Lecture 5:

Heteroclinic vs. HH the pheromone sub-system

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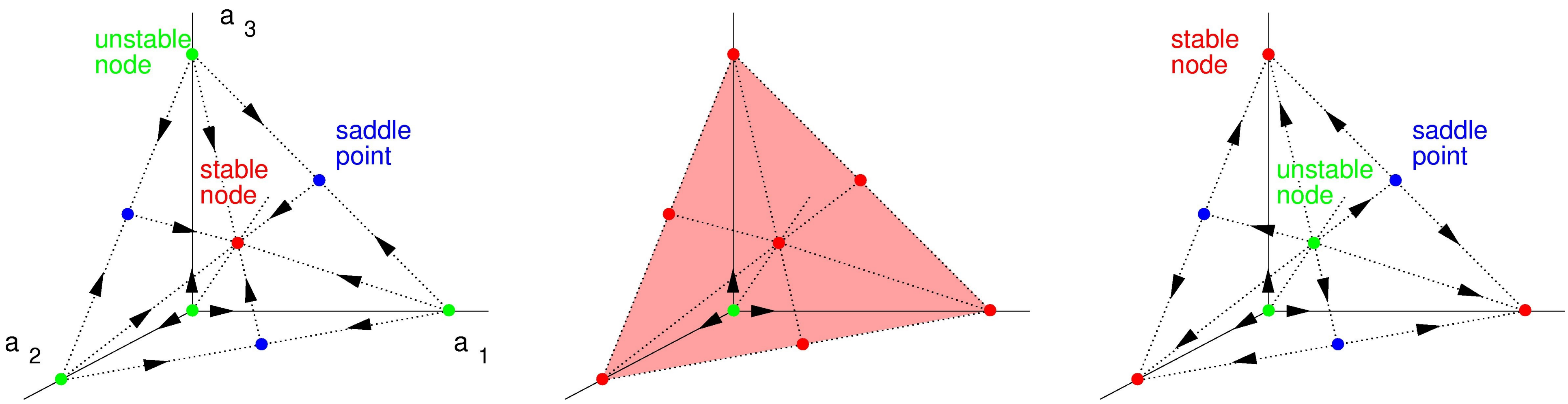
Yesterday ...

- Heteroclinic orbits exist in the generalized Lotka-Volterra model with asymmetric synaptic connections
- Under certain conditions the heteroclinic can be a global attractor
- Now ... does something like this exist for more realistic Hodgkin-Huxley neurons?

Reminder: Lotka-Volterra Model

$$\frac{a_i(t)}{dt} = a_i\left[1 - \left(a_i + \sum_{i \neq j}^{N} \rho_{ij} a_j\right)\right]$$

Symmetric connections: $\rho_{ij} = \rho$



$\rho < \rho_c$ $\rho = \rho_c$ $\rho > \rho_c$

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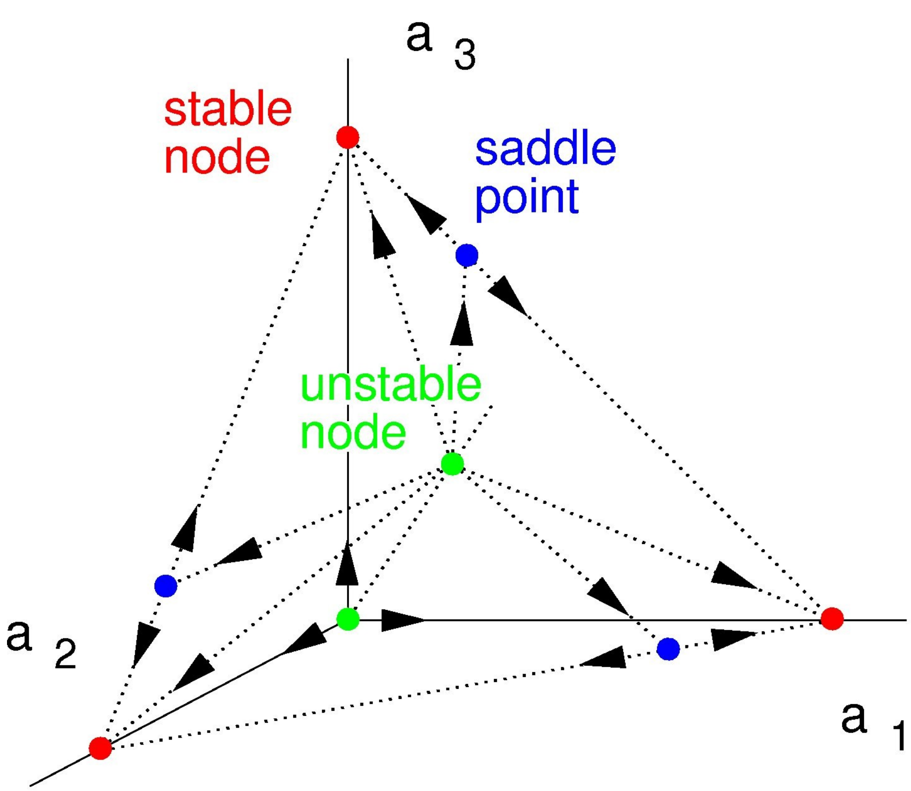
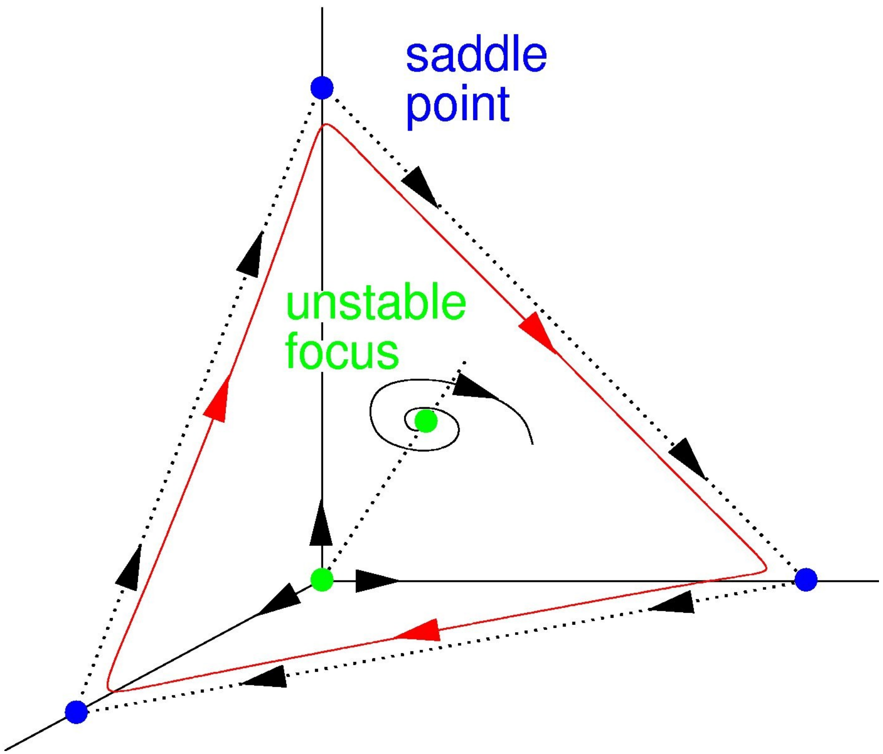
Reminder: Asymmetric connections



$$\rho_{\mathrm{fwd}} > \rho_{\mathrm{bwd}}$$



$$\rho_{\mathrm{fwd}} \gg \rho_{\mathrm{bwd}}$$

Heteroclinic for HH neurons?

- Problem: For HH neurons, saddle points are "saddle limit cycles" and analytically and numerically intractable
- Approach: Find an "equivalent rate model" that has heteroclinic structure:

(The following is published in:
Nowotny and Rabinovich, *Dynamical Origin of Independent Spiking and Bursting Activity in Neural Microcircuits,* Phys Rev Lett **98**, 128106 (2007))

Individual neurons

$$C\frac{dV(t)}{dt} = -I_{\mathrm{Na}} - I_{\mathrm{K}} - I_{\mathrm{leak}} + I_{\mathrm{syn}}$$

$$I_{\mathrm{Na}}(t) = g_{\mathrm{Na}}\, m(t)^3\, h(t)\, \big(V(t) - E_{\mathrm{Na}}\big)$$

$$I_{\mathrm{K}}(t) = g_{\mathrm{K}}\, n(t)^4\, \big(V(t) - E_{\mathrm{K}}\big)$$

$$I_{\mathrm{leak}}(t) = g_{\mathrm{leak}}\, \big(V(t) - E_{\mathrm{leak}}\big)$$

Traub and Miles, Neural Networks of the Hippocampus, Cambridge University Press, 1991

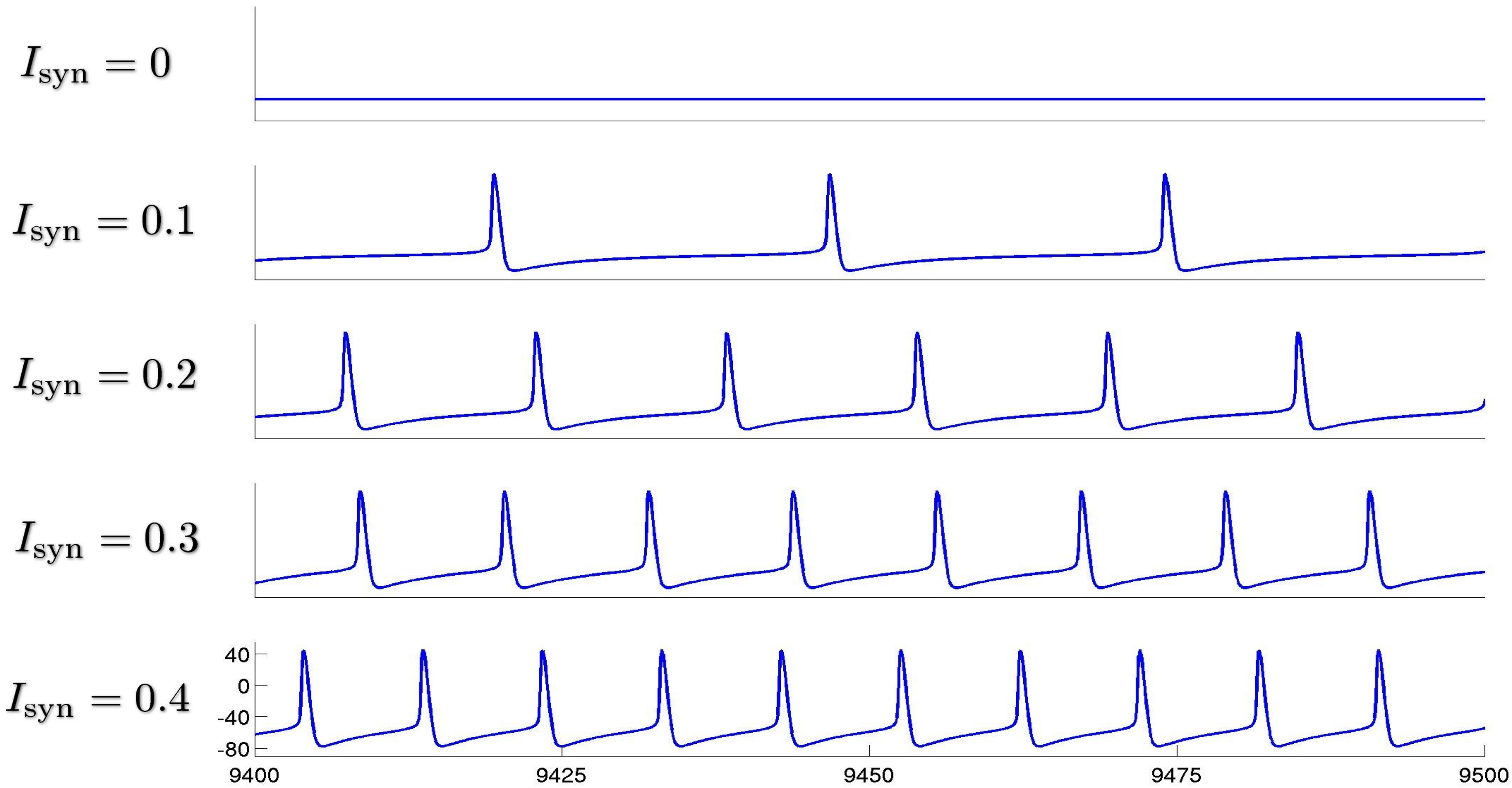
Individual neurons: Activation and Inactivation

