Design for Manufacture

6. Dynamics of Machine

Grass cutting

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Scissoring - Using fixed and moving blades, a cylinder mower cuts grass like a pair of scissors.

Impact cutting - A rotary mower rotates about a vertical axis with the blade spinning at high speed relying on impact to cut the grass, resulting in a rougher cut.

High speed blade tip approaches the grass first.

Cylinder lawn mower

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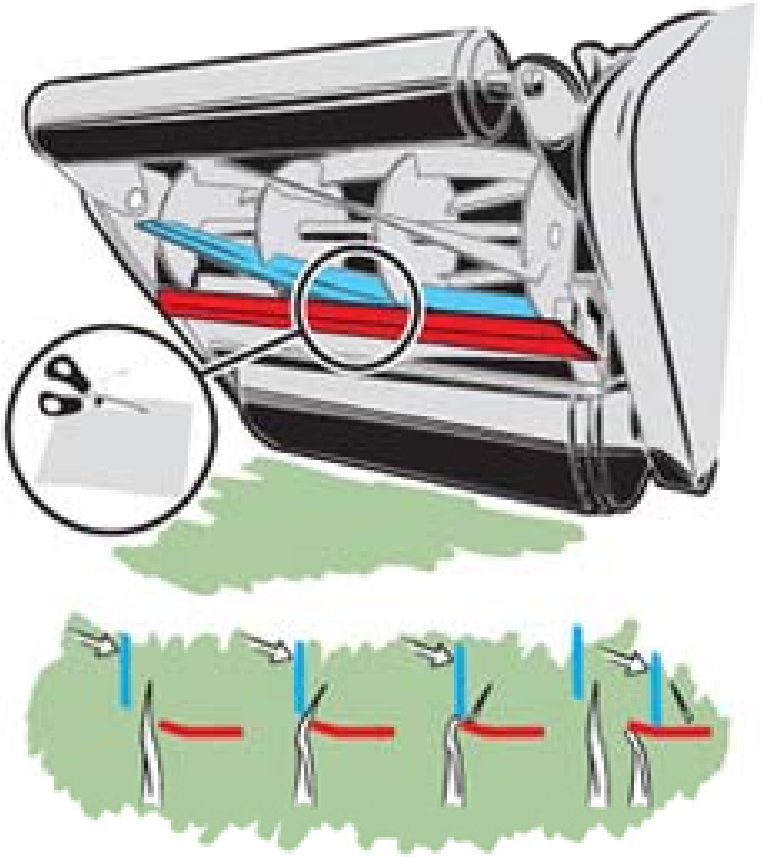

Each blade of grass is precisely cut with a scissor-like action between the spinning cylinder blade (blue) and the stationery bottom blade (red)

Rotary lawnmower

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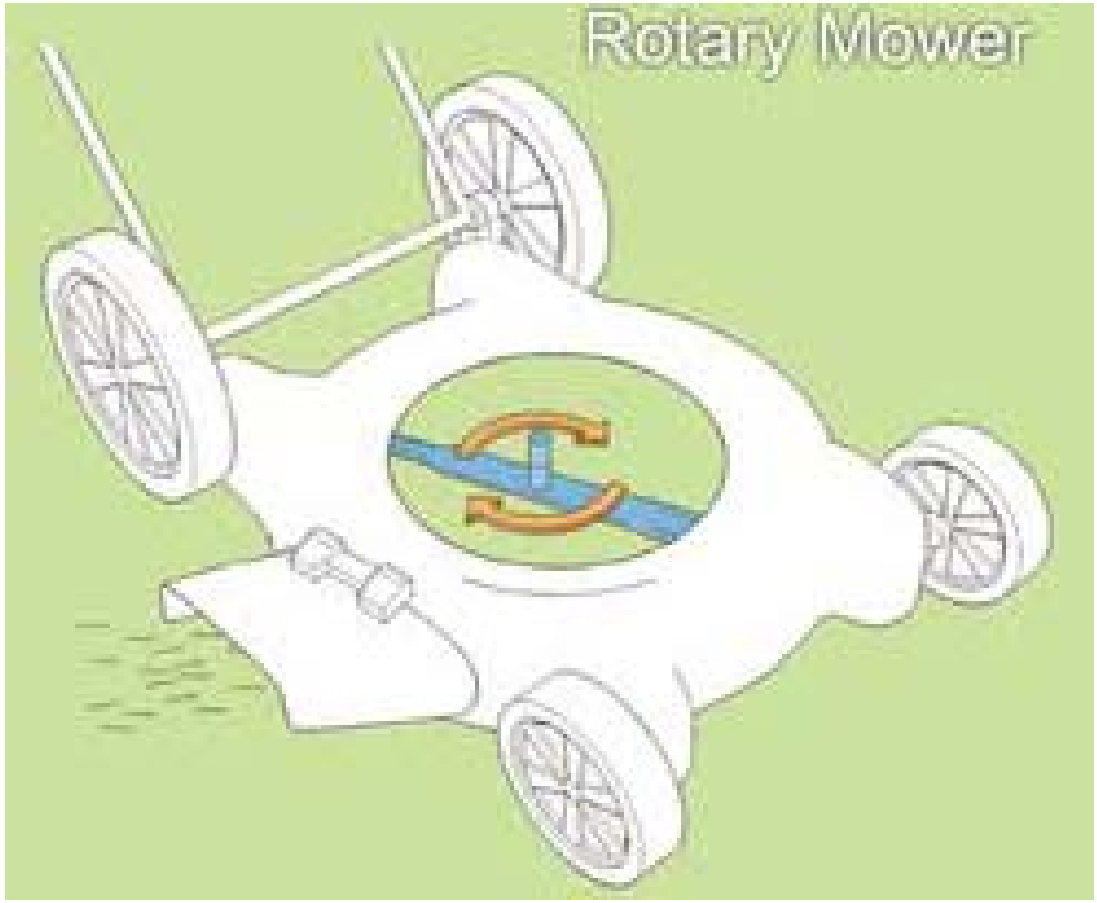

