

A Comparison of the Iterative Fourier Transform Method and Evolutionary Algorithms for the Design of Diffractive Optical Elements

Philip Birch, Rupert Young, Maria Farsari, David Budgett, John Richardson, Chris
Chatwin

School of Engineering,

University of Sussex

Brighton

BN1 9QT

United Kingdom

Tel: +44 (0)1273 606755

Fax: +44 (0)1273 690814

e-mail: P.M.Birch@sussex.ac.uk

R.C.D.Young@sussex.ac.uk

C.R.Chatwin@sussex.ac.uk

Abstract

Three methods of designing diffractive optical elements (DOE) are compared. The iterative Fourier transform algorithm (IFTA) is compared to an evolutionary strategy (ES) approach and a combination of both methods. It is shown that the combination of both methods produces a better solution than the iterative method and is faster than using the evolutionary strategy only.

Keywords

Diffractive Optics

Introduction

DOEs have many possible applications in modern optics. They have been used as lenses, imaging optics, gratings, and beam splitters for optical communications, etc. A common use is beam shaping. Lasers commonly produce a Gaussian intensity profile. For imaging, a uniform intensity pattern would be more useful. Although the centre of the beam can be sampled to achieve this, it is very light inefficient. To obtain a beam with only 10% intensity variation across its waist, 90% of the input light is lost. A more efficient method is to use a phase encoded DOE that transforms the Gaussian beam to a uniform intensity profile.

Analytic solutions do not exist for many designs of DOEs so an alternative method is

required. The traditional method of designing a DOE is using the IFTA method^{1, 2, 3}. For binary only devices, other methods exist such as direct searching⁴, and simulated annealing⁵, however they are too computationally expensive when real numbers are required. A common real number optimisation method is the use of an ES⁶. In this paper three methods of designing a DOE will be compared: the ITFA method; an ES, and a combination of both.

The Iterative Fourier Transform Algorithm

The basic algorithm for the IFTA is as follows. The input beam is given by

$$a_1(\mathbf{x}) = a_{0,1}(\mathbf{x}) \exp[i\phi_1(\mathbf{x})] \quad [1]$$

where \mathbf{x} is position co-ordinate, a_0 is the amplitude function and ϕ is the phase function.

This function is then propagated forwards by means of a Fourier transform to

$$A_i(\mathbf{k}) = A_0(\mathbf{k}) \exp[i\Phi(\mathbf{k})] \quad [2]$$

$A_0(\mathbf{k})$ is then replaced with the desired output amplitude $B_0(\mathbf{k})$ and the new complex amplitude function is propagated back to the input plane with an inverse Fourier transform

$$a_2(\mathbf{x}) = a_{0,2}(\mathbf{x}) \exp[i\phi_2(\mathbf{x})] \quad [3]$$

$\phi_2(\mathbf{x})$ then replaces $\phi_1(\mathbf{x})$ in equation [1] and the process is repeated until $A_0(k)$ equals $B_0(k)$.

The major problem with this method is that the algorithm can stagnate before a solution is reached. There is not necessarily any phase screen that can exactly transform a Gaussian beam into one with the desired intensity pattern. Stagnation is due to the possibility that the algorithm will find one of any stopping points but the stopping point may not necessarily be the global optimum⁷.

Evolutionary Strategies

ES are commonly grouped together with other evolutionary algorithms such as evolutionary programming and genetic algorithms (GA). ES are usually for real number optimisation. Each algorithm has a biological analogy. In the case of the ES, it is as follows. The population consists of a set of vectors. Each vector represents an individual of the species; the elements of the vectors represent the phenotypic traits of the individuals (not the genes). Each of the individuals competes against each other, and the fittest individuals survive to sexually recombine to produce the next generation. The major difference between a GA and an ES is that the ES is mainly dependant on mutation of the data while the GA depends on different combinations of the same data. The GA is more suited to optimising binary systems since a real number array would require a very

large number of initial trial solutions if it were dependant on recombination only.

The algorithm is as follows

1. Generate a set of μ parent vectors (where μ is number typical in the range $5 < \mu < 20$).
These are usually taken as random numbers between the range of possible solutions (0 to 2π in this case).
2. Recombine the μ parents to produce λ offspring (where λ is a number greater or equal to μ). In this case two parents were chosen at random and the mean of each vector was used as the offspring. This was repeated until λ offspring were produced.
3. Mutate the offspring. If x_i is the i^{th} element of an offspring vector, the mutated element x_i' is given by⁶

$$\sigma_i' = \sigma_i \exp[\tau' N(0,1) + \tau N_i(0,1)] \quad [4]$$

$$x_i' = x_i + \sigma_i' N(0,1) \quad [5]$$

where $N(0,1)$ is a random number with normal distribution, σ_i is the mutation parameter of the previous generation, and

$$\tau = \frac{C}{\sqrt{2\sqrt{n}}} \quad [6]$$

$$\tau' = \frac{C}{\sqrt{2n}} \quad [7]$$

n is the number of elements in a vector and C is a constant, usually equal to 1.

4. Select the μ best offspring to be the next generation.
5. Return to step 2 using the new generation as the parents until some stopping criterion is reached.