$$\frac{dy(t)}{dt} = \alpha_y\left(V(t)\right)\left(1 - y(t)\right) - \beta_y\left(V(t)\right) y(t)$$

$$\alpha_m = 0.32(-52 - V)/(\exp((-52 - V)/4) - 1)$$
$$\beta_m = 0.28(25 + V)/(\exp((25 + V)/5) - 1)$$
$$\alpha_h = 0.128\exp((-48 - V)/18)$$
$$\beta_h = 4/(\exp((-25 - V)/5) + 1).$$

$$\alpha_n = 0.032(-50 - V)/(\exp((-50 - V)/5) - 1)$$
$$\beta_n = 0.5\exp((-55 - V)/40)$$

Example waveforms



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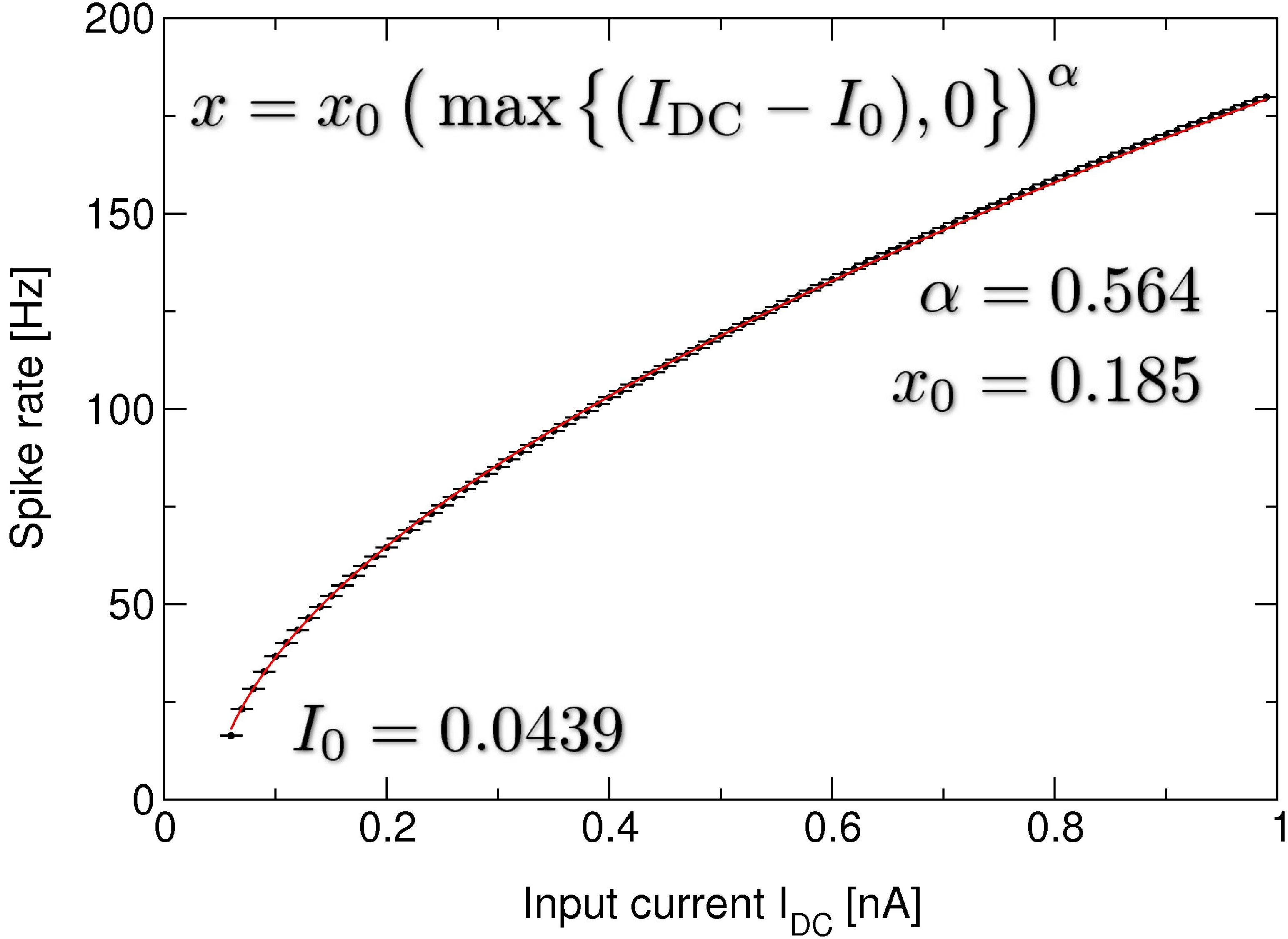
Spike rate



$x = x_0 \left(\max\{(I_{DC} - I_0), 0\} \right)^\alpha$
$\alpha = 0.564$
$x_0 = 0.185$
$I_0 = 0.0439$
Spike rate [Hz]
200
150
100
50
0
0
0.2
0.4
0.6
0.8
1
Input current I_{DC} [nA]

Synapses

$$I_{\mathrm{syn},ji} = g_{ji}\, S_i\, (V_j - V_{\mathrm{rev}})$$

$$\tau \frac{dS_i}{dt} = (R_i - \kappa S_i) \frac{S_{\max} - S_i}{S_{\max}}$$

$$\tau = 50\,\mathrm{ms}$$

$$\tau \frac{dR_i}{dt} = \Theta(V_i - V_{\mathrm{th}}) - R_i,$$

Modified from: W. Rall, J Neurophysiol 30:1138 (1967).

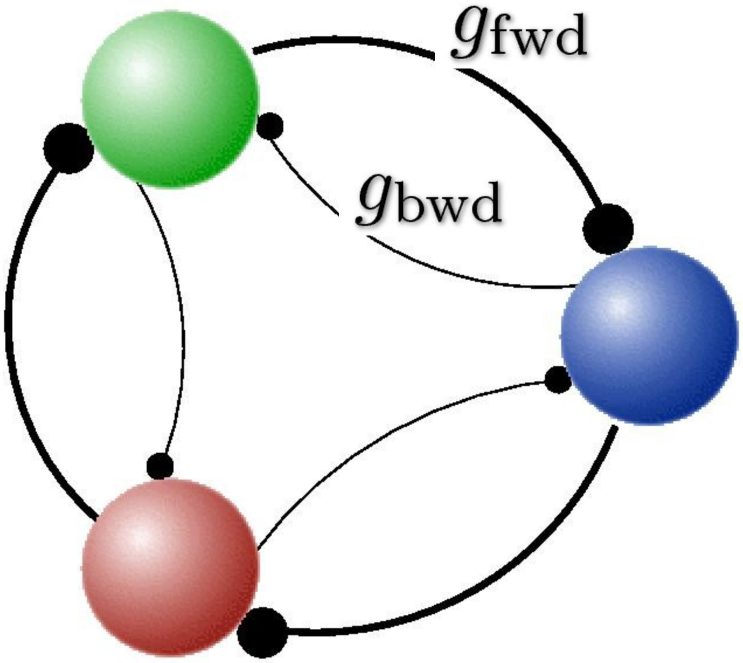
Summary

Neurons: $V_i(t),\ m_i(t),\ h_i(t),\ n_i(t)$ $\quad i = 1 \ldots 3$

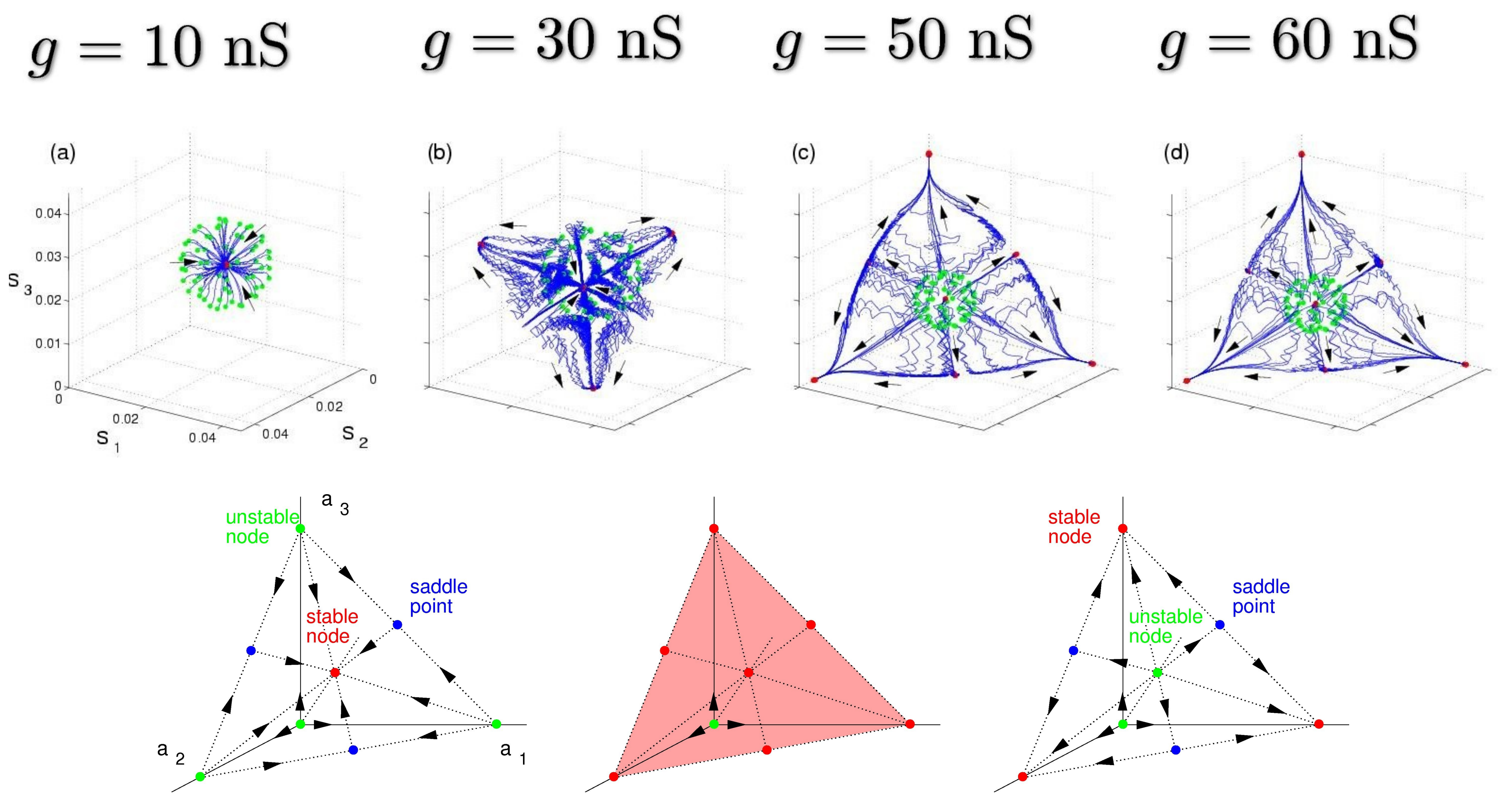
Synapses: $R_i(t),\ S_i(t)$ $\quad i = 1 \ldots 3$

