Abstract

We present results of a correlation filter utilising a computer generated hologram using an analogue ferroelectric liquid crystal spatial light modulator (SLM). The SLM amplitude modulates light and can induce a 0/π phase shift, which is equivalent to modulating along the real axis. Two pixels are combined into a macropixel using a phase detour technique enabling full complex modulation. The method is used as the filter for a conventional optical correlator and in a digital/optical hybrid correlator.

Keywords: correlator, spatial light modulator, image recognition

Introduction

Correlators enable high-speed image and character recognition. One of the usual configurations for such a device is based on the Vander-Lugt design. Images are captured by a CCD camera and displayed on an intensity modulating spatial light...
modulator (SLM). This is then optically Fourier transformed onto the filter plane SLM. In order to test many templates, the filter SLM must work at a much higher speed than the input SLM. Several kilohertz are often required if the input camera SLM is updated at normal video rate. A common technique to achieve this is to use a conventional bistable smectic C* ferroelectric liquid crystal (FLC) SLM. However, this type only produces a binary phase only filter (BPOF) which are susceptible to input noise, object rotation and scale. It would therefore be more desirable to produces a filter that can not only be updated at high speed but also is capable of full complex modulation. In this paper we will discuss the construction of such a filter and demonstrate its use in a correlator system.

In modern 4-f optical correlators, the two Fourier transforms and the mixing of the input signal with the reference signal is performed optically and the data handling and control is usually managed by a computer. The filter spectrum is generally phase modulated by an electrically addressed SLM in the frequency domain. The BPOF FLC SLM controlled by digital electronics has proved an effective solution. This method is the approach that has been preferred by many system developers recently, several well engineered prototype systems having being developed largely, but not exclusively, for missile terminal guidance applications. Extensive research has shown that binarisation of the Fourier plane phase generates a good correlation response, the correlation peak being remarkably resistant to such gross frequency plane quantisation. However, binarisation of the input scene is less effective, particularly for unconstrained scenes in which the lighting levels are unknown and variable. Thus, SLMs capable of binary amplitude modulation can only be used as the input modulator to an optical correlator if the scene acquired from the input camera has been digitally pre-processed in a sophisticated manner. Even then, input plane binarisation still restricts the effectiveness of the correlation based pattern recognition.

In general, the input scene does not need to be up-dated at the kilohertz frame rates achievable with binary effect SLMs, often the 25 -30 Hz frame up-date rate associated with standard video sources being adequate. Thus, SLMs capable of generating a grey
level response may be used as the input device to the optical correlator, the input scene being introduced to the correlator with minimal pre-processing. However, there are many problems associated with doing this\(^4\). A particularly important development for the optical processing community in recent years has been the availability of analogue ferroelectric (AFLC) liquid crystal based devices\(^5\). With appropriate configuration of polariser, the devices can be made to generate a bi-polar analogue response (along the real axis of the complex plane) and so are useful both in the input and frequency plane of a coherent optical correlator\(^6\). The capability to use identical devices with exact pixel matching in both spatial and frequency planes will greatly facilitate the design of all optical correlators.

**Compact Optical Correlator Design Problems**

The specific characteristics of the SLM devices employed are all important in the design problems faced in realising a compact and robust optical correlator. The main problem is to accurately match the Fourier transform of the scene displayed on the input SLM to the spectrum displayed on the frequency plane SLM, together with the maintenance of this match in the face of, maybe severe, environmental disruption. Thus, all optical correlators that meet demanding military and industrial requirements present far from trivial electro-optic and opto-mechanical design problems which have only been partially solved by the current state of the art. The problems inherent in the design of an all optical correlator can be summarised as follows:

- The input SLM is required to produce a high dynamic range analogue image with high optical quality and a high data transfer rate.

- The mechanical stability of the optical system must be very high. The pixel size of a typical current device is \(\sim 40\mu m\) and this number is constantly been reduced as pixels become smaller. For optimum performance the optical spectrum must kept aligned to less then one pixel displacement. This is a problem in the
commercial development of correlator systems when the end-user might not have the technical knowledge to align the system.

