Representational Interpretive Structure: Theory and Notation

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Abstract. A cognitive theory of the interpretive structure of visual representations (*RIST*) was proposed by Cheng (2020), which identified four classes of schemas that specify how domain concepts are encoded by graphical objects. A notation (*RISN*) for building RIST models as networks of these schemas was also introduced. This paper introduces common RIST/RISN network structures – *idioms* – that occur across varied representations. A small-scale experiment is presented in which three participants successfully modelled their own interpretation of three diverse representations using RIST/RISN and idioms.

Keywords: Cognition, representations, interpretation, schemas, idioms.

1 Introduction

To advance the study of Diagrams, and visual representations in general, the field requires a comprehensive cognitive account of how readers of representations *interpret representations*. Such a theory is needed for multiple reasons.

(A) Although it is tempting to assume, say, for the sake of theoretical analysis, that a representation has one 'correct' reading, this mask the full diversity of the readers' interpretations. It is unlikely that two readers of a given representation will naturally construct identical interpretations. So, some approach to systemically describe those varied interpretations could be valuable; for example, the mastery of visual representations is critical in STEM subjects, so there is pedagogic utility in being able to characterise what differs between novice and competent readers of a target representation.

(B) The particular content of any given topic can be encoded in quite distinct representations, with dramatic differential impacts on problem solving and learning across those representations (e.g., [3], [10], [22]). Thus, an approach to estimating the relative cognitive benefits of alternative interpretations of representations could be useful. For instance, such measures could be deployed in the development of automated systems

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to select effective representations tailored to individuals and classes of problems (e.g., [8], [20]).

(C) Related to the previous point, but more fundamental, is the issue of how even to compare representations with substantially different formats that encode the same informational content. Conventionally, comparison of alternative representations involves laborious task analyses (e.g., [2], [3]) or cognitive modelling (e.g., [10], [11]), or empirical studies (e.g., [2], [3], [22]). Instead, an approach at an intermediate level of abstraction could obviate the toil of ultra-fine-grained analyses and costly experiments. The approach will require the formulation of generic, format-independent, theoretical constructs that are applicable to all representations. Such constructs could serve as "natural" explanatory entities for interpretations. For these reasons, a cognitive theory of the structure of interpretations of representations is a worthy goal.

A contrast with linguistics is instructive. Linguistics has produced accounts of the interpretation of natural language which specify cognitive structures and processes of meaning extraction from verbal representations (e.g., [9], [16]). Many accounts of the nature of diagrams address structure (e.g., [10], [17], [18], [21], [22]) but comparatively less attention has been paid to how individuals interpret or comprehend diagrams ([11], [12]).

Our purpose here is to take the next step towards a general cognitive theory of the interpretation of representations, by testing the "sketch" of the theory developed by Cheng [4], which we will call *Representational Interpretative Structure Theory* (RIST). The RIST sketch proposed that the human interpretation of representations deploys four elementary types of mental schemas. Critically, the schemas coordinate information about concepts from a target topic with information about how those concepts are encoded in the graphical components of the representations. To operationalise RIST, Cheng [4] also outlined a *graphical notation* for constructing models of interpretations under RIST, which we will call *RISN* (RIS Notation). RIST and RISN[1] are described in Section 2 of this paper.

In Section 2 we introduce RIST and RISN, and take the opportunity to increase the precision of the definition of RIST's components and to more tightly specify how RISN captures particular interpretive constructs. In Section 3, we introduce and describe patterns of elementary schemas – *idioms* – that commonly occur in interpretations, which we discovered in RIST/RISN networks across diverse representations. Idioms have the potential to meet the requirement that RIST identifies "design patterns" as standard interpretive structures for constructing RISN models [4]. As noted above (reason A), different readers of a given representation will naturally construct alternative interpretations of that representation, so the requirement that RIST accounts for, and for RISN to model, alternative interpretations is investigated in a small-scale experiment in Section 4. Drawing these advances together, in Section 5, we will briefly consider how RIST and RISN may yield estimates of the cognitive cost of making alternative interpretations of a representation (reason B), and how RIST and RISN may provide a neutral approach to the cognitive analysis of representations that is independent of the particular format of representations (reason C).

[1] Pronounced like "wrist" (/ˈɹɪst/) and "risen" (/ˈɹɪzən/), respectively.