Total: 18 dimensional system

Main parameters: g_{fwd} $\quad g_{\text{bwd}}$



Symmetric inhibition



Lotka-Volterra model

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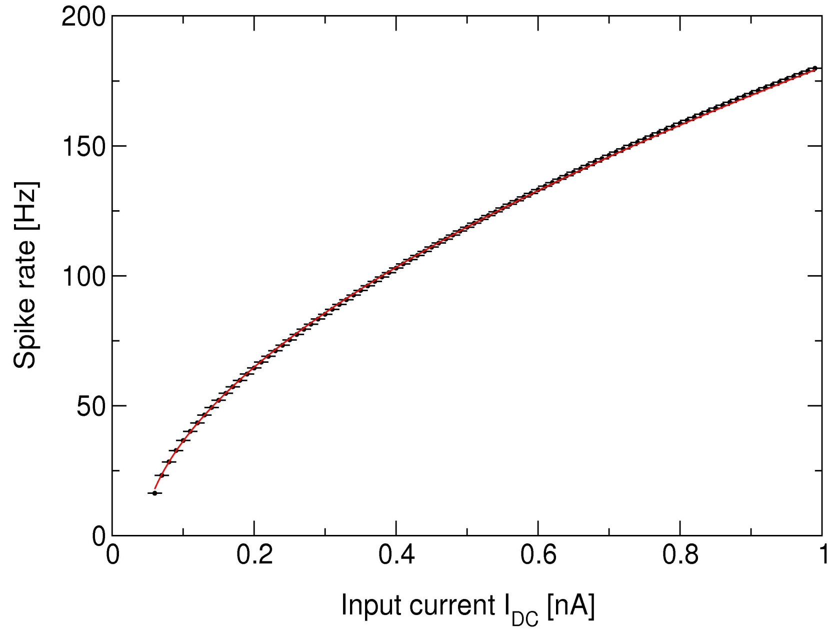
Reduction to a rate model



$$x = x_0 \left(\max\left\{ (I_{\mathrm{DC}} - I_0), 0 \right\} \right)^{\alpha}$$

Ansatz: $\tau \dfrac{dr_i}{dt} \approx a(x_i) - r_i$

Then require that for tonic spiking the spiking model and rate model match, leading to

$$a(x_i) = \frac{1 - \exp(-\tau_{\mathrm{spike}}/\tau)}{1 - \exp(-1/(x_i \tau))} \approx c\, x_i\, \tau$$

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Equivalent rate model

Pre-synaptic activation

Post-synaptic activation

$$\tau \frac{ds_i}{dt} = (r_i - \kappa s_i) \frac{S_{\max} - s_i}{S_{\max}}$$

$$\tau \frac{dr_i}{dt} = \tilde{x}_0 \left(\tilde{I} - \sum_j \tilde{g}_{ij} s_j \right)_+^{\alpha} \tau - r_i$$

The variables in this “rate model” are the synapse activation variables!

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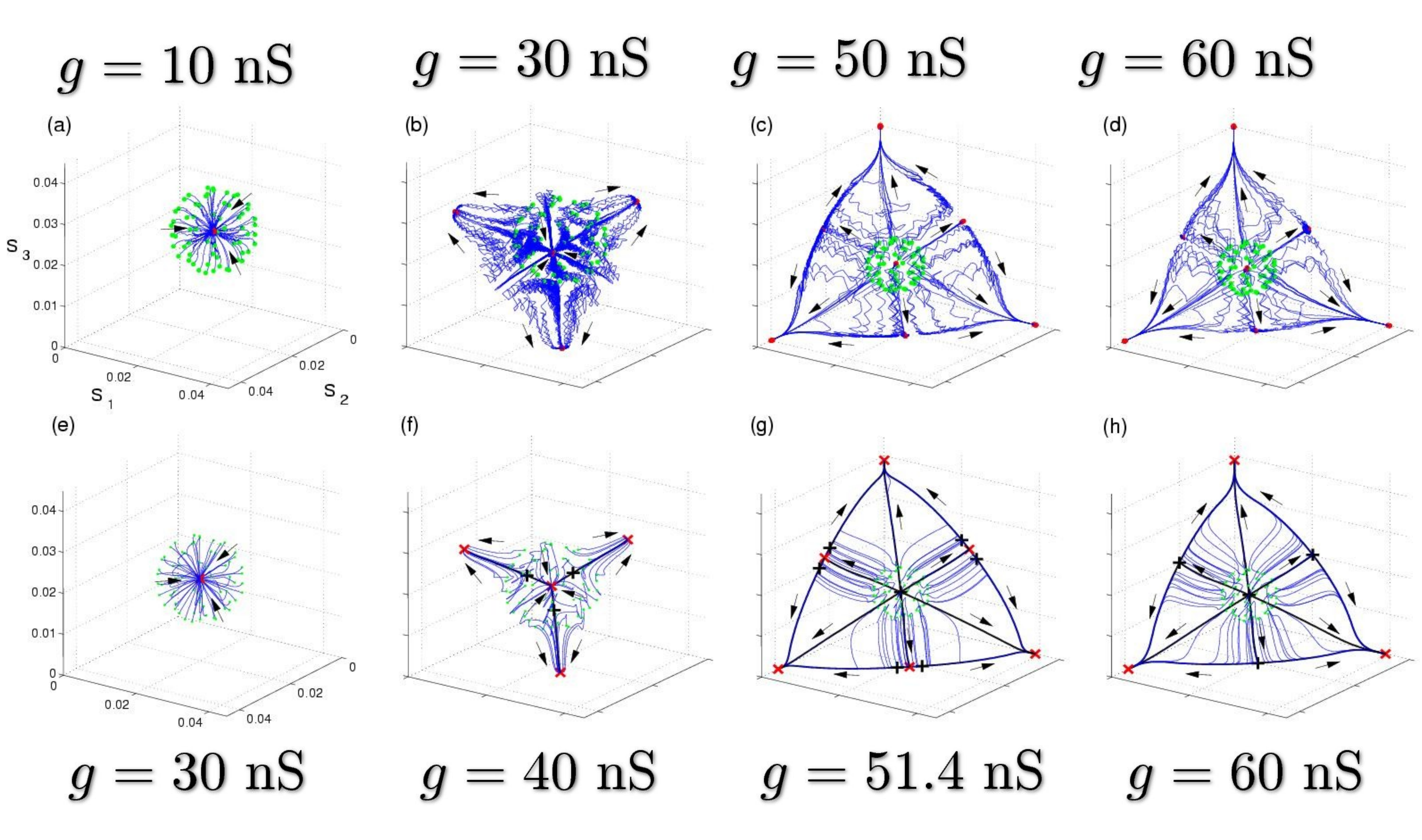
Comparison