- The small pixel size is also demanding on the optical design of the Fourier transform lens systems. The lens design requirements are complex if a compact correlator arrangement is to be realised. This is further complicated by the need to uniformly illuminate the input SLM. This turns out to be a non-trivial problem. A laser usually gives a Gaussian beam profile which needs to be either sampled, which is very wasteful of beam power, or diffractive optical elements are required which introducing a random phase to the input and also require highly stable laser sources.

**Hybrid Digital/Optical Design**

The hybridisation with digital electronics can be taken a stage further. Digital signal processing can be used to overcome several of the problems listed above by replacing the first Fourier transform of a conventional correlator system with its electronic equivalent. This arrangement is shown schematically in figure 1. To overcome problems with the input SLM, the scene is captured with a CCD camera and electronically fast Fourier transformed (FFT). The complex data is then mixed with reference filters and placed onto a SLM. This SLM is then optically Fourier transformed onto a high-speed camera that records the results. This method has several advantages over conventional correlators: the input SLM is avoided so the spectrum of the input signal is no longer degraded by imperfections of the SLM. The power levels required of the laser are less and there is no longer a need for uniform illumination. The optical design of the system is greatly simplified since only one Fourier transform lens is required. This in turn increases the compactness of the system and greatly increases the mechanical robustness.

The computer in this system has replaced two functions previously performed optically: the Fourier transform of the input signal, and the mixing of the complex spectrum with the filter functions. The FFT is computationally more intensive than the mixing function.
However, for an image recognition task only video rates (25Hz) are usually required for the input while the mixing is limited to either the frame speed of the SLM or the frame speed of the camera. The basic principle of the system is to grab the input signal, FFT this at video rate and then mix this data with predesigned filter functions and place these onto the SLM. The SLM is uploaded at a faster rate than the FFTs are performed so that a large number of filters can be tested for each input FFT. The concept of combining a digital and optical Fourier transform into a hybrid correlator system was first reported a number of years ago by the authors but a high-speed hardware realization has only recently been possible. The design and construction of such a correlator is discussed in reference 4 and is summarised as follows.

The system uses eight SHARC floating point DSP processors that are capable of performing a 512x512 FFT at a rate of 25 frames per second. The input scene camera is a digital DALSA camera and the data is fed into the SHARC processors via a field programmable gate array (FPGA) chip. A further two SHARCs and FPGA is used to perform the multiply between the input spectra and the template filters and write this data to the SLM controller. A high frame rate DALSA camera (128x128 pixels and ~800Hz) is synchronised with the SLM and this records the correlation image. Another FPGA and SHARC processor interface with this camera to perform the peak detection.

The system was originally designed to use a BPOF. A fully complex filter would perform closer to the ideal filter and this paper assesses the design of such a filter. The filter is demonstrated experimentally in a conventional 4-f correlator and in the hybrid mode of operation.

Several authors have demonstrated full complex modulation before. One method of achieving this is by cascading two SLMs together, one amplitude modulating SLM and another phase modulating SLM, or by two different twisted nematic SLMs. Two SLMs proved a simple solution but unfortunately increase the bulk of the system and its cost. Single SLM methods have involved blurring together pixels of deformable mirrors and using four pixels of the SLM. Several authors have used the phase detour technique.
However, not all of these methods can be easily realised on a pixelated SLM. Lee\textsuperscript{12} proposed a four-pixel phase detour technique where the complex datum is split into its positive real, positive imaginary, negative real and negative imagery components. Each component is then placed on adjacent pixels. The complex datum point will then be realised off axis where a $\pi/2$ phase lag exists between each pixel. This was later reduced to three pixels\textsuperscript{13} by sampling the 0°, 120° and 240° components. Unfortunately, SLM design is limited in resolution and so ideally the least number of pixels as possible should be used to represent a complex datum point. There is also a large dc associated with the Lee technique. The method used in this paper reduces the number of pixels required to two effectively doubling the resolution and removes the dc term as explained below.