Rotary lawnmower

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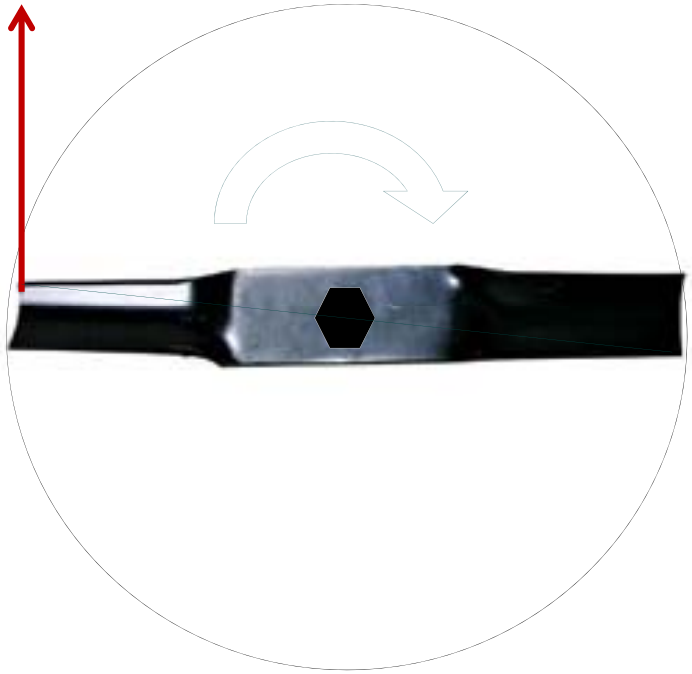
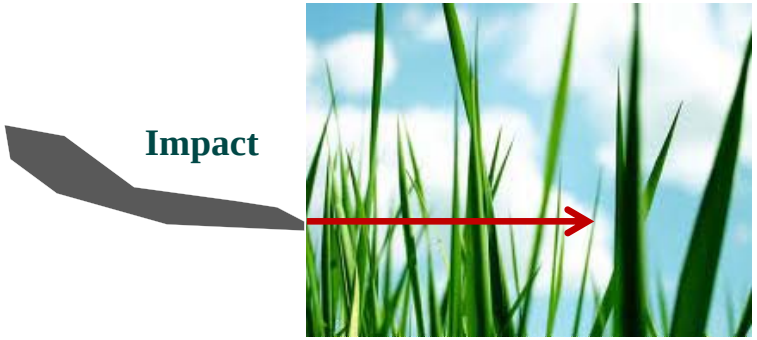
$V=\omega r$



Impact



Grass and air flow

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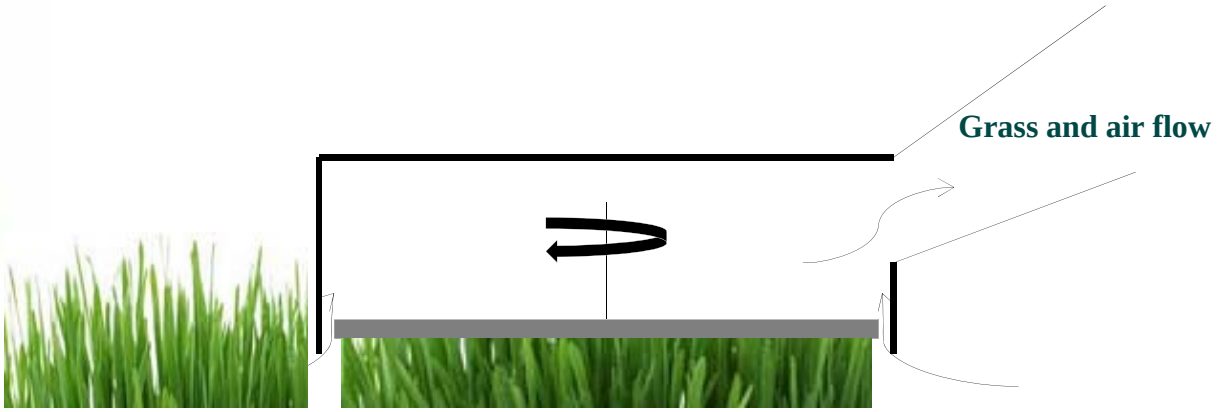
Grass exit



