Tyre Dynamics

Wheels have been used for at least 5700 years.
Near the northern side of the Caucasus several graves were found,
in which since 3700 BC people had been buried on wagons or carts.



Tires or tyres are pneumatic enclosures used to protect and enhance the effect of wheels.



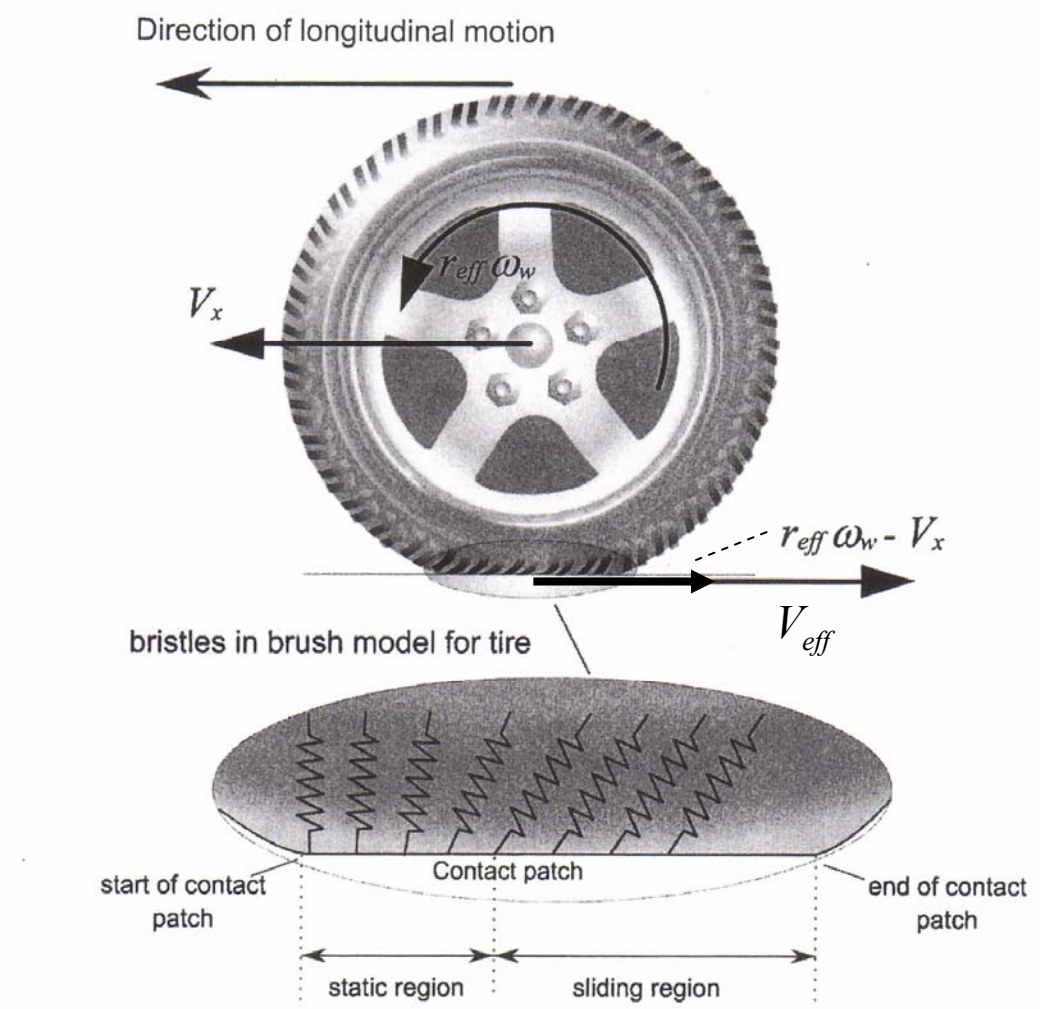
The earliest tires were bands of iron made by by skilled craftsman, known as a wheelwright.

The first practical pneumatic tire was made by the Scot John Boyd Dunlop for his son's bicycle, in an effort to prevent the headaches his son had while riding on rough roads.

The pneumatic tire also has effect of vastly reducing rolling resistance compared to a solid tire on rough ground. It is able to "absorb" bumps in the road as it rolls over them without experiencing a reaction force opposite to the direction of travel, as is the case with a solid (or foam-filled) tire.