**Theory**

**Real Axis SLM Modulation**

The SLM used in this paper is an analogue ferroelectric liquid crystal (AFLC) SLM. The device was constructed by Boulder Nonlinear Systems. It has 128x128 pixels and a frame rate of 102 $\mu$s. The device is capable of modulating along the real axis, in an analogue and bipolar (i.e. positive and negative) manner.

The SLM pixels are approximately equivalent to quarter-wave plates with an electronically controllable optical axis. Since the device is reflective, the device is operated in double pass, so it effectively acts like a half-wave plate. The SLM is placed behind a polarising beam splitter. The input polarisation state is linear and aligned to bisect the maximum angles of the optical axes achievable by the SLM. If the optical axis of the SLM pixel is at angle $\theta$ to the plane of polarisation of the incident light, the reflected light will have been rotated by angle $2\theta$. The polarising beam splitter is then used to select the polarisation component orthogonal to the incident polarisation, hence giving amplitude modulation. If the optical axis of the pixel is rotated -$\theta$, the outgoing light will be rotated in the opposite direction leading a negative amplitude modulation.
compared to the positive $\theta$ example. This allows analogue modulation along the real axis in both positive and negative directions.

**Complex Modulation Technique**

Complex modulation can then be achieved using a phase detour technique. Combined phase and amplitude modulation can be represented using only two pixels since the pixel value can go negative. This is an improvement in the technique described in reference 12, which requires 4 pixels and since they are all positive results in a large DC bias. Our technique has been previously demonstrated producing computer generated holograms\textsuperscript{14}.

The complex function to be encoded on the SLM, $F(x,y)$, is represented by an $N\times N$ array and is split into real and imaginary parts. A $2N\times 2N$ array is produced and the real data is written to every $2n$ element in the $x$ direction, whilst the imaginary data is written to every $2n+1$ element $(n=0,1,2,3\ldots)$. Every second pair is negatated to maintain a continuous $\pi/2$ phase lag over the four pixels.

If each SLM pixel is square and of width $b$, the complex function, $f(x,y)$ can be represented by

$$f_s = \text{comb}\left(\frac{y}{b}\right)\sum_{m=-\infty}^{\infty} \delta(x-na)\exp(i\pi n)\text{Re}[f(x,y)] +$$

$$\text{comb}\left(\frac{y}{b}\right)\sum_{m=-\infty}^{\infty} \delta\left(x-a\left(n+\frac{1}{2}\right)\right)\exp(i\pi n)\text{Im}[f(x-a^2/2, y)]$$

where $\text{Re}[.]$ is real part, $\text{Im}[.]$ is the imaginary and $a=2b$. This Fourier transforms to become

$$F_s(\omega,\nu) = \frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} F\left(\omega - \frac{n}{2a}, \nu - \frac{m}{b}\right)(1 + \exp(-i\pi a \omega))(1 - \exp(i\pi n)) +$$

$$\frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} F\left(-\omega - \frac{n}{2a}, -\nu - \frac{m}{b}\right)(1 - \exp(-i\pi a \omega))(1 - \exp(i\pi n))$$
$F_s(\omega, \nu)$ only exists when $n$ is odd. The $(1 \pm \exp(-i\pi \omega))$ modulation term causes the normal and conjugate terms to be alternatively blanked when the device is used as a computer generated hologram. In the hybrid correlator system the function $F(\omega, \nu)$ is the result of the FFT of the input signal mixed with the filter. In the hybrid method the result is the intensity pattern from [2], $F_s(\omega, \nu)F_s^*(\omega, \nu)$, where in this case $f(x, y)$ is the FFT of the input scene to the correlator multiplied by the filter.