Grass and air flow

Torque

Polytropic process

$pv^n = \text{constant}$

$$\frac{p_x}{p_0} = \left(\frac{v_0}{v_x}\right)^n$$

then $$p_x = \left(\frac{v_0}{v_x}\right)^n p_0$$

Force

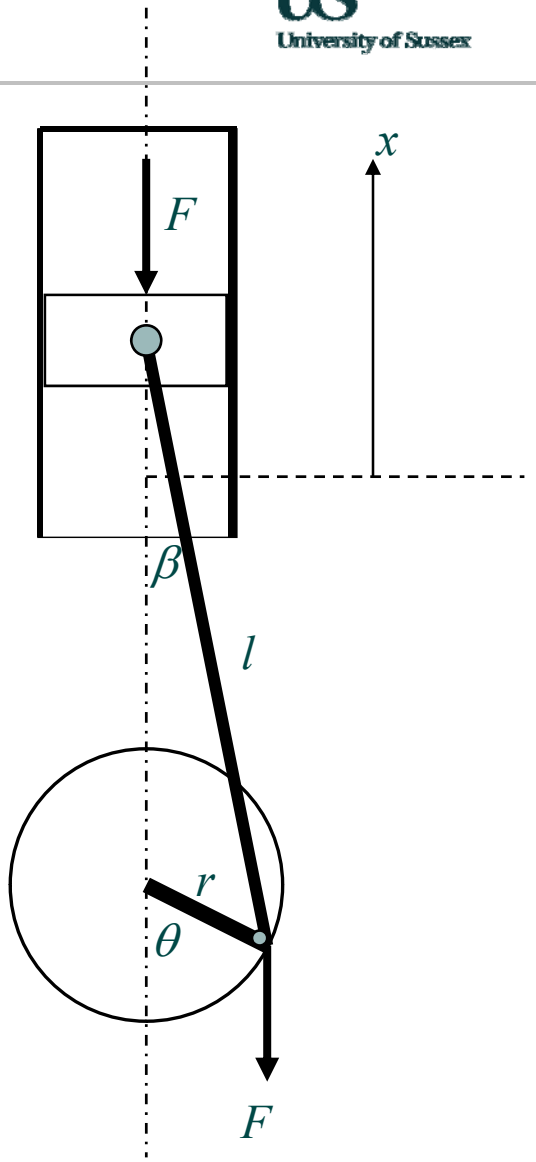
$$F = p_x A = \left(\frac{v_0}{v_x}\right)^n p_0 A$$

Torque:

$$T = Fr\sin\theta = p_x Ar\sin\theta$$

i.e.

$$T = p_0\left(\frac{v_0}{v_x}\right)^n Ar\sin\theta$$



Torque

$$x = l(\cos\beta - 1) + r(1 - \cos\theta)$$

$$v_x = v_0 - Ax = v_0 - A\left(l(\cos\beta - 1) + r(1 - \cos\theta)\right)$$

$$l\sin\beta = r\sin\theta$$

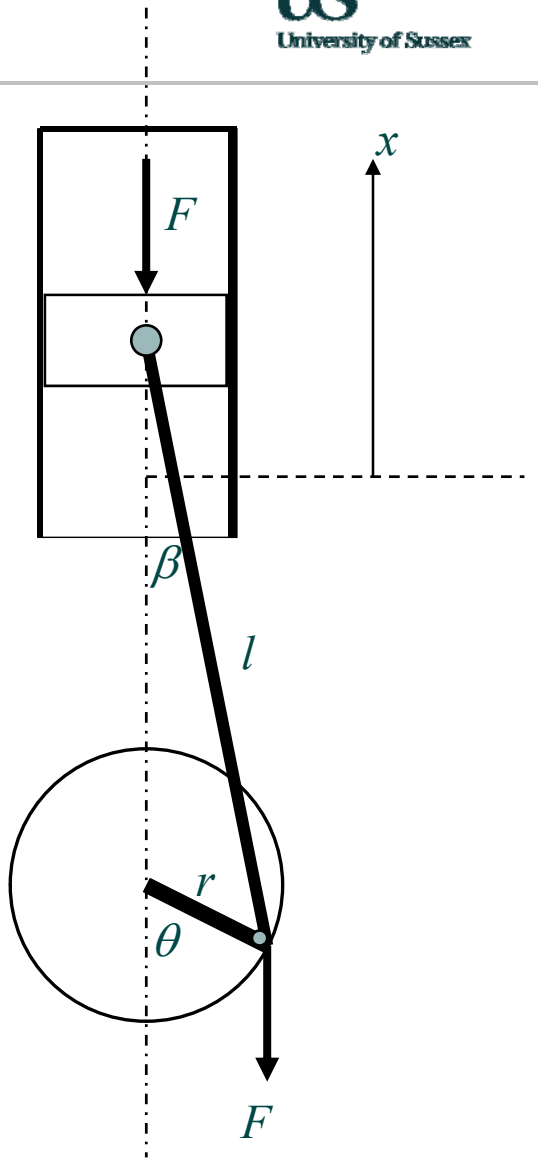
$$\sin\beta = \frac{r}{l}\sin\theta, \quad \cos\beta = \sqrt{1 - \left(\frac{r}{l}\sin\theta\right)^2}$$