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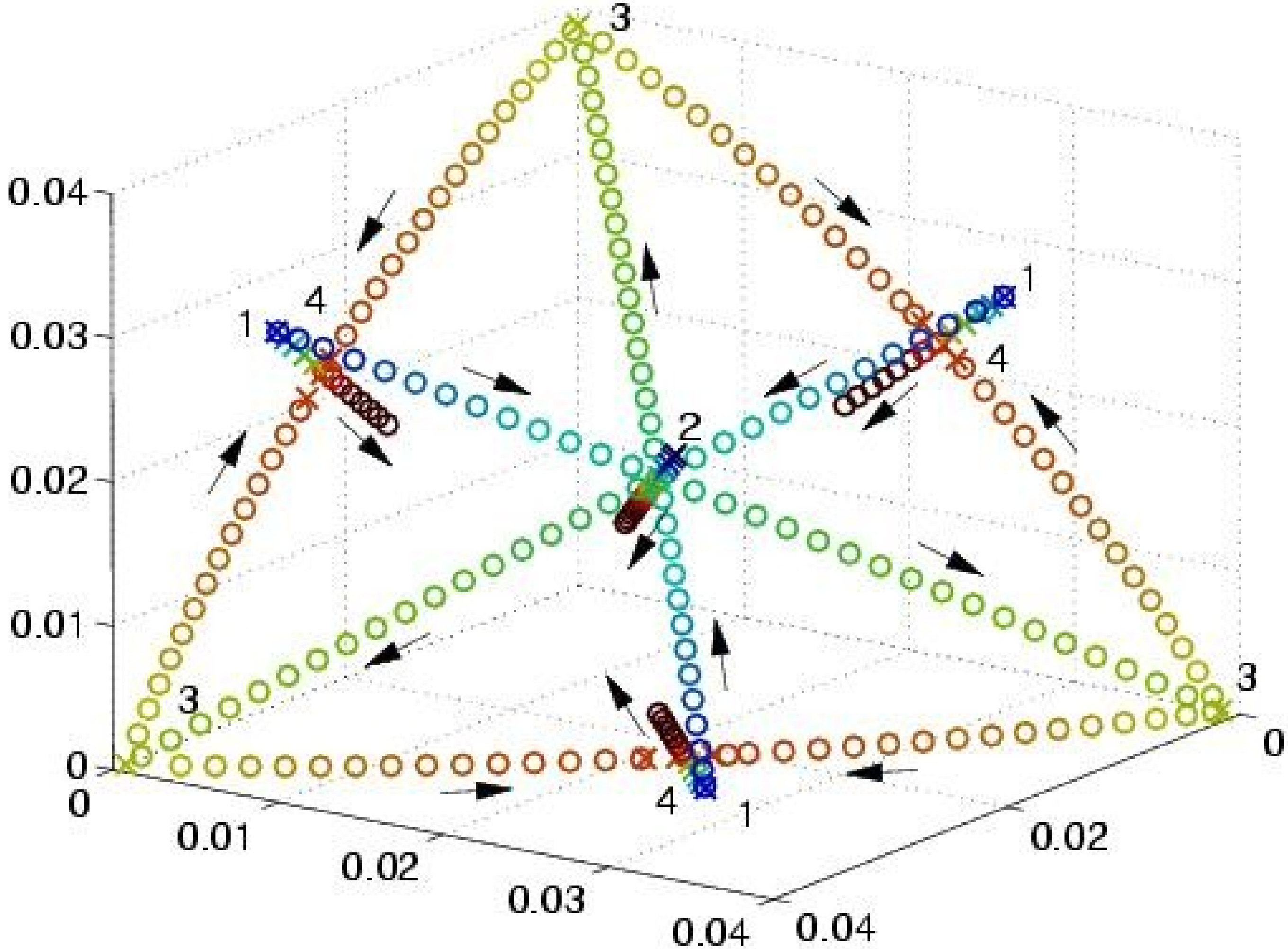
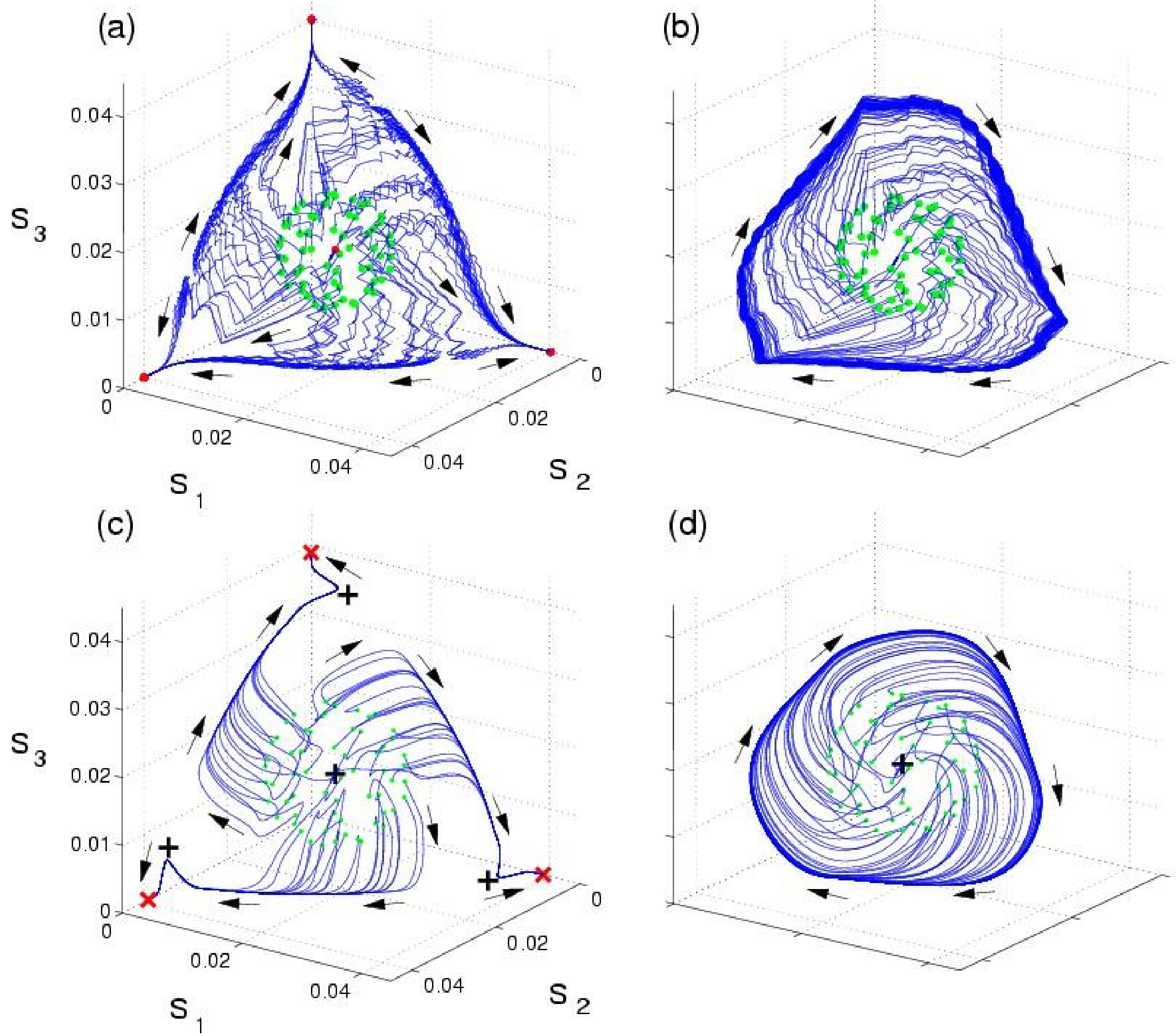
Numerical bifurcation analysis



Asymmetric connections



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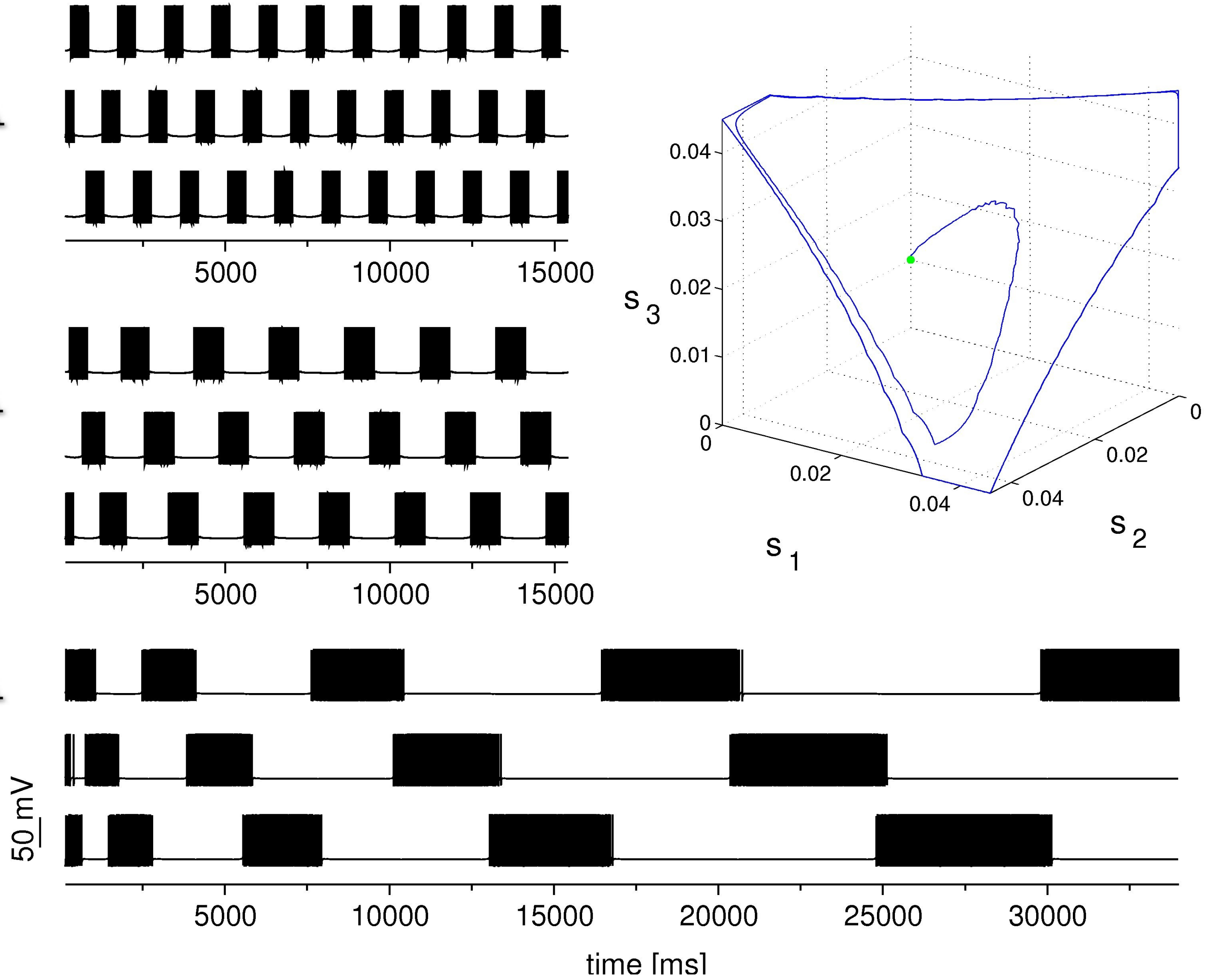
Stable heteroclinic orbit (?)

$I_{\mathrm{DC}} = 0.08\ \mathrm{nA}$

$I_{\mathrm{DC}} = 0.16\ \mathrm{nA}$

$I_{\mathrm{DC}} = 0.22\ \mathrm{nA}$



Conclusions: Heteroclinic in HH model

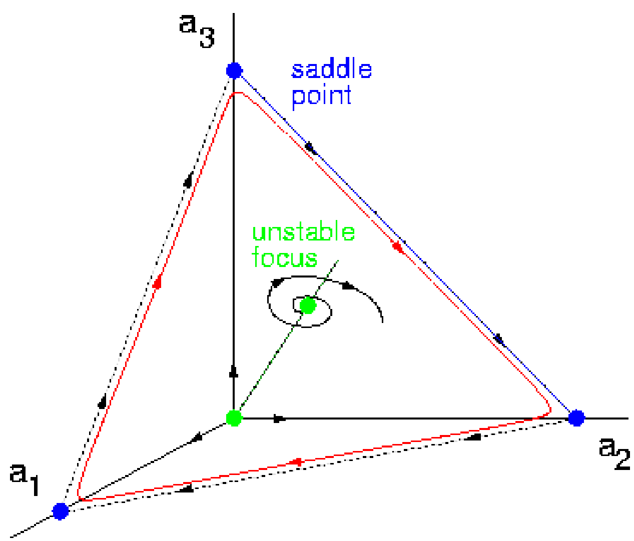
- The reduction from the spiking model (18D) to the rate model (6D) seems successful
- The bifurcation analysis in 6D shows the expected structures, similar to LVm
- The spiking model can be pushed into a regime where its trajectory seems to approach a heteroclinic.