Pneumatic tires are made of a flexible elastomer material such as rubber with reinforcing materials such as fabric and wire.

Tire companies were first started in the early 20th century.
Today over 1 billion tires are produced annually.


Direction of longitudinal motion
$r_{eff}\,\omega_w$
V_x
$r_{eff}\,\omega_w - V_x$
V_{eff}
bristles in brush model for tire
Contact patch
start of contact patch
end of contact patch
static region
sliding region

Effective tyre radius

r_{eff} – radius which relates ω_w to the linear longitudial velocity of the wheel V_{eff}

$V_{eff} = r_{eff}\omega_w$ (it is faster than V_x when in traction)

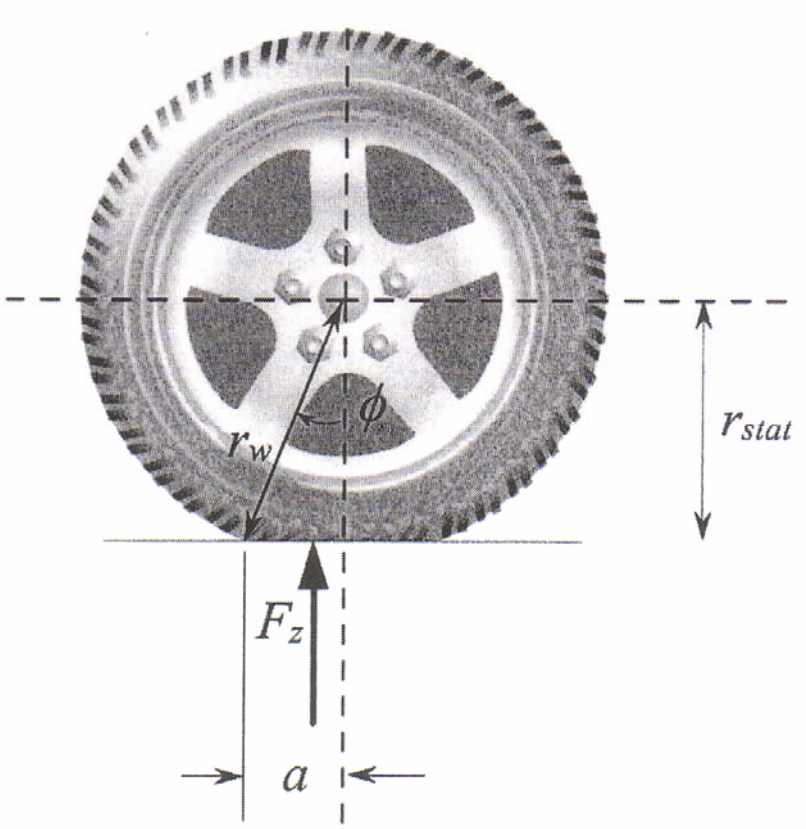
$V_x = r_{eff}\omega_{eff}$

$$\phi = \frac{a}{r_{eff}}$$

$$r_{eff} = \frac{a}{\phi}$$

$$\omega_w = \frac{\phi}{t},$$

t – time taken by an element of tyre to move through half the contach patch

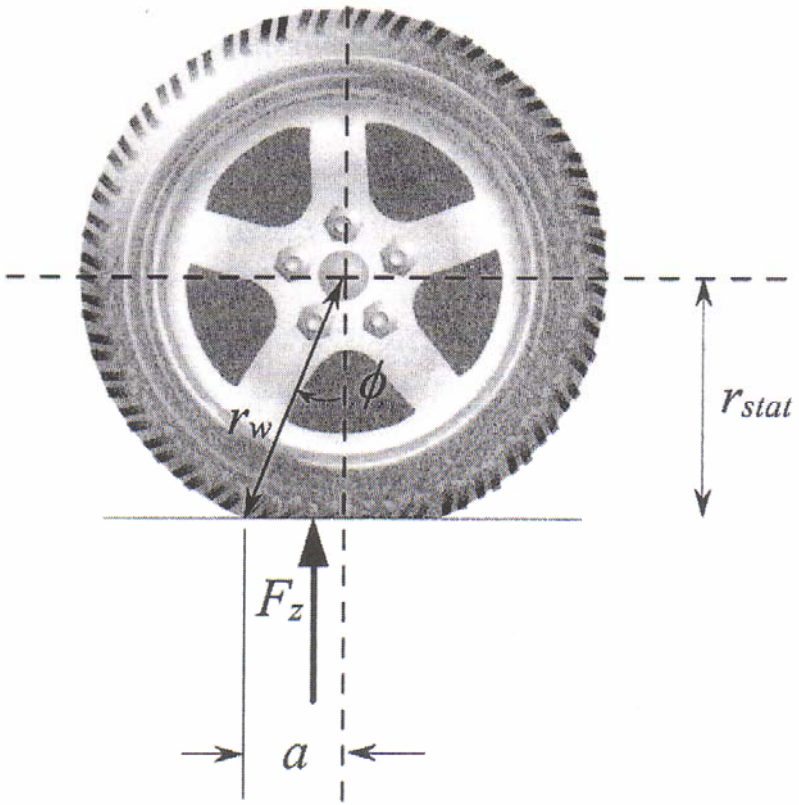


$$V_{eff} = r_{eff}\,\omega_w = \frac{a}{t},$$

$$r_{stat} = r_w - \frac{F_z}{k_t}$$

k_t – virtical tyre stiffness

$$r_{stat} = r_w \cos\phi, \qquad a = r_w \sin\phi$$



Therefore

$$r_{eff} = \frac{a}{\phi} = \frac{r_w \sin\phi}{\phi} \qquad \text{and} \quad \phi = \cos^{-1}\left(\frac{r_{stat}}{r_w}\right),$$

i.e.

$$r_{eff} = r_w \frac{\sin\left(\cos^{-1}\left(\frac{r_{stat}}{r_w}\right)\right)}{\cos^{-1}\left(\frac{r_{stat}}{r_w}\right)}$$