Force

$$F = \left(\frac{v_0}{v_0 - A\left(l(\cos\beta - 1) + r(1 - \cos\theta)\right)}\right)^n p_0 A$$

Torque

$$T = \left(\frac{v_0}{v_0 - A\left(l(\cos\beta - 1) + r(1 - \cos\theta)\right)}\right)^n p_0 Ar\sin\theta$$



Forces on piston and bearing

Force acting on the crank bearings may include 3 forces

$$F = p_x A + \left(m_p + \frac{1}{3} m_r \right) \ddot{x} + m_{p+r+s} g$$

$$x = l(\cos\beta - 1) + r(1 - \cos\theta)$$

$$\dot{x} = -l\dot{\beta}\sin\beta + r\dot{\theta}\sin\theta$$

$$\ddot{x} = -l\left(\dot{\beta}^2\cos\beta + \ddot{\beta}\sin\beta\right) + r\left(\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta\right)$$

since

$$\sin\beta = \frac{r}{l}\sin\theta$$

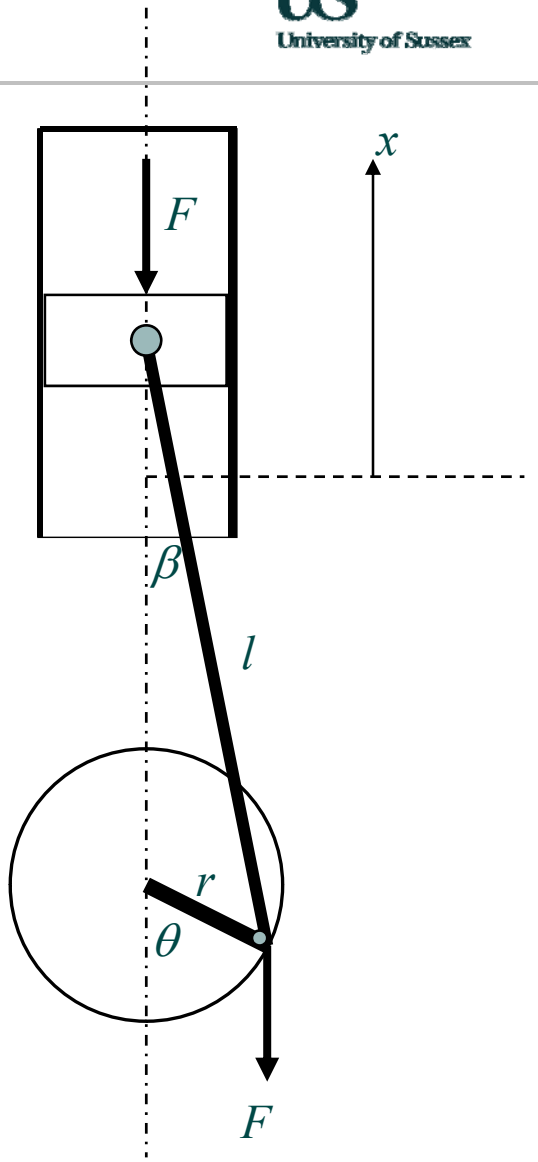
$$\dot{\beta} = \frac{r}{l}\frac{\cos\theta}{\cos\beta}\dot{\theta}$$

$$\theta = \omega t, \qquad \dot{\theta} = \omega, \qquad \dot{\beta} = \frac{r}{l}\frac{\cos\theta}{\cos\beta}\omega, \qquad \ddot{\theta} = 0.$$