- However, *the last point is contingent on the modification of the synapse model*

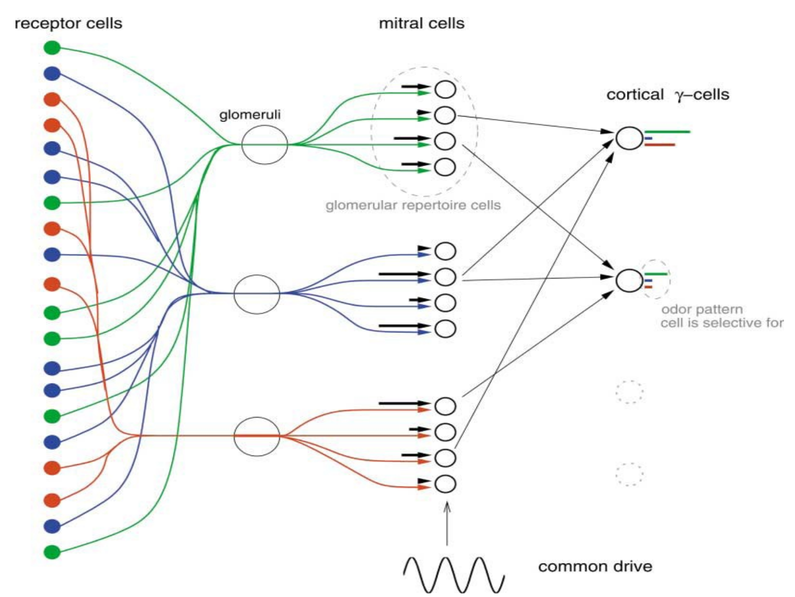
Pheromone Subsystem

Last time ... general olfactory system

Winnerless competition



Hopfield's model of olfaction



- This was all about the general olfactory system
- Today: Pheromone sub-system

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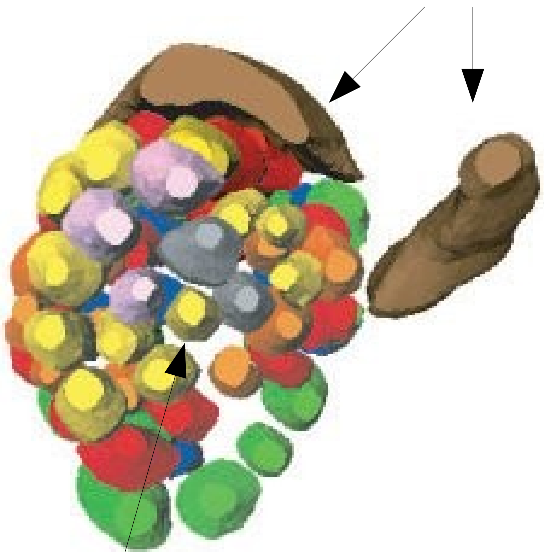
Role of pheromones

- Pheromones are substances that animals secrete to communicate with each other
- Examples
  - Pheromone trails laid by ants
  - Pheromones in urine to mark territory
  - Sexual pheromones to communicate mating status
  - Sexual pheromones to attract mates
  - ...

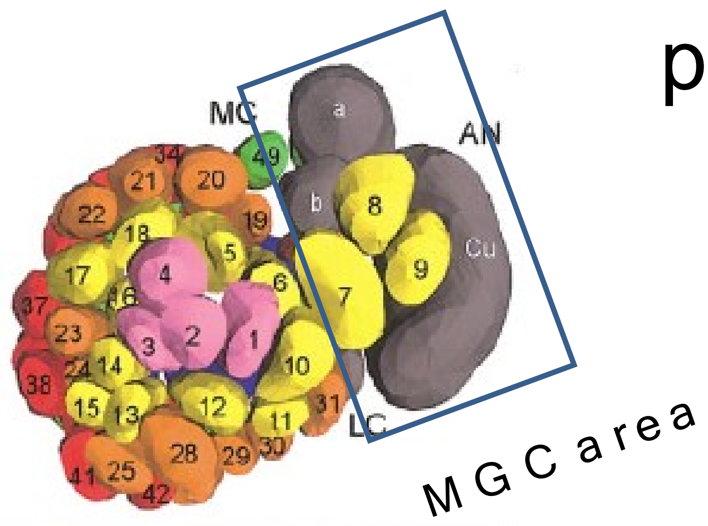
Pheromone sub-system: Anatomy

Pheromone sub-system:
Macro-glomerular Complex (MGC)



In insects there are typically 2-3 large glomeruli exclusively dedicated for pheromone processing

General antennal lobe



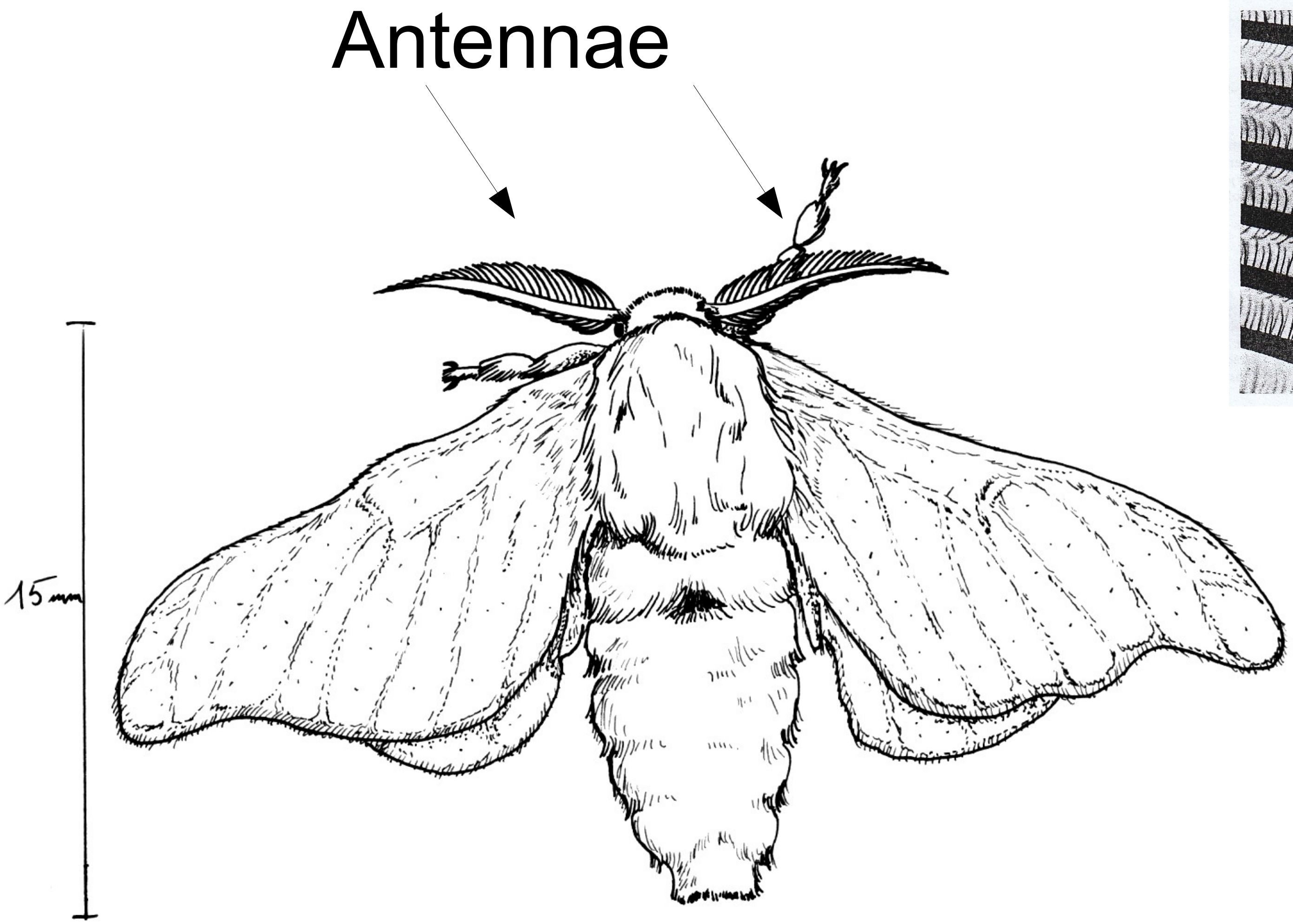
Pheromone sub-system in moths

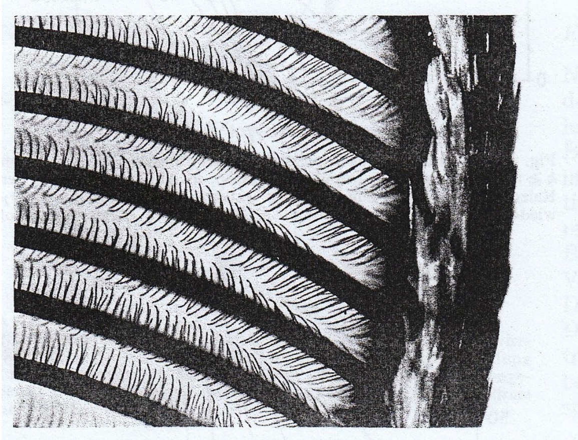
- Here I will concentrate on the pheromone system of moths
- Pheromone is secreted by females to attract males
- Males can smell the pheromone at distances of up to 2-3 miles
- Pheromone blend consists of several chemicals (components), in a typical ratio
- Related (but distinct) species may use the same chemicals but in a different ratio (!)