In the optical correlator system only the filter is placed onto the SLM, so just after the filter the signal is represented by

\[
H_x(\omega, \nu)G(\omega, \nu) = \frac{1}{2} \left( \text{comb} \left( \frac{\omega}{2a} \right) - \text{comb} \left( \frac{\omega - a}{2a} \right) \right) H(\omega, \nu)G(\omega, \nu) + \\
\frac{1}{2} \left( \text{comb} \left( \frac{\omega}{2a} \right) - \text{comb} \left( \frac{x - a}{2a} \right) \right) H^\times(\omega, \nu)G(\omega, \nu) + \\
\frac{1}{2} \left( \text{comb} \left( \frac{\omega - a}{2a} \right) - \text{comb} \left( \frac{\omega + a}{2a} \right) \right) H(\omega - \frac{a}{2}, \nu)G(\omega, \nu) - \\
\frac{1}{2} \left( \text{comb} \left( \frac{\omega - a}{2a} \right) - \text{comb} \left( \frac{\omega + a}{2a} \right) \right) H^\times(\omega - \frac{a}{2}, \nu)G(\omega, \nu)
\]

where $g(x, y)$ is the input signal, $G(\omega, \nu)$ is its Fourier transform, $H(\omega, \nu)$ is the Fourier space filter function and $h(x, y)$ is its Fourier transform. The output field is represented by the Fourier transform of [3]:

\[
h(x, y) \otimes g(x, y) = \frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h \left( x - \frac{n}{2a}, y - \frac{m}{b} \right) \left( 1 + \exp(-i\pi \omega) \right) \otimes g(x, y) + \\
\frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h^* \left( -x - \frac{n}{2a}, -y - \frac{m}{b} \right) \left( 1 - \exp(-i\pi \omega) \right) \otimes g(x, y)
\]

[4]
Again the \((1\pm\exp(-i\pi ax))\) term causes the conjugate and normal terms to be alternatively blanked so that a series of correlation spots are created off axis.

**Effect of Modulation Term**

The modulation term causes the filter function \(f(x,y)\) to be modified from the ideal. The amplitude of the function drops off as \(|x|\) increases. There is also a phase wedge introduced across \(f(x,y)\). Computer simulations of this effect were performed to assess its effect. Figure 2 (dotted line) shows a computer simulation of an all optical correlator using the complex filter. A spot was moved across the input field and correlated with a single spot. The maximum intensity of the resulting correlation is shown. A correlation peak was produced as expected. However, this was shifted off axis. The simulation was also performed for the hybrid correlator. The normalised peak intensity as the spot is moved is shown in figure 2 (solid line). In the conventional all optical correlator there is no intensity variation across the output field as the spots are shifted. However, there is a clear drop off in performance off axis for the hybrid correlator.

The lack of intensity drop off in the conventional correlator system is because to the reference object used to generate the actual filter contains is on axis and it is \(g(x,y)\) that contains the shift. Since \(g(x,y)\) is optically Fourier transformed it is not effected by any short comings the complex data encoding on the SLM.

In the hybrid correlator the intensity drop off as \(|x|\) increases comes from the intensity modulation term in equation 2. In addition to this, a conjugate term will also start to appear as the spot is moved off axis. This does not effect the hybrid correlator when real only or binary phase only filters are used. An example simulation of the hybrid correlator is shown in figure 3. In this case an on axis letter A is correlated with itself. Two large peaks are visible. These are the terms generated when \(n=\pm1\) in equation 2. These two terms are the correlation peaks produced by the first positive and first negative diffraction orders.
Comparison to ideal filter

Computer simulations comparing the phase detour encoding of a complex filter to an ideal matched complex filter that could represent complex data with a single pixel were carried out. Figure 4 shows the normalised maximum intensity of the correlation of the letter A that is rotated through 180°. A phase only filter simulation is also shown for comparison. The phase detour filter response is identical to the ideal filter graph. As expected the phase only filter is more intolerant to object rotation.

Experimental

The filter was set up in an all optical correlator. The input filter was a photographic negative and the light source was a 514.5nm Ar ion laser. The SLM is controlled using a IBM compatible PC. Data is written into an 8bit frame store located on the SLMs driver board. This is done over the ISA bus on the PC. The frame store hold a total of 16 images and these can be written to the actual SLM with a maximum frame rate of 102µs although a second negative frame is required to maintain a zero DC electric field across the liquid crystal.