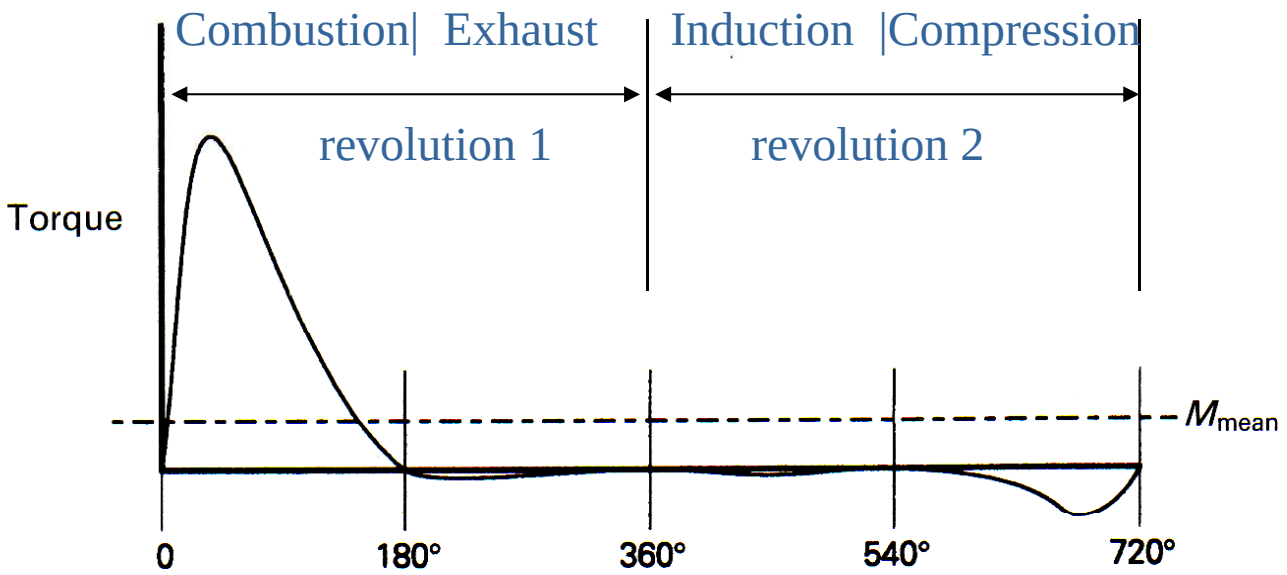
$$\ddot{\beta} = \frac{r}{l}\left(\frac{\cos\theta}{\cos\beta}\ddot{\theta} + \frac{\dot{\theta}\cos\beta\sin\theta - \dot{\beta}\cos\theta\sin\beta}{\cos^2\beta}\dot{\theta} \right)$$

$$= \frac{r}{l}\left(\frac{\omega^2\cos\beta\sin\theta - \left(\frac{r}{l}\frac{\cos\theta}{\cos\beta}\omega\right)\omega\cos\theta\sin\beta}{\cos^2\beta} \right)$$

$$= \frac{r}{l}\omega^2\left(\frac{\sin\theta}{\cos\beta} - \frac{r}{l}\frac{\cos^2\theta\sin\beta}{\cos^3\beta} \right)$$



Vibration Source



Vibration Source

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Rotating masses are balanced

Reciprocating masses cannot be balanced

Piston inertia force:

$$f = ma = m\left(\omega_c^2 r\cos\theta + \frac{\omega_c^2 r^2\cos 2\theta}{l}\right)$$

$$m - piston_mass + 1/3_connecting_rod_mass$$

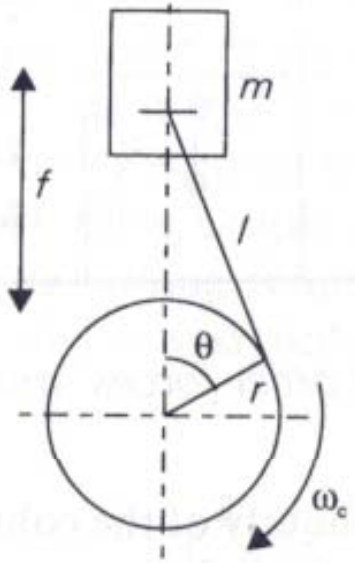
$$\theta = \omega_c t \qquad \dot{\theta} = \omega_c \qquad \ddot{\theta} = 0$$

$$l\sin\alpha = r\sin\theta$$

$$l\dot{\alpha}\cos\alpha = r\dot{\theta}\cos\theta$$

$$\dot{\alpha} = \frac{r\dot{\theta}\cos\theta}{l\cos\alpha} = \frac{r\dot{\theta}\cos\theta}{l\sqrt{1-\left(\frac{r}{l}\right)^2\sin^2\theta}} = \frac{r\dot{\theta}\cos\theta}{\sqrt{l^2 - r^2\sin^2\theta}}$$

$$x = (r+l) - r\cos\theta - l\cos\alpha$$



$$\theta = \omega_c t \qquad \dot{\theta} = \omega_c \quad \ddot{\theta} = 0$$

$$l \sin\alpha = r \sin\theta \rightarrow \sin\alpha = \frac{r}{l}\sin\theta$$

$$l\dot{\alpha}\cos\alpha = r\dot{\theta}\cos\theta \rightarrow \dot{\alpha} = \frac{r\dot{\theta}\cos\theta}{l\cos\alpha} = \frac{r\dot{\theta}\cos\theta}{\sqrt{l^2 - r^2\sin^2\theta}}$$

$$x = (r+l) - r\cos\theta - l\cos\alpha$$

$$\dot{x} = r\dot{\theta}\sin\theta + l\dot{\alpha}\sin\alpha = r\dot{\theta}\sin\theta + l\frac{r\dot{\theta}\cos\theta}{\sqrt{l^2 - r^2\sin^2\theta}}\frac{r}{l}\sin\theta = r\dot{\theta}\sin\theta + \frac{r^2\dot{\theta}\sin 2\theta}{2\sqrt{l^2 - r^2\sin^2\theta}}$$