Pheromone subsystem is different

- General olfactory system
  - ORN broadly tuned
  - One ORN type – one glomerulus
  - Need to recognize pure odors, mixtures, concentrations
- Pheromone sub-system
  - ORN very narrowly tuned to 1 chemical
  - One ORN type – one glomerulus
  - Need to recognize one specific mixture of pheromone components

Bombyx Mori (Silk Moth)





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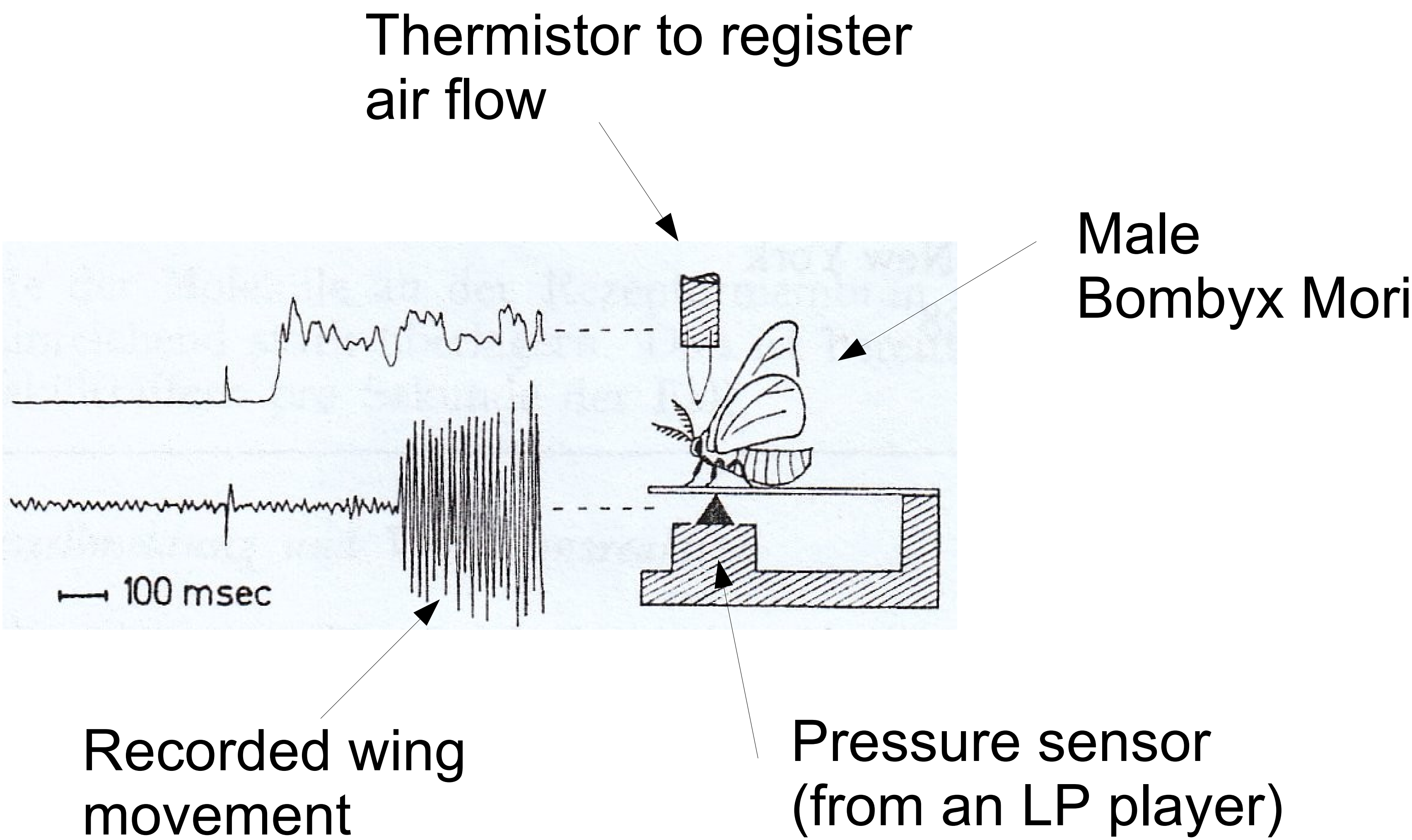
Classic results on Bombyx Mori

In the following we will have a look at classic results from

Kaissling K-E and Priesner E, “Die Riechschwelle des Seidenspinners”, Die Naturwissenschaften **57**(1): 23-28, 1970.

Type of experiments: Behavioral

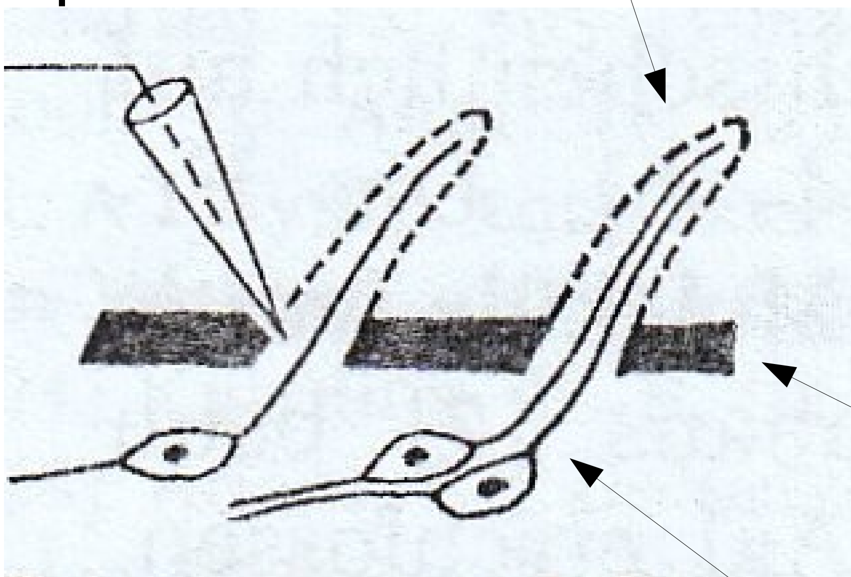


Kaissling & Priesner

Type of experiments: Electrophysiology

Sensillum

To Amplifier



Cuticle

ORN

Extra-cellular recordings

Only spikes visible

Kaissling & Priesner

ORN spikes

Stimulus

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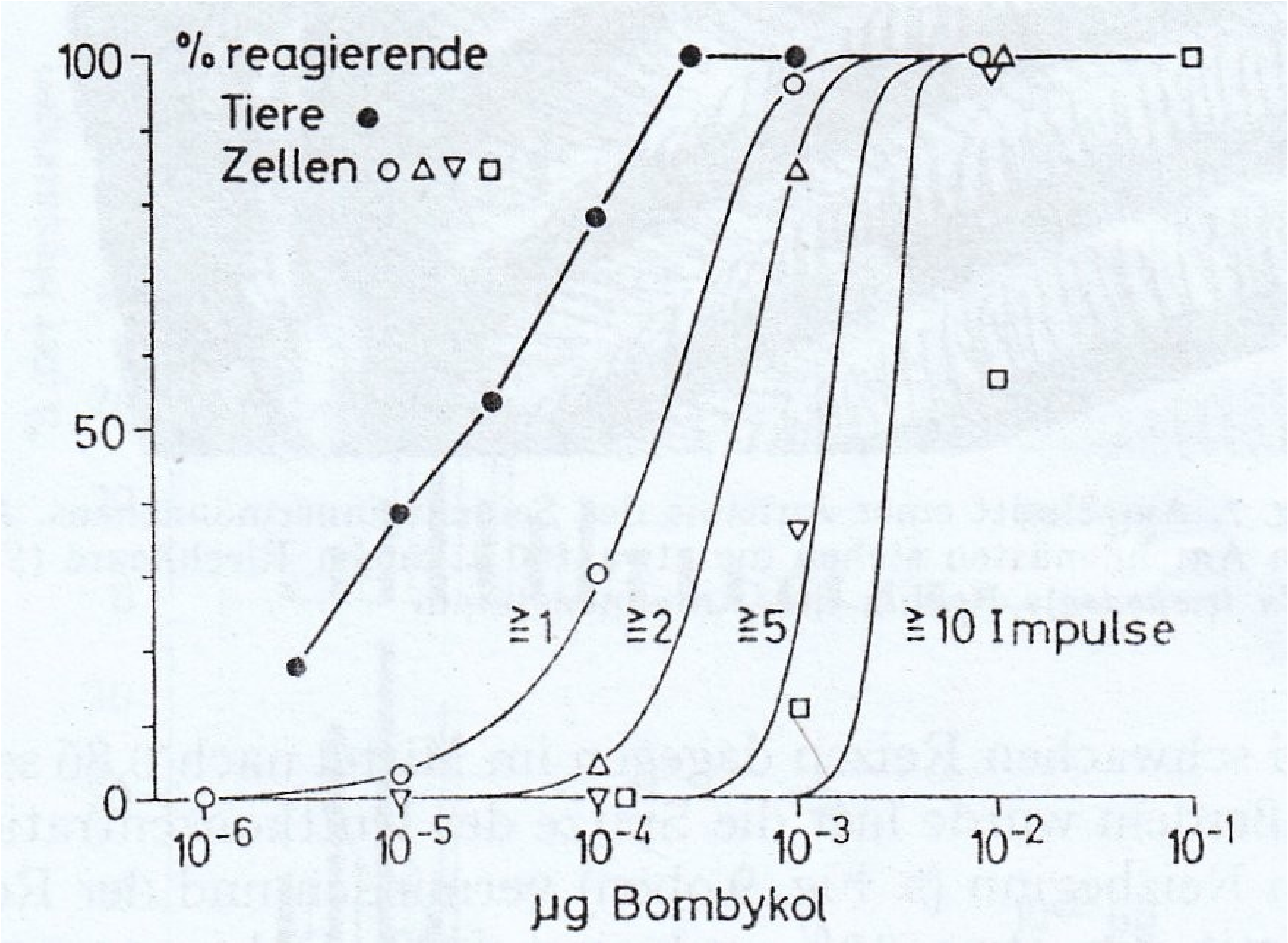
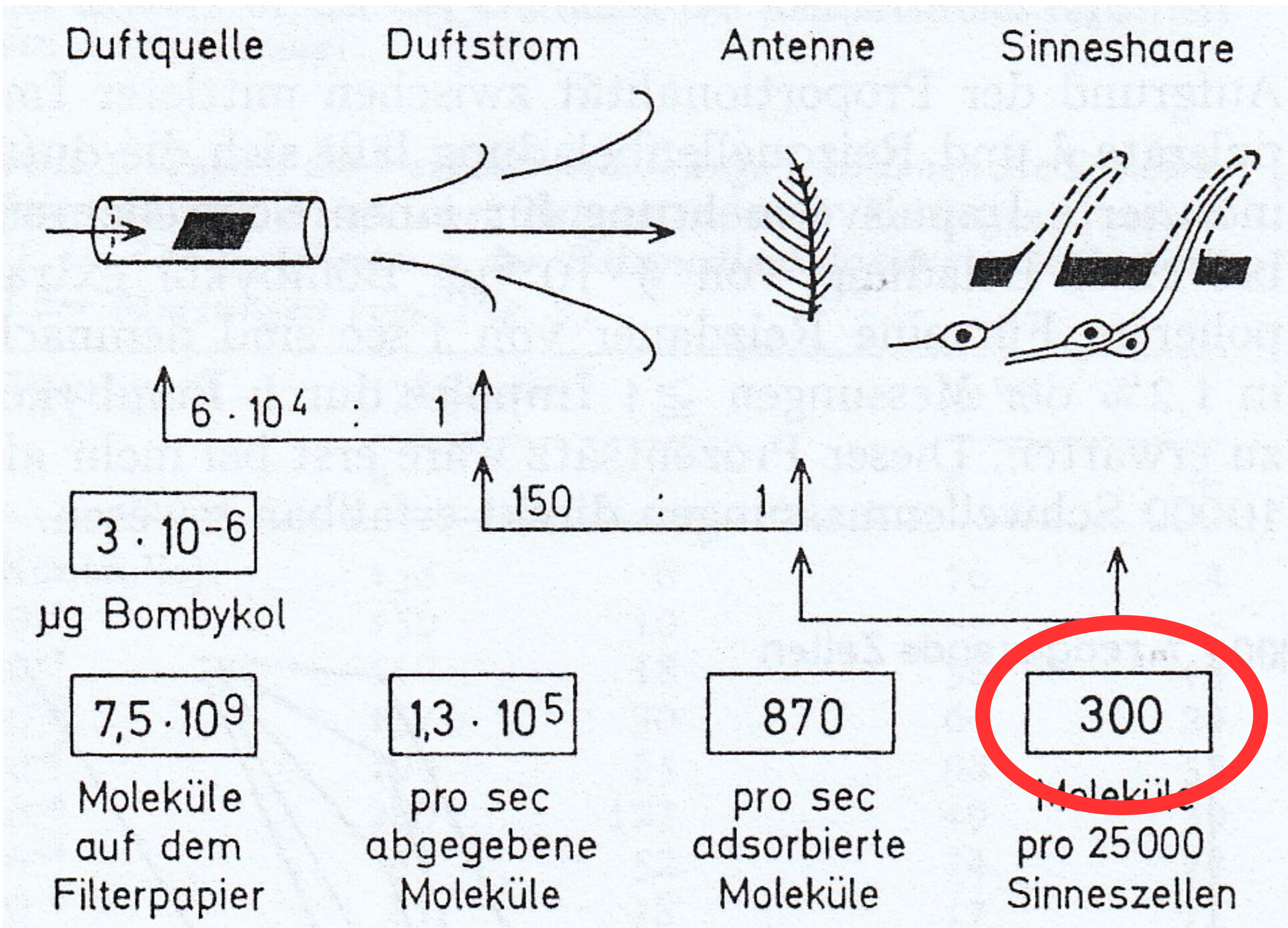
Sensitivity (behavior and ORN response)