Since

$$r_{eff} = \frac{a}{\phi} = \frac{r_w \sin\phi}{\phi}, \qquad r_{eff} < r_w$$

On the other hand, since

$$\frac{a}{r_{stat}} = \tan\phi,$$

then

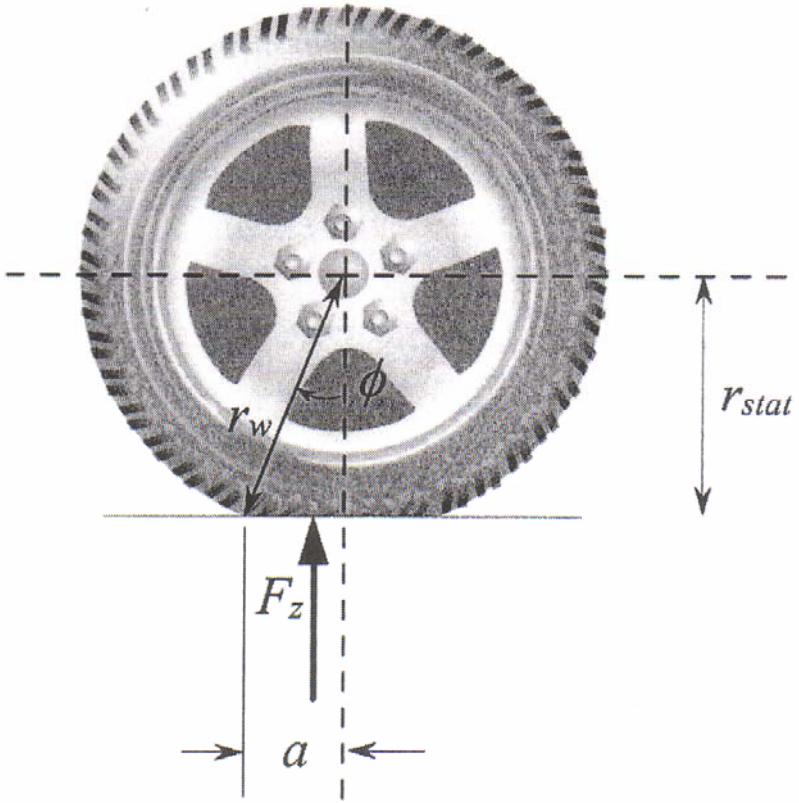
$$a = r_{stat} \tan\phi,$$

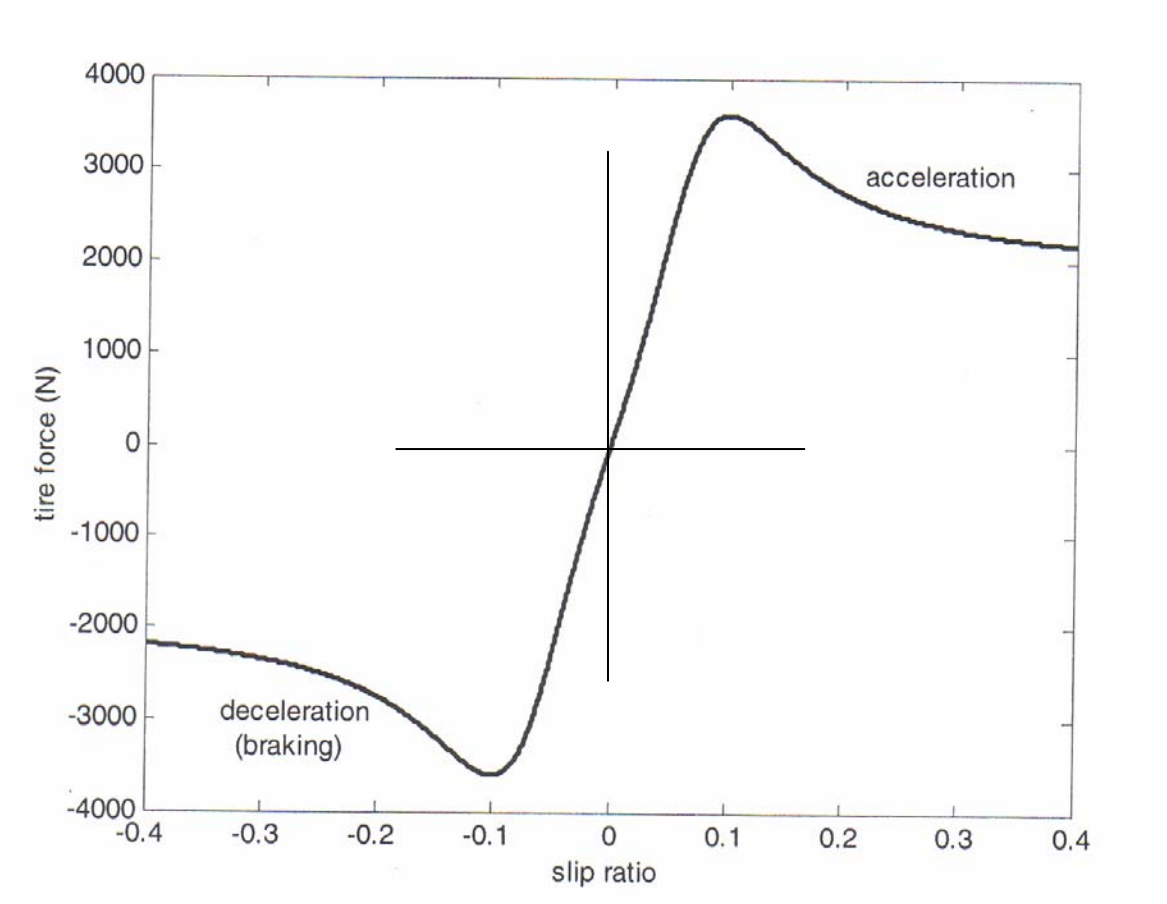
or

$$r_{ref}\phi = r_{stat} \tan\phi$$

$$r_{eff} = \frac{\tan\phi}{\phi} r_{stat}, \qquad \text{therefore} \quad r_{eff} > r_{stat} \qquad (\tan\phi > \phi)$$

$$r_{stat} < r_{eff} < r_w$$





Longitudinal tyre force is a function of slip ratio

It is linear near slip ratio = 0

$$F_{xf} = C_{\sigma f}\sigma_{xf}$$
$$F_{xr} = C_{\sigma r}\sigma_{xr}$$

Longitudinal tyre force

Longitudinal tyre force F_{xf} and F_{xr} depends on

a) slip ratio

b) normal load

c) friction coefficient of tyre - road interface

Longitudinal Slip :

