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Bifurcations on a spring-pendulum oscillator

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In this paper we study experimentally the dynamics of a non linear system: a spring pendulum coupled to an oscillator. The system was tested using a hybrid technique called Real Time Dynamic Substructuring (RTDS)[1, 2]. RTDS is a testing technique that involves splitting the system under study into two subsystems: one will be physically tested (physical substructure) and the other will be simulated in the computer (numerical model). These substructures interact in real time through a set of transfer systems. RTDS is a very powerful experimental methodology that not only allows full scale and real time testing but also real-time bifurcation tracking in complex engineering systems [3, 4]. In our hybrid experiment the spring pendulum is taken to be the physical substructure while oscillator is the simulated numerical model.

1 Description of the system

The complete system is shown schematically in figure 1. It consists of a spring pendulum with its pivot point connected to the mass *M*, belonging to the mass-spring-damper, characterized by *M*, *K* and *C*. The pendulum mass, *m*, is assumed to act at a single point and it is connected to the pivot point by a spring, k. Due to this spring connection the length of the pendulum is variable.

The system has three degrees of freedom and its dynamics are characterized by four frequencies: the external exciting one $\omega = \alpha sin(2\pi f t)$, and the natural frequencies of the linearized system in MSD, pendulum and spring modes $\omega_1 = \sqrt{\frac{K}{M}}$, $\omega_2 = \sqrt{\frac{g}{L}}$ and $\omega_3 = \sqrt{\frac{k}{m}}$, respectively.

2 Experimental results

To implement the real-time tasks a dSpace DS1104 RD controller board was used. MATLAB/Simulink was used to build the hybrid model. The transfer system consists of an electrically driven ball-screw actuator with an in line mounted synchronous servo motor controlled by a servo drive which applies a displacement to the pendulum pivot point in the vertical direction.

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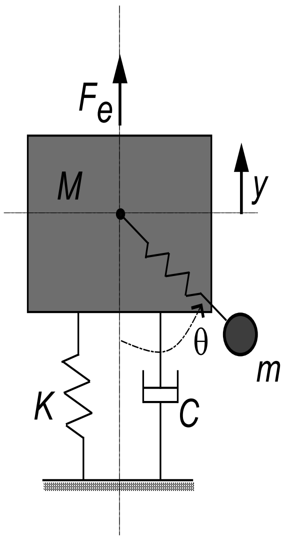

Fig. 1 Schematic representation of spring pendulum oscillator.

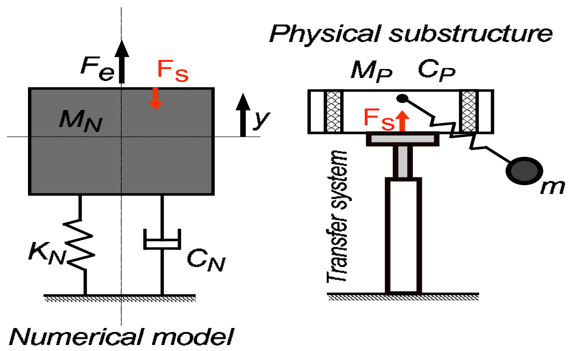

Fig. 2 Schematic representation of the substructures.

Fig. 3 Picture of the physical substructure.

Figure 3 shows a photograph of the experimental apparatus. The instrumentation used consists of a load cell to measure the force acting at the spring pendulum pivot, a LVDT displacement transducer connected to the platform to be able to track and control the actuator movement, a digital incremental encoder used to record both angular displacement and angular velocity of the pendulum and a computer vision system used to track the movement of the pendulum bob. Figure 4 shows experimental results for three different external forces. y denotes vertical displacement of the oscillator, θ is the pendulum angle and l is the pendulum length. From all this different solutions bifurcation diagrams can be made.

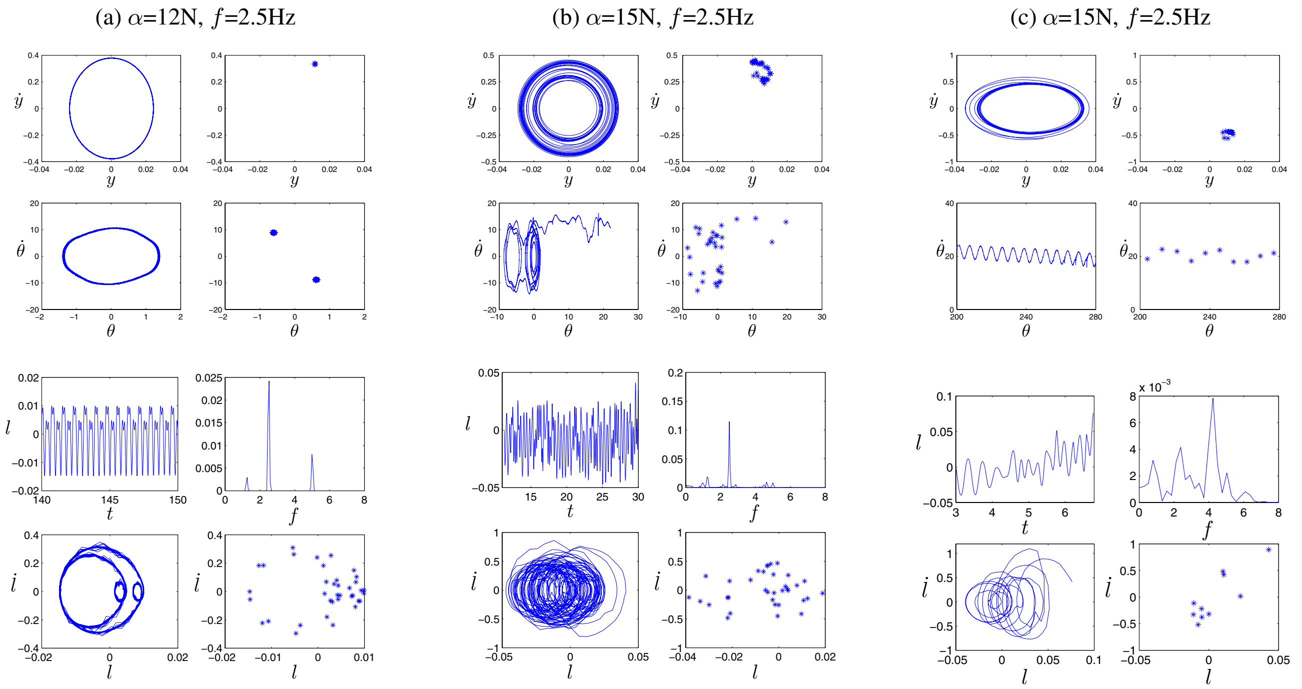

Fig. 4 Hybrid experiment results. Phase portraits and $Poincaré$ sections.

3 Conclusions

This paper describes how real time dynamic substructuring is used to test nonlinear systems and how experimental bifurcations diagrams can be built.

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