$$\ddot{x} = r\ddot{\theta}\sin\theta + r\dot{\theta}^2\cos\theta + \frac{r^2}{2}\frac{\sqrt{l^2 - r^2\sin^2\theta}\left(\ddot{\theta}\sin 2\theta + 2\dot{\theta}^2\cos 2\theta\right) - \dot{\theta}\sin 2\theta\dfrac{-r^2 2\dot{\theta}\sin\theta\cos\theta}{2\sqrt{l^2 - r^2\sin^2\theta}}}{l^2 - r^2\sin^2\theta}$$

$$= r\dot{\theta}^2\cos\theta + \frac{r^2}{2}\frac{2\dot{\theta}^2\cos 2\theta\sqrt{l^2 - r^2\sin^2\theta} + \dfrac{r^2\dot{\theta}^2(\sin 2\theta)^2}{2\sqrt{l^2 - r^2\sin^2\theta}}}{l^2 - r^2\sin^2\theta}$$

$$= r\dot{\theta}^2\cos\theta + \frac{r^2\dot{\theta}^2\cos 2\theta}{\left(l^2 - r^2\sin^2\theta\right)^{1/2}} + \frac{r^4\dot{\theta}^2(\sin 2\theta)^2}{4\left(l^2 - r^2\sin^2\theta\right)^{3/2}}$$

i.e.

$$\ddot{x} = r\omega_c{}^2\left[\cos\omega t + \frac{\cos 2\omega_c t}{\left(n^2 - \sin^2\omega_c t\right)^{1/2}} + \frac{1 - \cos 4\omega_c t}{8\left(n^2 - \sin^2\omega_c t\right)^{3/2}}\right] \qquad (n = l/r)$$

Piston first and second inertia exciting forces

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$$\theta = \omega_c t$$

$$f = m\omega_c^2 r \cos\omega_c t + \frac{m\omega_c^2 r}{n}\cos 2\omega_c t$$

$$First_order_inertia_force:$$

$$m\omega_c^2 r \cos\omega_c t$$

$$Second_order_inertia_force:$$

$$\frac{m\omega_c^2 r}{n}\cos 2\omega_c t$$

Noise sources

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- Exhaust noise is the main noise source.
- Structural borne noise: vibrating frequency is greater than 100hz.

If main shaft speed 3600 rpm = 60hz its basic harmonic does not generate audible structural borne noise noise but 2nd and higher harmonics do.

Flywheel

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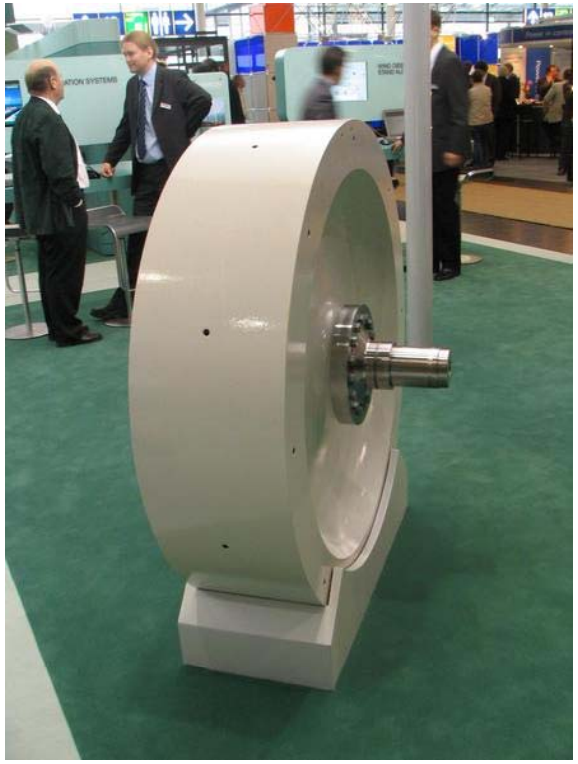
A flywheel is a rotating mechanical device that is used to store rotational energy.

Flywheels have a significant moment of inertia and thus resist changes in rotational speed.

Flywheel







Moment of inertia :