$s = r_{eff}\omega_w - V_x$

r_{eff} – effective radius

Slip ratio are defined as

$$\sigma_x = \frac{s}{r_{eff}\omega_w} = \frac{r_{eff}\omega_w - V_x}{r_{eff}\omega_w}$$ for tyre on wheel in traction mode

$$\sigma_x = \frac{s}{V_x} = \frac{r_{eff}\omega_w - V_x}{V_x}$$ for tyre on wheel in braking mode or on driven wheel

For small linear region,

$F_{xf} = C_{\sigma f}\sigma_{xf}$

$F_{xr} = C_{\sigma r}\sigma_{xr}$

where $C_{\sigma f}, C_{\sigma r}$ are called tyre stiffiness

Longitudinal tyre force

Brush model

Net velocity at treads :

$$s = r_{eff}\omega_w - V_x$$

for driving wheel, the tread element is bending

$s > 0$ in direction of backwards

The bending defection generate longitudinal force

- proportional to s
- proportional to time duration for which the tread element stays in the contact patch, or inversely proportional to the velocity

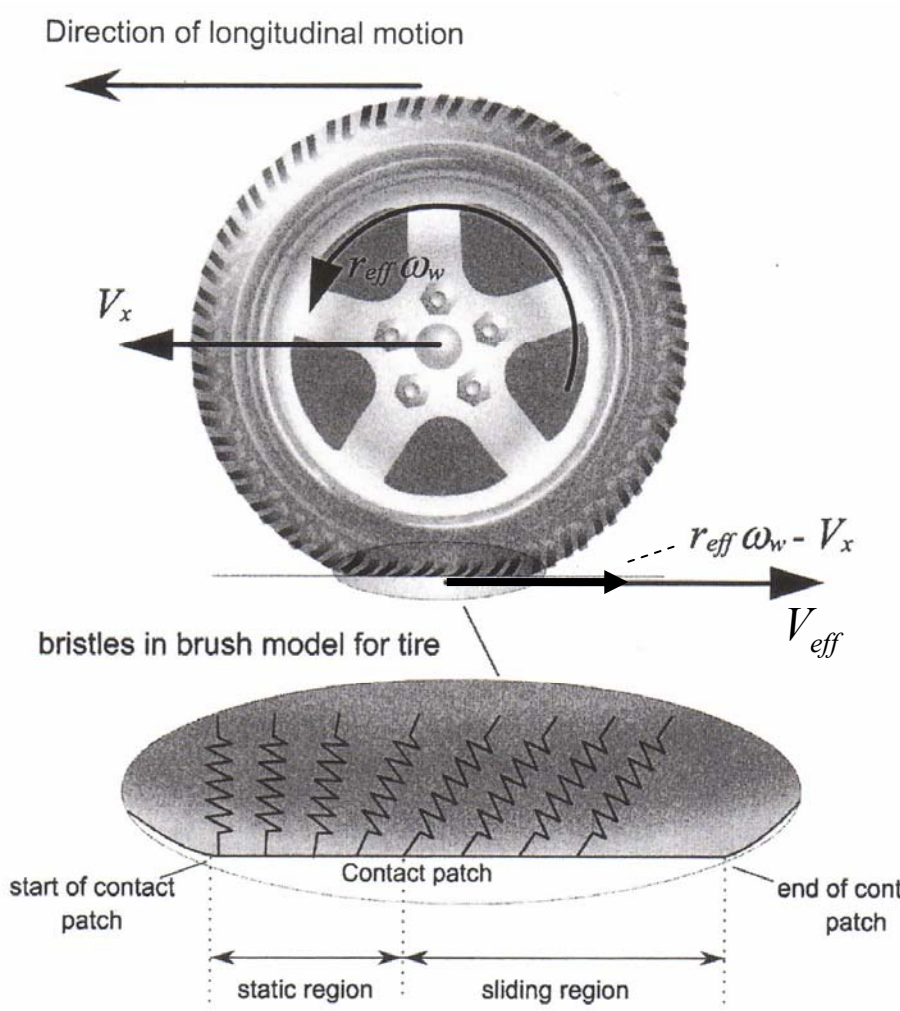
Therefore,

$$F_{xf} = C_{\sigma f}\frac{s}{r_{eff}\omega_w}$$

For driven wheel

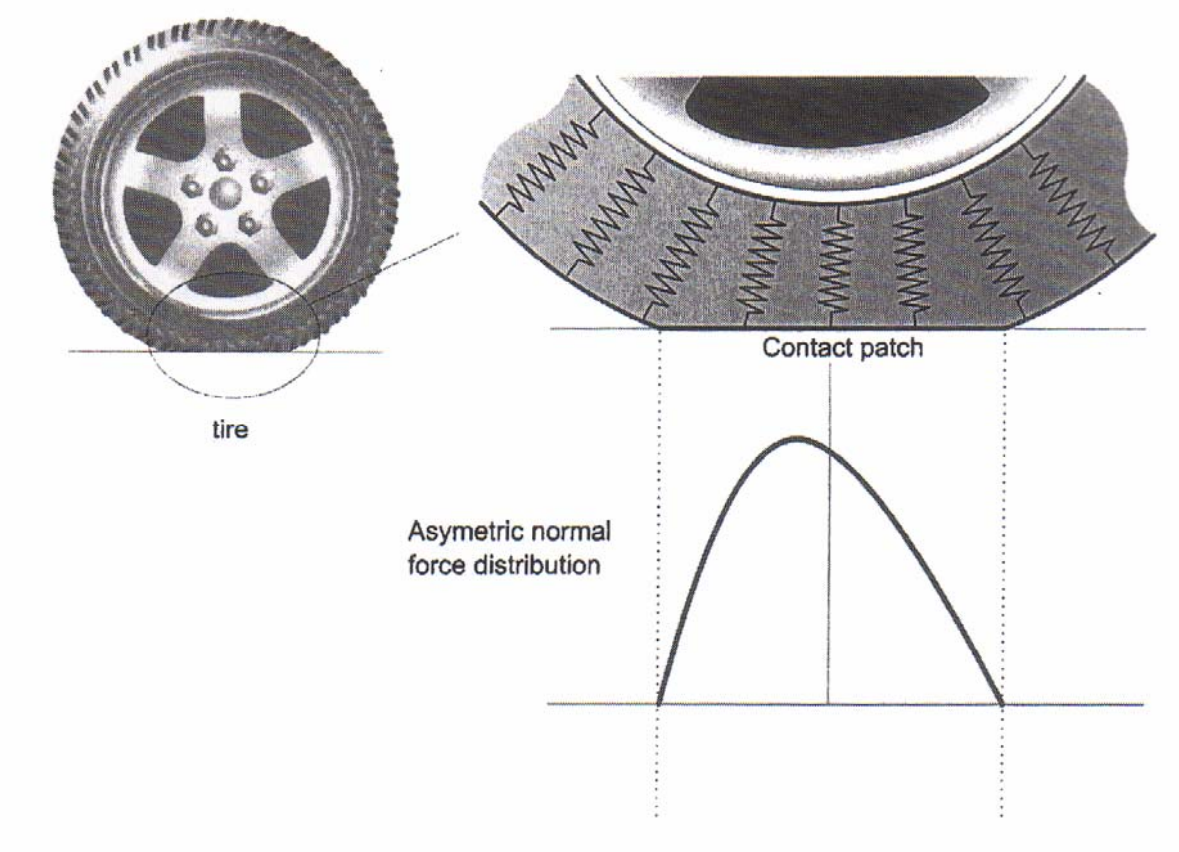
$s = r_{eff}\omega_w - V_x < 0$

for driven wheel, the tread element is bending in direction of backwards. Tyre is subject to backwards ground force (resistance).



