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COMPARISON AND CONVERGENCE TO EQUILIBRIUM IN A NONLOCAL DELAYED REACTION-DIFFUSION MODEL ON AN INFINITE DOMAIN

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Abstract. We study a nonlocal time-delayed reaction-diffusion population model on an infinite one-dimensional spatial domain. Depending on the model parameters, a non-trivial uniform equilibrium state may exist. We prove a comparison theorem for our equation for the case when the birth function is monotone, and then we use this to establish nonlinear stability of the non-trivial uniform equilibrium state when it exists. A certain class of non-monotone birth functions relevant to certain species is also considered, namely birth functions that are increasing at low densities but decreasing at high densities. In this case we prove that solutions still converge to the non-trivial equilibrium, provided the birth function is increasing at the equilibrium level.

1. **Introduction.** In a recent paper, Gourley & So [1] derived a partial differential equation satisfied by the total number of mature adult members of a population on an infinite one-dimensional domain. Their formulation involves a distribution of possible ages of maturation and uses a probability density function on which ecologically realistic assumptions were made. In the situation when no positive equilibrium exists, they proved global attractivity of the zero equilibrium. Motivated by this study, we shall consider their equation and prove the attractivity of the non-zero steady-state when it exists.

The equation considered by Gourley and So [1] is

$$w_t - Dw_{xx} + dw = \int_0^\infty f(a)e^{-da} \int_{-\infty}^\infty b(w(t-a, x-y))\Gamma(a,y)\, dy\, da, \qquad (1)$$

where $w(t,x)$ is the total number of mature adults at (t,x), $D > 0$ and $d > 0$ are the diffusion and death rates respectively, $b(w)$ is the birth function, which satisfies $b(0) = 0$, $f(a) \geq 0$ is a probability density function satisfying

$$\int_0^\infty f(a)\, da = 1$$

and

$$\Gamma(a,x) = \frac{1}{\sqrt{4\pi Da}} e^{-x^2/4Da}. \qquad (2)$$

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