary) fielder and the ball, y is the vertical distance of the ball above the ground, g is the acceleration due to gravity, Q is the constant of proportionality for drag, v is the instantaneous velocity of the ball, and θ is the instantaneous angle that velocity makes to the horizontal.

These equations cannot be solved analytically, so trajectories must be obtained by numerical integration; for example over small time increments, by using the third derivative to improve the approximation (Brancazio 1985). Starting with initial values of x, y (initially zero), dx/dt , and dy/dt , we determine the trajectory making the following calculations in sequence every time step:

$$v^2 = \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2, \quad (\text{A1.3})$$

$$\frac{d^2x}{dt^2} = -Qv \frac{dx}{dt}, \quad (\text{A1.4})$$

$$\frac{d^2y}{dt^2} = -g - Qv \frac{dy}{dt}. \quad (\text{A1.5})$$

Expressions (A1.4) and (A1.5) are obtained from (A1.1) and (A1.2) by eliminating θ .

$$\frac{dv}{dt} = \frac{1}{v} \left(\frac{dx}{dt} \frac{d^2x}{dt^2} + \frac{dy}{dt} \frac{d^2y}{dt^2} \right), \quad (\text{A1.6})$$

$$\frac{d^3x}{dt^3} = -Q \left(\frac{dv}{dt} \frac{dx}{dt} + v \frac{d^2x}{dt^2} \right), \quad (\text{A1.7})$$

$$\frac{d^3y}{dt^3} = -Q \left(\frac{dv}{dt} \frac{dy}{dt} + v \frac{d^2y}{dt^2} \right), \quad (\text{A1.8})$$

$$\Delta x = \frac{dx}{dt} \Delta t + \frac{1}{2} \frac{d^2x}{dt^2} \Delta t^2 + \frac{1}{6} \frac{d^3x}{dt^3} \Delta t^3, \quad (\text{A1.9})$$

$$\Delta \left(\frac{dx}{dt} \right) = \frac{d^2x}{dt^2} \Delta t + \frac{1}{2} \frac{d^3x}{dt^3} \Delta t^2, \quad (\text{A1.10})$$

$$\Delta y = \frac{dy}{dt} \Delta t + \frac{1}{2} \frac{d^2y}{dt^2} \Delta t^2 + \frac{1}{6} \frac{d^3y}{dt^3} \Delta t^3, \quad (\text{A1.11})$$

and

$$\Delta \left(\frac{dy}{dt} \right) = \frac{d^2y}{dt^2} \Delta t + \frac{1}{2} \frac{d^3y}{dt^3} \Delta t^2. \quad (\text{A1.12})$$

Then at the next time step calculations cycle back to equation (A1.3) again, until y returns to zero. According to Daish (1972) and our own empirical results, the value for Q for a cricket ball is about 0.007 m^{-1} and this is the value used in the simulations reported in the text. The value of the time step was 0.05 s (reducing the time step to 0.01 s produced results that agreed to within 1%).

If drag is proportional to velocity, then

$$\frac{d^2x}{dt^2} = -L \frac{dx}{dt}, \quad (\text{A1.13})$$

$$\frac{d^2y}{dt^2} = -g - L \frac{dy}{dt}, \quad (\text{A1.14})$$

where L is the linear constant of proportionality. These equations can be solved with Laplace transforms to give the horizontal and vertical position of the ball as a function of time:

$$x = x_0 - V_x \frac{1 - e^{-Lt}}{L}, \quad (\text{A1.15})$$

$$y = V_y \frac{1 - e^{-Lt}}{L} - g \frac{e^{-Lt} + Lt - 1}{L^2}. \quad (\text{A1.16})$$

where x_0 is the initial horizontal distance between the fielder and the ball, V_x is the initial horizontal speed of the ball, V_y is the initial vertical speed of the ball. Differentiating these equations gives formulae for dx/dt , dy/dt , d^2x/dt^2 , and d^2y/dt^2 .