$$I = mk^2$$

k : Radius of gyration

Quantum Power Engine flywheel

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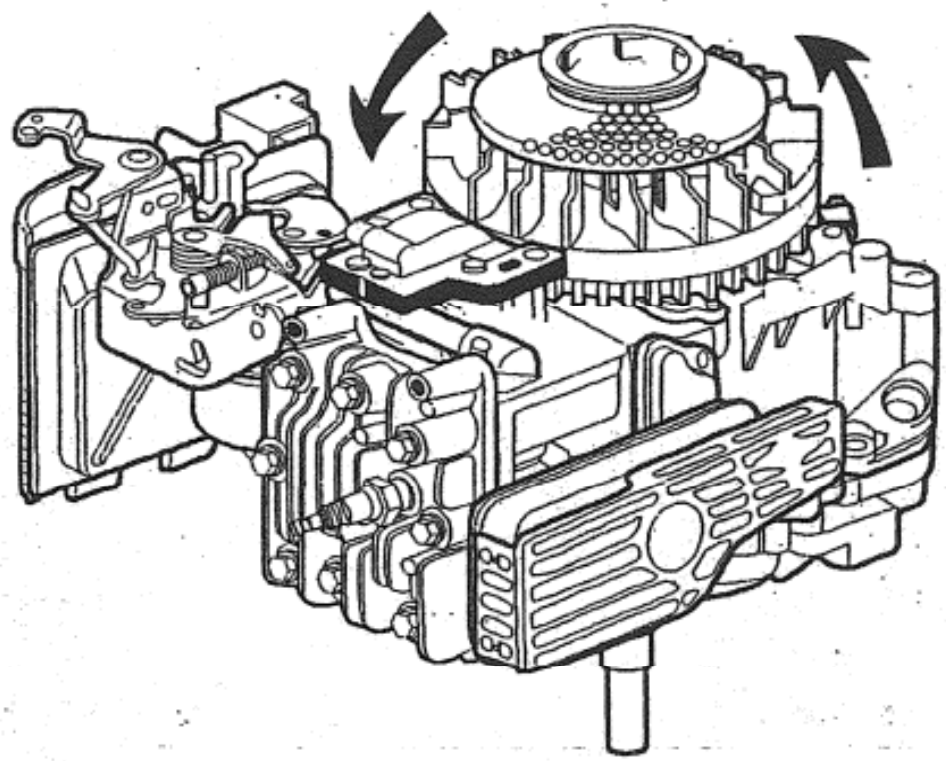
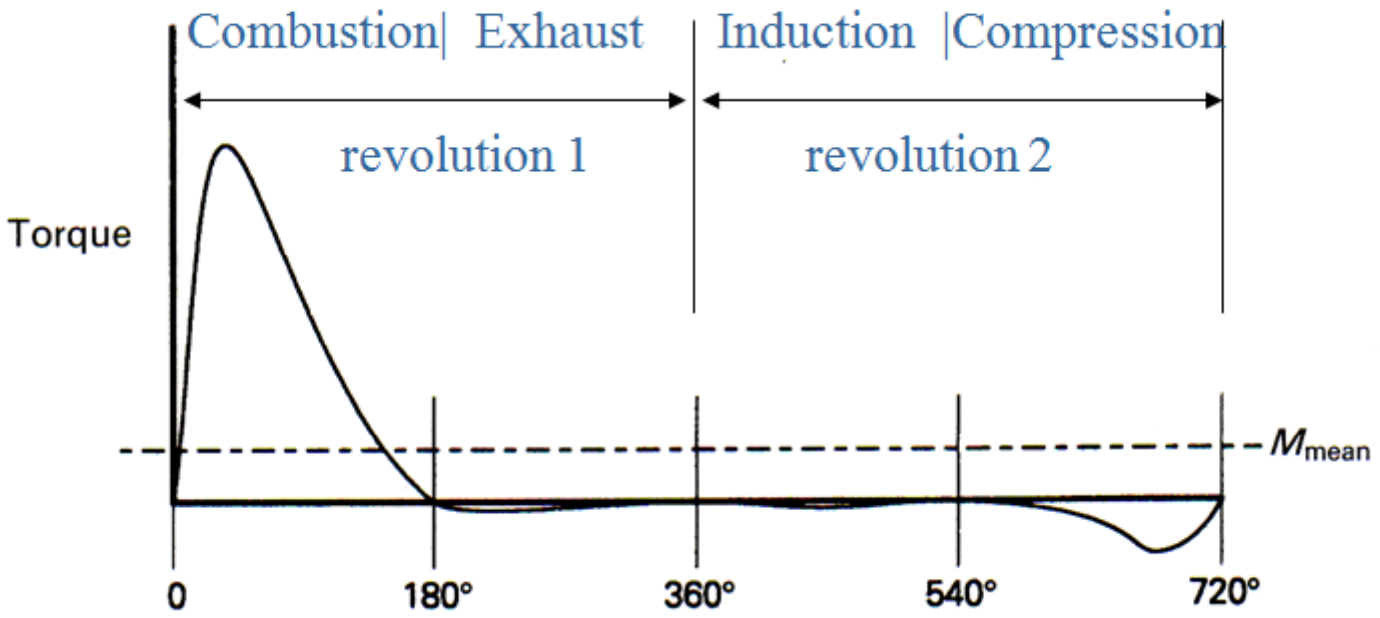
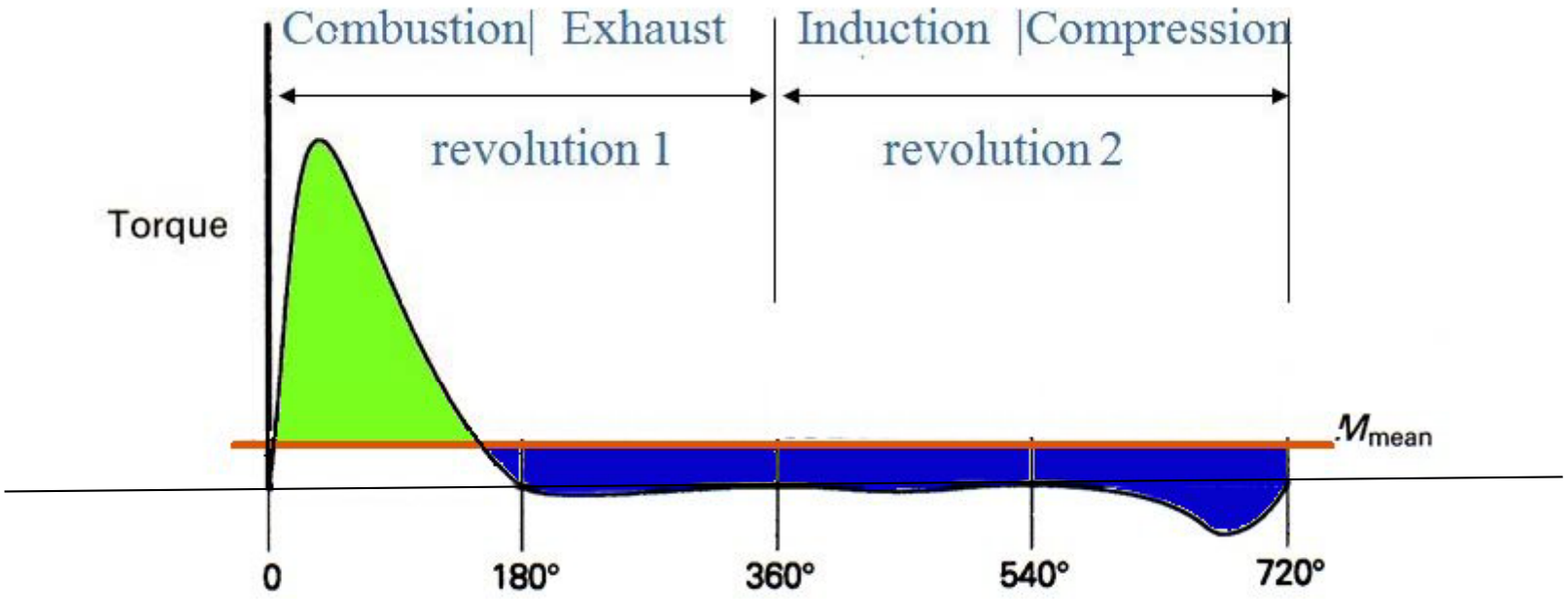