Kaissling & Priesner

Analysis with radio-actively labeled pheromone



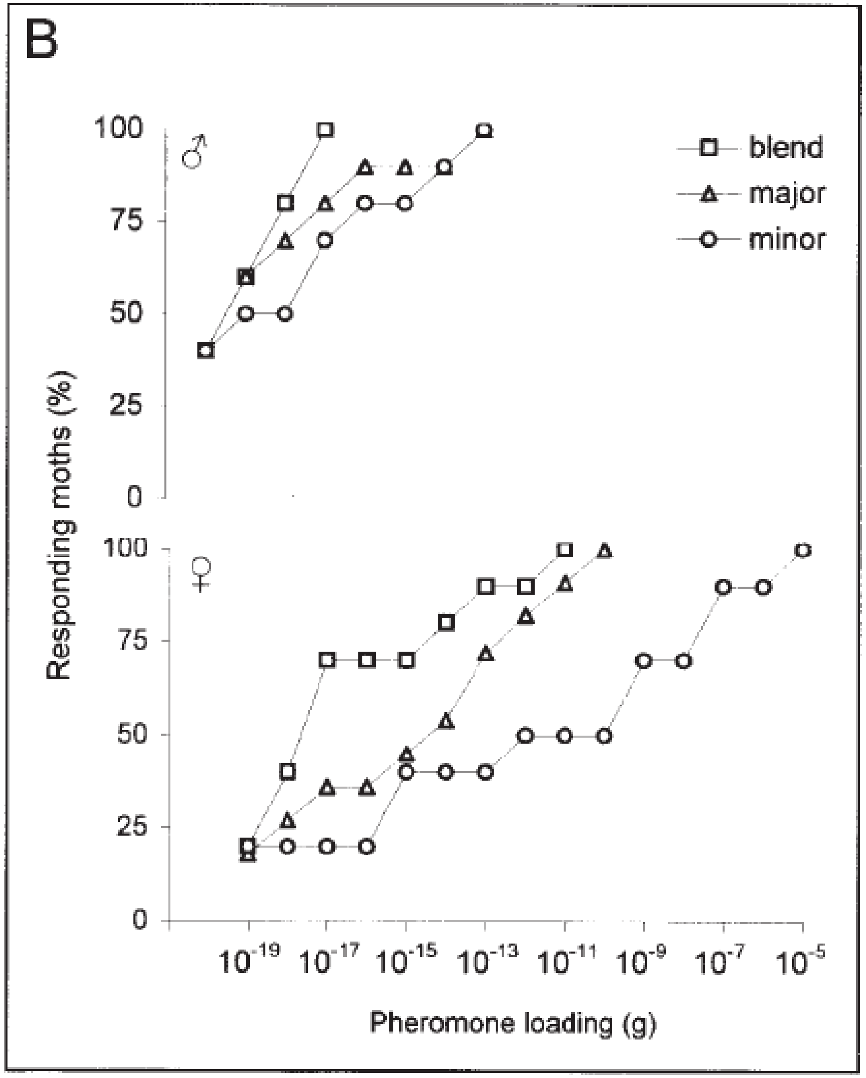
Kaissling & Priesner

More recent study

Angioy AM et al. “Extreme Sensitivity in an Olfactory System”, Chem Senses **28**:279-284 (2003)

- Similar experiments but measuring the heartbeat of the moths (rather than the actual behavior)
- Experiments on *Spodoptera Littoralis* (cotton leafworm)

Results



Male moths seem to respond from $< 10^{-18}$ g (10^{-9} ng);
6 molecules on antenna (!)

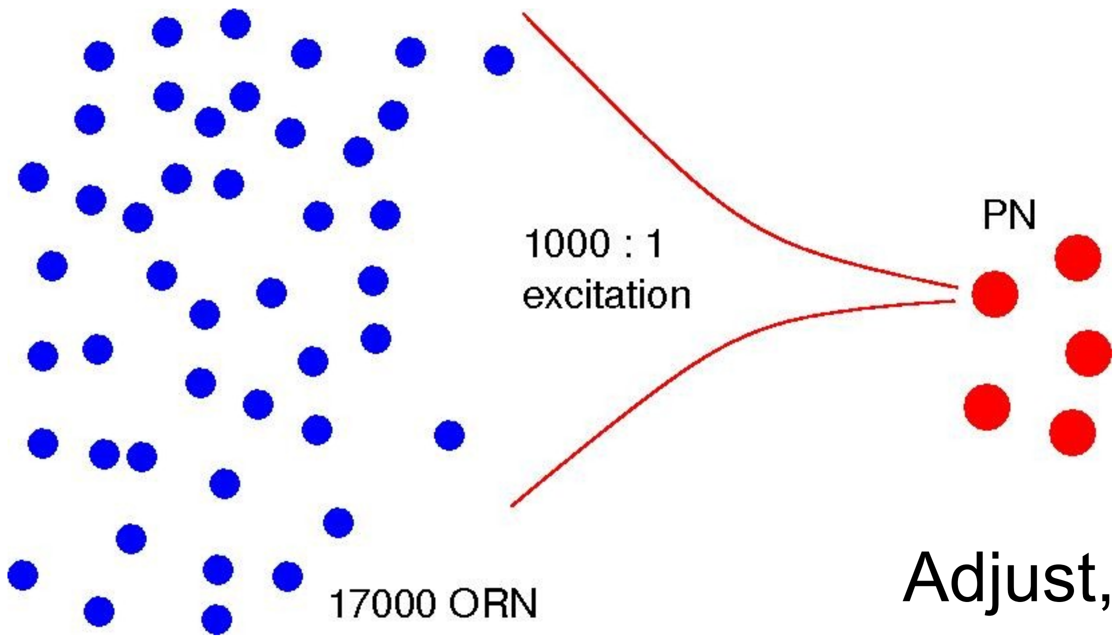
(Kaissling: $3 \cdot 10^{-3}$ ng;
300 molecules on antenna)

Angioy et al, 2003

Sensitivity analysis

- Is the convergence of ~20000 ORN to less than 100 PN sufficient to explain response threshold at 300 molecules?

Simple statistical model:



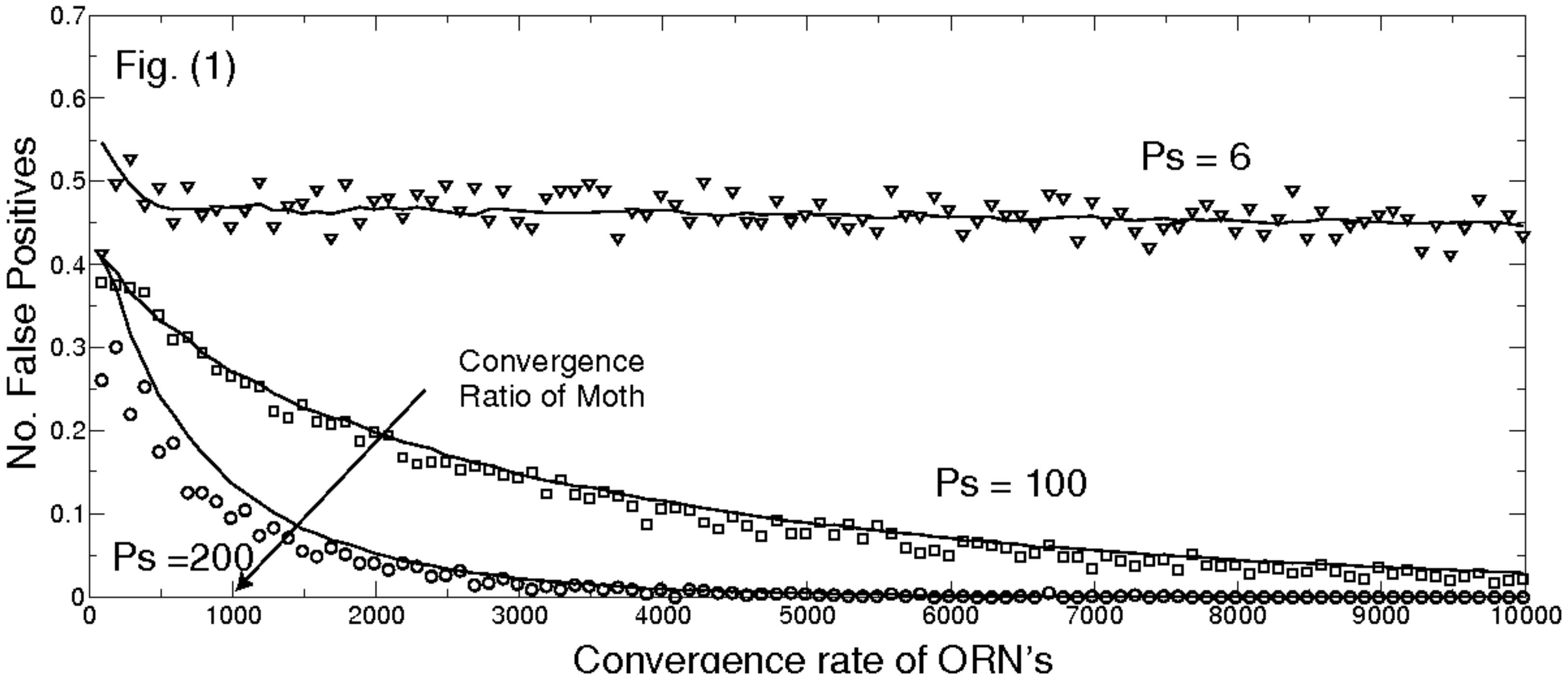
1600 spikes/s at baseline

1600+P_S spikes/s with pheromone

Adjust, such that 50 % response

Result on convergence sufficiency

Calculating the number of false positives:



Dr. Chris Buckley (Sussex)

http://www.informatics.sussex.ac.uk/research/projects/PheroSys

Conclusion so far

- s convergence enough to explain 200 molecule threshold? - maybe

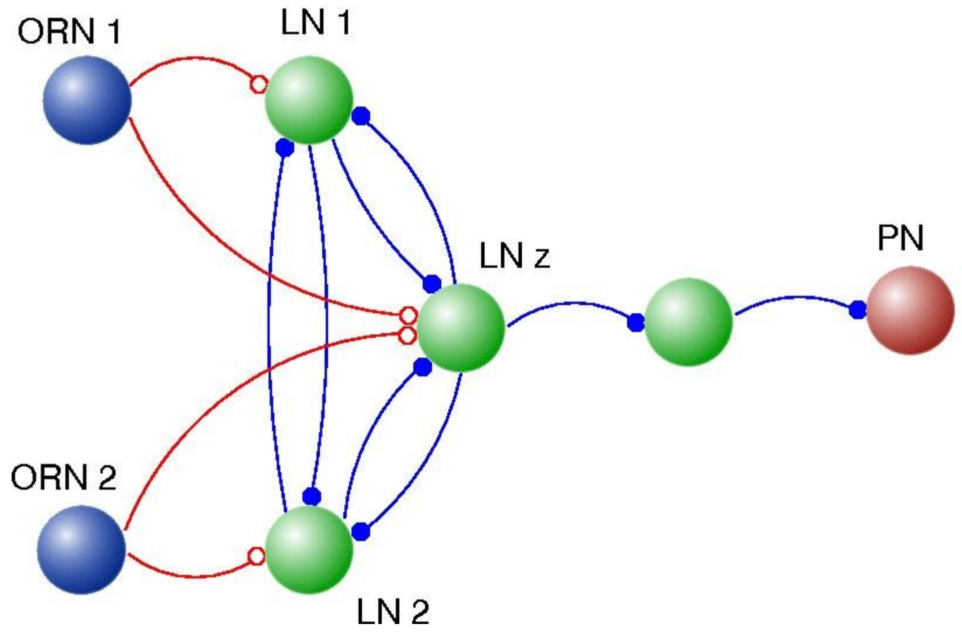
- Is convergence enough to explain 6 molecule threshold? - no!

- ... to be continued

http://www.informatics.sussex.ac.uk/research/projects/PheroSys/

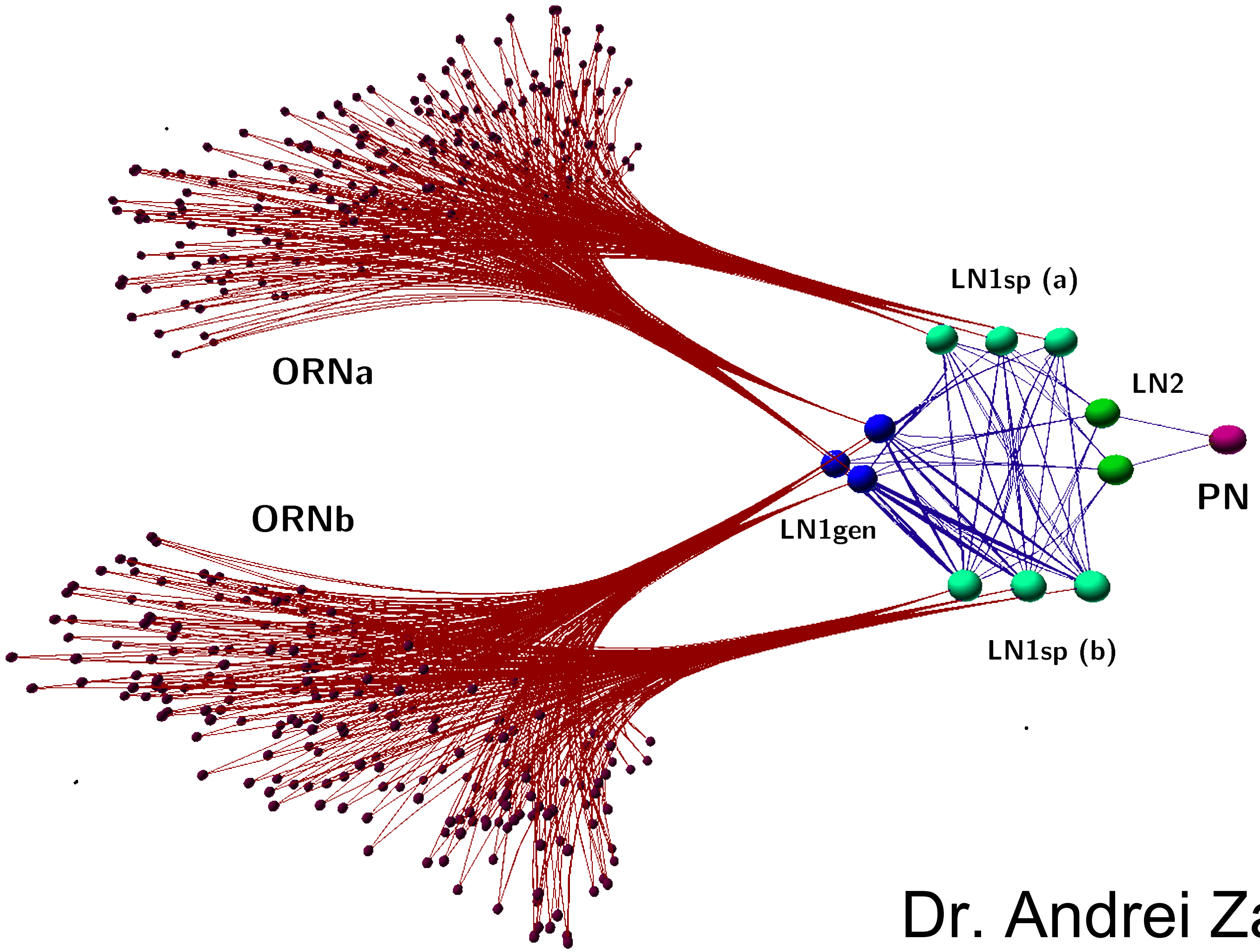
Ratio coding

- It is essential that moths can recognize the correct ratio over a large range of different concentrations
- In principle, this can be solved by winner-take-all competition, e.g.



See, e.g. Kwok YC, Encoding of Odor Blends in the Moth Antennal Lobe, PhD Thesis, University of Leicester, 2007

HH based model



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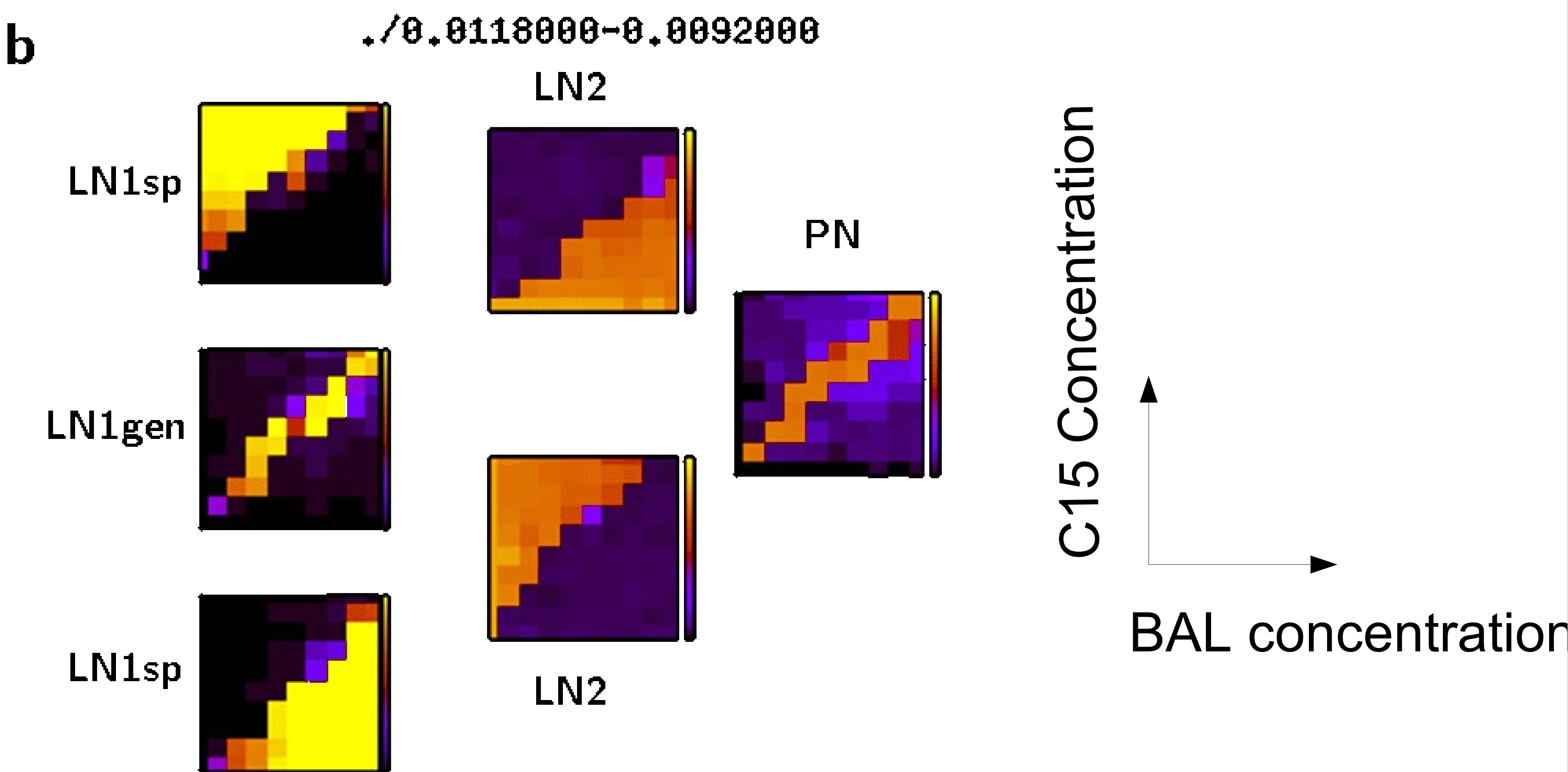
Preliminary Results



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Ongoing work

- Optimize model with automated parameter estimation
- Analyze emerging synchronization phenomena
- Different ratios
- Generalization to multiple ratios

http://www.informatics.sussex.ac.uk/research/projects/PheroSys/

Discussion

- Existence of human sexual pheromones still debated
- Note: If they exist, it is still unclear what role they may play: It is unlikely that it is to find females like moths do