Computational Simulations

Three methods were used to design the DOE. The code was written in Matlab and run on a 300MHz Pentium 2 PC with 128Mb of RAM. For the sake of speed the arrays were limited 64x64 elements, although this could be increased at the expensive of computational cost. To access the best offspring and to compare the different methods a fitness parameter was measured for each trial phase screen. This was calculated by combining the amplitude function of input Gaussian laser beam to the trial phase screen and fast Fourier transforming the complex data. The intensity in the Fourier plane, $|A|^2$, was then calculated and a reference image, ζ , was subtracted from this. The fitness was then the root mean square of the remainder, i.e.,

$$\Theta = \sqrt{\frac{\sum_{i=1}^n \alpha |A_i|^2 - \zeta_i}{n}} \quad [8]$$

where α is a scaling factor which is different for each ζ , and is used to normalise the intensity of the image in the Fourier plane with the reference image. In the case of the IFTA, α was calculated by minimising equation [8] about α . Although this method provides the best result, it was too slow to be used for the ES where many more evaluations of fitness are required.

The simulations were carried out using the following methods to design a letter H in the output plane, (see figure 1).

1. The IFTA was used by itself.
2. The results from the IFTA were used by the ES (with $\mu=15$ and $\lambda=105$) as the initial parents instead of using randomly filled arrays. Several different values for μ and λ were tried but these values appeared to give the fastest results. In this case, α in equation [8] was calculated by taking the mean intensity over the white area of the letter H in figure 1. This method was considerably faster than minimising equation [8].
3. The ES was run by itself. To use the fast mean intensity method for calculating α , as in method 2, it is required that the majority of the energy in the output is plane is already formed into ζ . Since it is known that the IFTA method produces reasonably

good results, the value of α which was used by the IFTA is used for the first 1000 generations. After that, the same method as in 2 is used. This allows for fine tuning of α which in turn reduces the noise in the output plane.

Results

The final output produced by the IFTA method is shown in figure 2 and the phase screen to produce this is shown in figure 3. The fitness versus the number of iterations is shown in figure 4. The ES was then used to improve the phase screen shown in figure 3. The output after 4000 generations is shown in figure 5 and the phase screen to produce this is shown in figure 6. Figure 7 shows the final output using the ES alone. The phase screen is shown in figure 8. Figure 9 shows the fitness, calculated with equation [8], versus generations for both ES methods.

Discussion

It can be seen from figure 2 that the IFTA method produces an uneven image in the output plane. This is because of the IFTA stagnating before the solution is reached. The IFTA and ES methods combined produces a very flat response in the output plane by comparison. However, the two methods involving the ES described in this paper differ in the amount of processing time required. When the IFTA is used to generate the initial parents for the ES, the time to reach a better solution is faster than using the ES with random initial parents. This is because the IFTA generates a trial solution that is nearer to

the optimum solution than the purely random initial solution of the ES alone. It is therefore recommended that the IFTA and ES method be used together to find the best solution in the shortest amount of time. The method which only used an ES was stopped after 20 000 generations when it became clear that the method was much slower than the IFTA and ES combined.

The criterion that controls when the ES stops can be altered by the choice of parameters, unlike the IFTA. These parameters are C from equations [6] and [7] and the values of σ_i for the first generation. Lower values will cause the ES to find a better solution although possibly more generations would be required.

Since this simulation was running in Matlab, the computation times are slow. When running in C on the same computer a speed of 2.4 generations per second was achievable. The size of the array has been limited to 64 by 64 elements in this paper so that the software works reasonably fast. There is no reason why a higher size of array can not be used. However, a large array increases the processing time. In the case of the IFTA, this time increase will be dominated by the time required to Fourier transform the data. The ES methods also require a Fourier transform of each parent vector. The number of generations required will also increase due to the increase in the size of the search space.

Since only the intensity information in the Fourier plane is required for the ES to work, it should be possible to use an optical Fourier transform with a nematic liquid crystal (NLC) spatial light modulator (SLM) as the DOE and a CCD camera to measure the

quality. 128x128 NLC SLMs that have a switching speed of about 4ms are currently available⁸. Coupled with a normal CCD camera a generation per second should be achievable or faster with a higher speed camera and frame grabber. This method also has the advantages that it can remove any aberrations present in the optical system and calibration errors of the SLM. It is however unlikely to produce a faster convergence than the calculation on the PC, so the fastest method would probably be to use the IFTA technique on the PC and transfer to an optical system for the ES method.

Conclusions

Three different methods of designing a DOE have been described and compared. This was the conventional IFTA method, the IFTA with an ES, and an ES alone. The ES was used to overcome the stagnation problems associated with the IFTA method. The fastest method was to use the IFTA combined with the ES.

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Figure 1. The mask used as the template.

Figure 2. The simulation of the output plane of the DOE system designed by the IFTA only.

Figure 3. The phase screen of the DOE designed by the IFTA only. Legend in radians.

Figure 4. The fitness versus the number of iteration of the IFTA.

Figure 5. The simulation of the output plane of the DOE system designed by the IFTA and improved using the ES.

Figure 6. The phase screen of the DOE designed by the IFTA and improved using the ES. Legend in radians.

Figure 7. The simulation of the output plane of the DOE system designed by the ES only after 20 000 generations.

Figure 8. The phase screen of the DOE designed by the ES only. Legend in radians.

Figure 9. The fitness versus the number of generations for both the IFTA with the ES method (dotted line) and the ES only method (solid line).

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