Figure 5 shows an example result of the all optical correlator. The input signal is shown in figure 6 and filter selects one of the ‘A’s. The correlation peak is quite large since a 800mm focal length lens was used. It is also located off axis. Figure 7 shows the same set up but the imaginary data has been blacked. Since it now a real only filter the correlator can not differentiate between the two ‘A’s.

Figure 8 shows an example result with the same input and filter function except using the hybrid correlator. The correlation spot is smaller in this case since a faster lens can be used because there is no longer need to match the spectrum of the input signal to the filter.

Discussion

The optical correlator produces a correlation peak off axis that comes from the -1/2a term in equation [4]. The hybrid system also produces a correlation peak off axis. In this case,
the peak appears quarter way between the zeroth order and the first orders produced by
diffraction from the SLM pixelation.

The hybrid system was considerably easier to set up than the optical system since only
one optical Fourier transform is required. However, it has a reduced off axis performance
compared to the all optical correlator. This is caused by the appearance of a conjugate
image in the output field and a reduction in intensity of the correlation peak. The
reduction in intensity was limited to 60% in computer simulations and may not be a
problem for some applications. Despite this limitation the hybrid method still has many
of the advantages over the all optical method such as robustness and compactness
described above.

**Pixel shape**

The SLM pixels are all square so if the data is arranged into a 2x1 macro pixel there will
be a rectangular output field. For the all optical correlator this was overcome by doubling
the macro pixels up, making a 2x2 square. Equally valid methods to overcome this
problem could be to have rectangular pixels or modify the filter design so that there was a
non symmetric sampling rate in both directions. In the case of the hybrid correlator, the
pixels could simply be left with a 2x1 ratio, which would give a rectangular output field.

**Computational Overheads**

In the hybrid system there are computational overheads to be considered when choosing
which filters to implement. The overheads for the FFT system have previously been
discussed in reference 4 and experimentally demonstrated to perform a 512x512 FFT in
under 40ms. For the filter, the simplest case is a binary phase only filter. For a 128x128
SLM, 16 384 ‘exclusive or’ (XOR) operations are required, increasing to 262 144 for a
512x512 device. The XOR is a relatively simple binary operation and can be performed
using the two SHARC chips dedicated to the process. Each XOR operation takes 3 cycles
of the 40MHz processors so it can be achieved in 10μs. The same number of operations
are required for analogue phase and real only filters, except in this case the operators are
additions and floating point multiplies respectfully, which are computationally more
intensive and take 300µs in the 128x128 pixel case. The filters will require the same number of addition operators plus a reminder operation to find the phase over the range 0 to \(2\pi\) or amplitude over \([-1, 1]\). A lookup table or polynomial multiply will then also have to be performed to get the correct drive voltage for the SLM pixel. The computationally most challenging filter type is the complex filter which will require 4 multiplies and two additions per datum. However, the number of pixels has been halved in one dimension so 1ms is required.

**Conclusions**

A complex filter design has been demonstrated using an AFLC SLM. The filter requires 2 pixels grouped together and uses a phase detour technique. An all optical correlation demonstration has been shown, along with results from a hybrid correlator. The hybrid correlator provides a simpler and more robust solution to implementing a traditional 4f optical correlator design.

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\[\text{References}\]


Figure Captions

Figure 1 Schematic layout of digital/optical hybrid correlator.

Figure 2. A simulation of the hybrid and 4f correlator results. The graph shows the peak intensity of the correlation as the input signal is moved across the field. Solid: hybrid; dotted: 4f.

Figure 3. Simulation result of the hybrid correlator.

Figure 4. Simulation result comparing and ideal matched filter, the SLM filter and the phase only filter (POF). The x axis represents the rotation angle of the input. The y axis is the correlation peak intensity (COPI).

Figure 5. The results of the optical correlator with a complex filter.

Figure 6. The input mask for the optical correlator.

Figure 7. The optical correlator with a real only filter.

Figure 8. The hybrid correlator result. Two correlation peaks are visible on each side of the DC term.
Figure 1  Birch
Figure 3  Birch
Figure 4 Birch
Figure 5    Birch
Figure 6  Birch
Figure 7  Birch
Figure 8  Birch