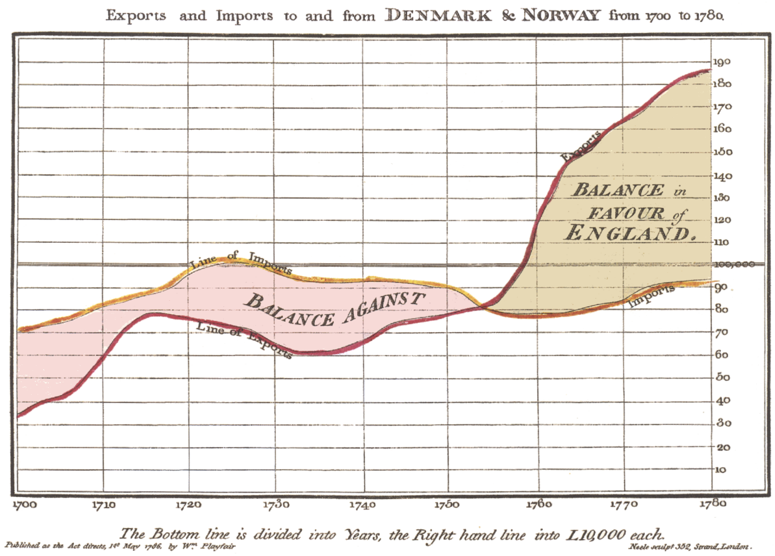

Fig. 1. William Playfair's line graph, in *Commercial and Political Atlas*, 1786.

2 Representation Interpretation Theory/Notation – RIST/RISN

To introduce RIST and RISN [4], we adopt a running example of the analysis of the interpretation of a famous diagram – Playfair's line graph, Fig. 1. Following Cheng's [4] analysis guidelines, Fig. 2 annotates the important graphical components of Playfair's line graph, and Fig. 3 is a RISN model of the graph[2].

2.1 Four schemas

RIST hypothesises that four *schemas* underpin our ability to interpret representations[3]. The fundamental purpose of these schemas is to tightly coordinate concepts from the target topic with the graphic objects in the representation that stand for those concepts. Networks of these schemas encode the rich hierarchical structure of the encoding relations that constitutes an interpretation of a representation. RISN is a system for modelling such networks; Fig. 3 is an example. At the highest level is the *Representation* schema, capturing an entire representation. *R-Scheme* schemas capture intermediate level sub-structures. *R-Dimension* schemas deal with varying quantities; they describe

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[2] Fig. 3 was drawn in a web browser tool, RIS Editor (RISE), that was specifically developed for creating RISN models. The tool will be presented in a paper to follow.

[3] A schema is a mental knowledge representation for a category defined by a set of attributes (slots) for which a particular instance of a concept is assigned values (fillers); e.g., [16].

R-symbol domains. The *R-symbol* schema identifies the 'unitary' concepts of the target topic. Their depiction in RISN models is shown in Fig. 4 and examples are scattered throughout Fig. 3. Let us consider them in turn, in reverse order.

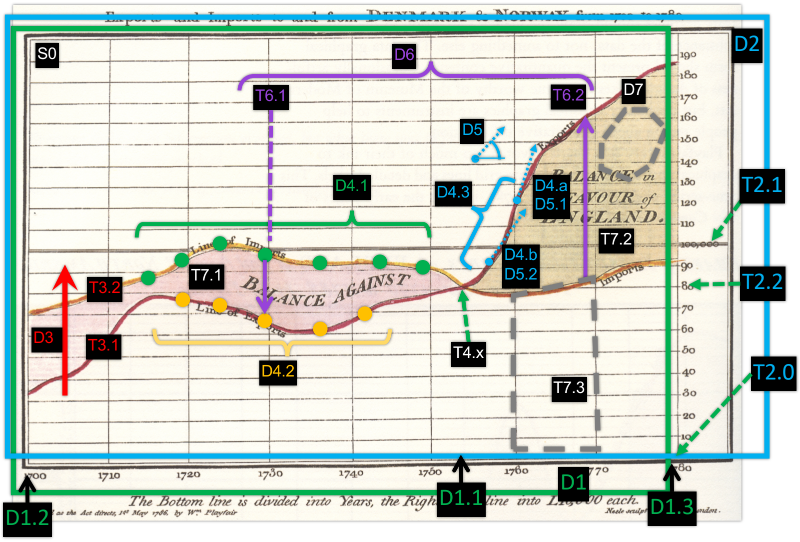

Fig. 2. Playfair's line graph as annotated for modelling (by R1).