Appendix 2. Trigonometric formulae

Successive differentiations of $\tan \alpha = y/x$ give the following formulae, used in the simulations presented in section 2:

$$\frac{d \tan \alpha}{dt} = \frac{1}{x^2} \left(x \frac{dy}{dt} - y \frac{dx}{dt} \right), \quad (\text{A2.1})$$

$$\frac{d^2(\tan \alpha)}{dt^2} = \frac{1}{x^3} \left[x^2 \frac{d^2y}{dt^2} - xy \frac{d^2x}{dt^2} - 2x \frac{dx}{dt} \frac{dy}{dt} + 2y \left(\frac{dx}{dt} \right)^2 \right], \quad (\text{A2.2})$$

$$\frac{d\alpha}{dt} = \frac{1}{x^2 + y^2} \left(x \frac{dy}{dt} - y \frac{dx}{dt} \right), \quad (\text{A2.3})$$

$$\frac{d^2\alpha}{dt^2} = \left(\frac{1}{x^2 + y^2} \right)^2 \left[\left(x \frac{d^2y}{dt^2} - y \frac{d^2x}{dt^2} \right) (x^2 + y^2) - 2 \left(x \frac{dy}{dt} - y \frac{dx}{dt} \right) \left(x \frac{dx}{dt} + y \frac{dy}{dt} \right) \right]. \quad (\text{A2.4})$$

Appendix 3: Why keeping a negative $d(\tan \alpha)/dt$ constant normally ensures interception

In the text we showed that the strategy of maintaining $d(\tan \alpha)/dt$ constant leads to interception of the ball if $d(\tan \alpha)/dt > 0$ when the fielder starts to run. Here we show that it is usually successful even if the fielder starts to run when

$$\frac{d(\tan \alpha)}{dt} < 0. \quad (\text{A3.1})$$

Now

$$\tan \alpha = \frac{y}{x}.$$

Differentiating, we obtain

$$\frac{d(\tan \alpha)}{dt} = \frac{1}{x} \left(\frac{dy}{dt} - \frac{dx}{dt} \tan \alpha \right). \quad (\text{A3.2})$$

When the fielder starts to run the following conditions hold:

- (i) $\tan \alpha > 0$,
- (ii) $y > 0$,
- (iii) $x > 0$.

We will also make the assumption that the ball is approaching the fielder when he starts to run, ie

$$\text{(iv)} \quad \frac{dx}{dt} < 0.$$

Given the above conditions and inequality (A3.1), both of the following are implied by expression (A3.2):

$$\text{(v)} \quad \frac{dy}{dt} < 0,$$

$$\text{(vi)} \quad \left| \frac{dx}{dt} \tan \alpha \right| < \left| \frac{dy}{dt} \right|.$$

As stated in the text, the requirement for a successful catch is that

$$-\tan \alpha \left/ \frac{d \tan \alpha}{dt} \right. > T. \quad (\text{A3.3})$$

Using relation (A3.2) we can express relation (A3.3) in terms of x and y and their derivatives

$$-y \left/ \left(\frac{dy}{dt} - \frac{dx}{dt} \tan \alpha \right) \right. > T. \quad (\text{A3.4})$$

Because equation (A3.3) is a requirement for a successful catch only if $d(\tan \alpha)/dt$ is a constant, equation (A3.4) is also a requirement for a successful catch only if $d(\tan \alpha)/dt$ is a constant.

Since dy/dt and $(dx/dt) \tan \alpha$ are both negative [conditions (i), (iv), and (v)] and $|(dx/dt) \tan \alpha| < |dy/dt|$ [condition (vi)], a sufficient condition to catch the ball is

$$-y \left/ \frac{dy}{dt} \right. > T. \quad (\text{A3.5})$$

That is, the $(dx/dt) \tan \alpha$ term reduces the denominator of the left-hand side of inequality (A3.4), so if (A3.5) holds then inequality (A3.4) will also hold. The left-hand side of expression (A3.4) will always be larger than that of expression (A3.5).

If the ball has reached terminal velocity, then $T = -y/(dy/dt)$. In all other cases, inequality (A3.5) will certainly hold because the ball will continue to accelerate downwards. Even if the ball has reached terminal velocity, inequality (A3.4) will hold, because the left-hand side of this inequality is always larger than that of inequality (A3.5). That is, if after a point in the flight of the ball towards a stationary fielder, the fielder moves so as to keep the acceleration of tangent zero, then the fielder will definitely intercept the ball.

Note that the above proof for the effectiveness of maintaining a negative $d(\tan \alpha)/dt$ assumed a negative dx/dt . This is crucial. If dx/dt is zero and the ball has reached terminal velocity, then the acceleration of tangent is always zero, even though the fielder will not catch the ball. If dx/dt is positive then, by relation (A3.2), $d(\tan \alpha)/dt$ can be kept negative even though the ball and fielder are moving further apart. The logic of the proof remains the same, however, if the fielder views the flight path from an angle, and x refers to the distance between the fielder and the ball as before. The ball is then approaching the fielder if it has a component of its velocity towards the fielder. That is, if the trajectory makes an angle β to the line connecting the fielder and the ball, then the strategy of maintaining a negative $d(\tan \alpha)/dt$ will work so long as β is less than 90° .