4 stroke engine torque



Work done per cycle



$$720^{o} = 4\pi$$

$$\text{work done per cycle} = T_{mean} \cdot 4\pi$$

Green area = blue area

Energy fluctuation

$$\text{Energy} = \frac{1}{2} I\omega^2 \qquad (Joul)$$

$$\text{Power} = \mathrm{T}\dot{\theta} = T\omega \qquad (Watt)$$

coefficient of fluctuation of speed :

$$\phi = \frac{\text{variation in speed}}{\text{mean speed}} = \frac{\omega_{\max} - \omega_{\min}}{\omega_{mean}}$$

coefficient of fluctuation of energy :

$$\beta = \frac{\frac{1}{2} I\omega_{\max}^2 - \frac{1}{2} I\omega_{\min}^2}{4\pi \mathrm{T}_{\text{mean}}} = \frac{I\left(\omega_{\max}^2 - \omega_{\min}^2\right)}{8\pi T_{mean}}$$

Energy fluctuation

$$\beta = \frac{I\left(\omega_{\max}^{2} - \omega_{\min}^{2}\right)}{8\pi T_{mean}}$$

$$I = \frac{8\beta\pi\ T_{mean}}{\omega_{\max}^{2} - \omega_{\min}^{2}}$$

Since

$$\omega_{\max}^{2} - \omega_{\min}^{2} = \left(\omega_{\max} + \omega_{\min}\right)\left(\omega_{\max} - \omega_{\min}\right) \approx 2\omega_{mean}\ \phi\omega_{mean} = 2\phi\omega_{mean}^{2}$$

Therefore

$$I = \frac{4\beta\pi\ T_{mean}}{\phi\omega_{mean}^{2}}$$

Example of flywheel

A 3.5 horse power, running at 2500rpm mean speed, 4 stoke single cylinder IC engine requires a flywheel with a fluctuation of speed ±2% and fluctuation of energy 5%. Its radius of gyration is 100mm. Find the mass of the flywheel.

Power: 3.5 ×745=2600 (Watts)
Mean speed: 2500rpm/60×2π=261.8rad/s
Mean Torque:2600/261.8=9.93(Nm)

$$I = \frac{4\beta\pi T_{mean}}{\phi\omega_{mean}^2} = \frac{4 \times 0.05 \times \pi \times 9.93}{0.04 \times 261.8^2} = 0.0023 \ (kgm^2)$$

mass of flywheel :

$$\mathrm{m} = \frac{I}{k^2} = \frac{0.0023}{0.1^2} = 0.23\,(kg)$$