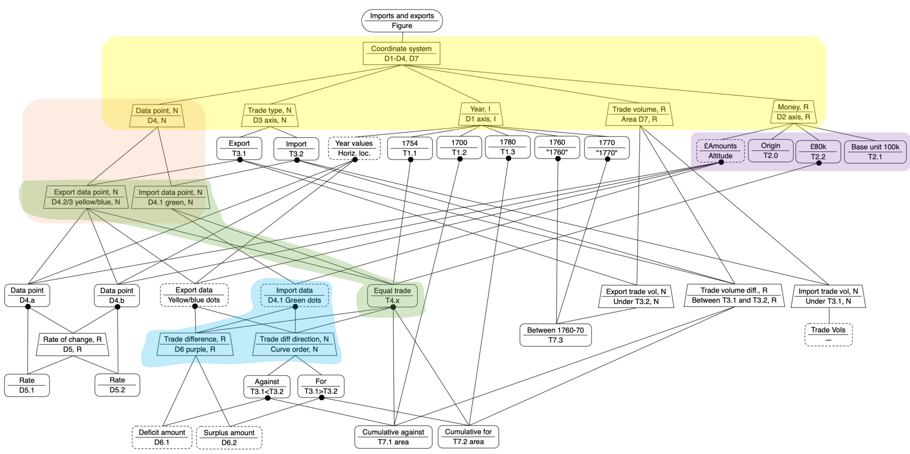

Fig. 3. Model of the interpretation of Playfair's line graph (by R1). Colour shadings are for reference and not part of the model.

R-symbols[4]**.** R-symbols are the 'fixed' elements of a representation. Their role is to code the association of concept with the graphic object representing it. In RISN, R-

[4]*R-symbol* supersedes *Token* used in [4] for reasons of notational and theoretical consistency.

symbols are rounded rectangles, with labels identifying the concept and graphical object (Fig. 4d). In Fig. 2, the overlaid annotations with labels beginning with a ''T' are instances of R-symbols, and these labels are written in the slots of the corresponding R-symbol icons in Fig. 3. The graphic object may also be described (e.g., *altitude*). For textual graphic objects, the text in quotes may be written in the R-symbol icons (e.g., "1770" in Fig. 3). Critically, through the structure of its R-symbol schema, RIST asserts the distinction between what is being represented, the concept, and what is it is being represented by, the graphical object: they should not be conflated. For example, in Playfair's line graph the graphic object "80" on the y-axis, labelled *T2.2* represents the concept '£80,000'.

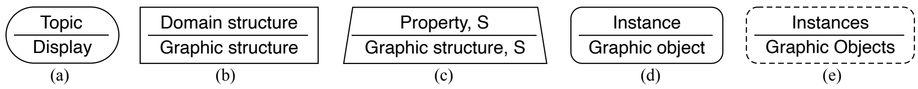

Fig. 4. The four schemas as icons: (a) Representation; (b) R-Scheme; (c) R-Dimensions (S=quantity scale alignment); (d) R-symbol; (e) class R-symbol.

R-Dimensions. This schema encodes concepts about attributes, features or dimensions of the topic that are variable in that they may be assigned alternative values. R-dimension concepts are more general than those encoded by R-symbols. These concepts concern the variability of some feature or attribute of the topic. In the schema for R-dimensions, RIST simultaneously distinguishes the concept of variable quantities from its graphic object whilst also declaring their association. R-dimensions are drawn as a trapezium, with labels for the concept and graphic object, Fig. 4c. In the line graph model, Fig. 3, five global R-dimensions are identified: *Year*-D1 (x-axis); *Money*-D2 (y-axis); *Trade type*-D3 (z-axis for trade curves); *Trade volume*-D7 (area); *Data point*-D4.

An R-dimension's concept is analogous to a mathematical type: R-dimensions range over R-symbols. R-symbols belong to at least one R-dimension; e.g., the *Year* R-dimension possess R-symbols for individual or a group of actual year values.

Given the underpinning role of quantity scales in inference, RIST requires that RISN models identify the *quantity scale* [19] for both the concept and the graphic object of each R-dimension. Whether each is a *nominal*, *ordinal*, *interval* or *ratio* scale is registered by a letter – *N*, *O*, *I* or *R,* respectively – appended to the concept and graphical object labels in the R-dimension icon (see Fig. 4c). Mismatches between concept and graph object quantity scales, which may hinder interpretation, are thus made apparent.

R-Schemes. R-Schemes capture complex structures within the representations, from large structures that span the entire representation, to local structures that organize just a few R-symbols. While R-Dimensions collect many R-symbols of a similar kind, R-Schemes are typically heterogeneous: they link together different R-Dimensions, R-symbols, or other R-Schemes, into some larger structure. R-Schemes are drawn as a rectangle in RISN (Fig. 4b). The RISN model (Fig. 3) for the interpretation of Playfair's graph (Fig. 1) has an overarching R-scheme composed of five R-dimensions.

Representations. At the highest level is the Representation schema. Representation schemas are drawn as lozenges (Fig. 4a). This schema defines a complete representation and a RISN model always has a Representation schema at its root. However, sub-Representations can occur in other parts of a RISN model, when there is a distinct nested representation within a larger representation (see anchoring below).