Rolling resistance

- Normal load causes tyre deformation ain contact patch
- Damping in the tyre material dissapates energy
- Viscousity causes forward half force stronger

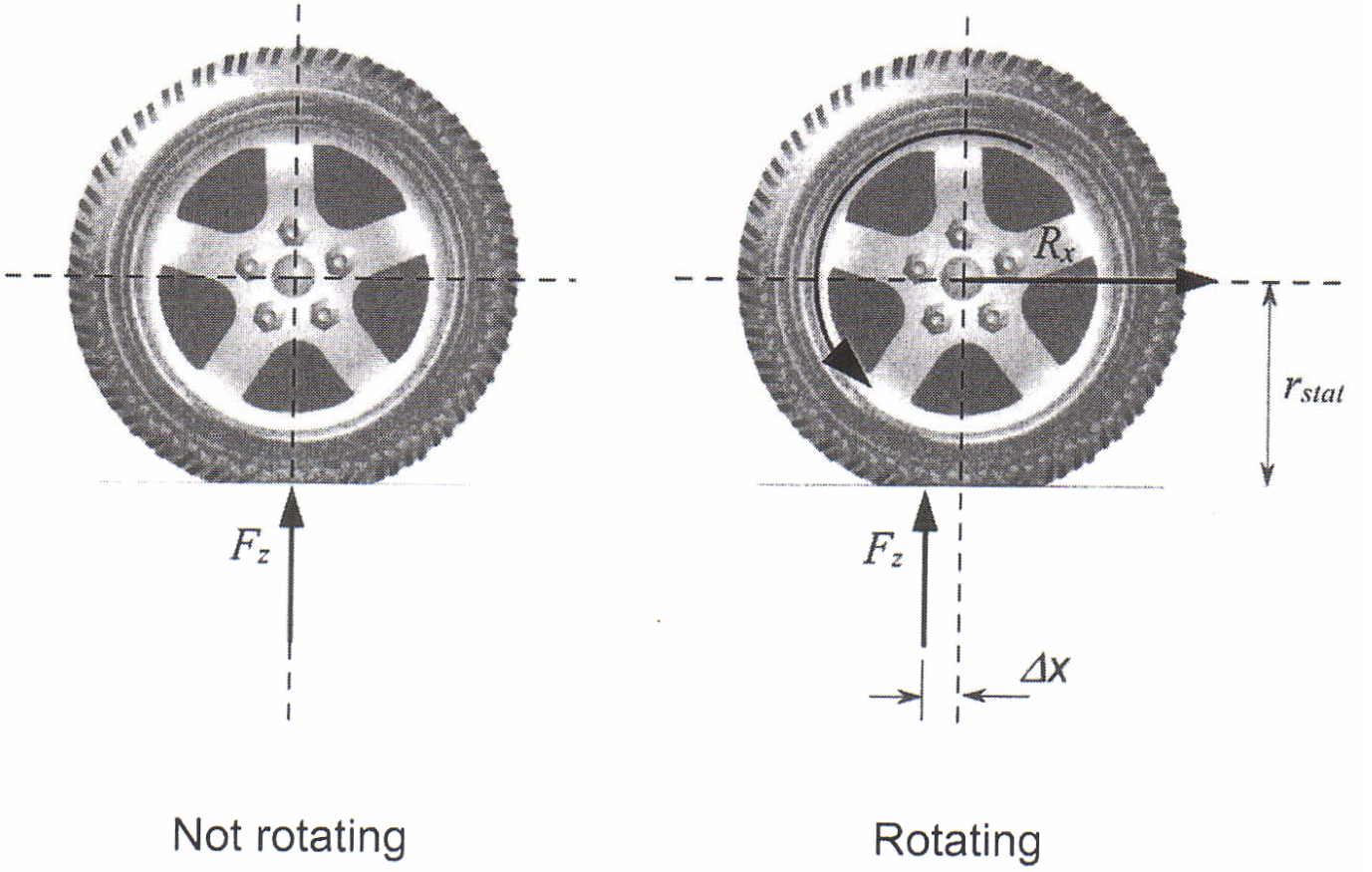


Rolling resistance

$$R_{xf} + R_{xr} = -f(F_{zf} + F_{zr})$$

$$f = 0.01 - 0.04$$

$$R_x \cdot r_{stat} = F_z \cdot \Delta x$$



Normal tyre forces

$$F_{zf} = \frac{1}{l_f + l_r}\left(-F_{aero}h_a - m\ddot{x}h\sin\theta + mgl_r\cos\theta\right)$$

$$F_{zr} = \frac{1}{l_f + l_r}\left(F_{aero}h_a + m\ddot{x}h\sin\theta + mgl_f\cos\theta\right)$$