2.2 Linking schemas

RIST conceptualizes interpretations of representations as rich hierarchical networks of relations among the four schemas. With the schemas defined, we can begin linking them together. RISN models must be connected. Here, we introduce a more precise definition of the three kinds of links proposed: *hierarchy*, *anchoring*, and *equivalence*.

Hierarchy. This most fundamental link asserts when one schema is conceptually enclosed by another. For example, R-symbols enclosed under R-Dimensions will represent a specific value from that R-dimension. The hierarchy link can be formed between any two schemas, with the following exceptions:

- The 'child' of a hierarchy link is never a Representation schema, because a Representation schema stands for a complete representation (but see anchoring below).
- An R-symbol schema can only be the parent of another R-symbol schema, because they are the base-level components of RIST/RISN (but see anchoring below).
- An R-dimension schema cannot be the parent of an R-Scheme schema, because R-dimensions only range over R-symbols.

We notate hierarchy using a thin solid line (no arrow heads). The hierarchy link is directed: the direction is indicated by connecting to the parent schema from below, and the child schema from above. Some subsequent properties of RISN models are:

- All schemas, except for the root Representation schema, must have at least one parent schema.
- All schemas must have at least one child, except for R-symbol schemas and non-root Representation schemas: they are the 'leaves' of a RISN model.
- A schema may not be the parent of any schemas that are its ancestors – that is, RISN models are acyclic. However, a schema may have multiple parents, and so parallel paths may exist.

Anchoring. *Anchoring* links denote a new substructure that exists as a direct result of the parent R-symbols. Anchoring is a rich relation where a new concept emerges. We denote anchoring using solid thin line, with a bullet terminal at the parent. The link is thus directed, with the direction being shown by the position of the bullet. The parents *must* be R-symbol schemas, but there is no restriction on the children except that they are not an ancestor of the parent – that is, anchors must not introduce cycles into the RISN model. For example, in Fig. 3 (left), the sequence of hierarchy and anchor relations from the D3 R-dimension through to the D5.1 R-symbol, via D4.2/3, D4.a and

D5, expresses the notion that export data points are identified by the export curve and that it is only meaningful to speak of a specific rate of change of the curve with reference to a particular data point. Anchoring is more than just a sub-R-symbol relationship, such as a segment of a line, or the digits in a number.

A (sub-)Representation schema may be anchored to an R-symbol; for example, a Representation Schema for Hindu-Arabic numbers can be added to some of the leaf nodes in Fig. 3, if we wish to elaborate the inner workings of that numeration system.

Equivalence. It is useful to register cases of repeated symbols for concepts (e.g., the two 'x' in $x\times2=x+3$), because of their potential impact to the cognitive efficacy of a representation. Further, a single sophisticated concept in a representation may be encoded by quite different subnetworks of schemas in a RISN model; for instance, imagine that the areas for trade *against* and *in favour* in Fig. 1 are equal. The *equivalence* link captures the 'mental bookkeeping' that occurs during such interpretations, in which the reader must hold in mind the relationships between different parts of the representation. It is not intended to capture "mathematical" equivalence – although it may do, if this is part of the mental bookkeeping. Equivalence links are undirected and represented by a thick, dashed line with no terminals. There are no restrictions on what can be connected via the equivalence relation, allowing cycles in RISN models.

That completes the summary of RIST and RISN. We have outlined RIST's "words" and "grammar" for composing "sentences" that express interpretations of representations. RIST makes strong claims about the fundamental mental knowledge structures we use to interpret representations (the four schemas) and how interpretation occurs (construction of networks of those schemas). In this paper, the adequacy of the theory has been enhanced by more rigorously specifying RIST's components; in particular, the circumstances under which each type of link is applicable. Some of the ambiguity in Cheng's original theory sketch [4] has been eliminated, which provides greater constraint on the permissible schema networks.

3 Idioms: higher-order structures

Consider an analogy. Chemical theory is successful because it identifies elements and has rules by which atoms may be composed into molecules, but moreover it provides general categories of structures and processes; benzene rings, alcohol groups, or multi-bonded carbon atoms are substructures of organic molecules, each providing local information about the molecule as a whole. Similarly, we observe substructures of schemes within RIST models. Through many applications of RIST to diverse representations, both sentential and diagrammatic, we observed repeated substructures capturing common ideas emerge naturally: we call these *idioms*. Idioms serve dual purposes: first, they are an aid to interpreting RISN models; second, they can serve as guides when building RISN models. Three particularly common classes of idioms are introduced and described here: *collections*, *R-dimension* idioms, and *coordinate systems*.

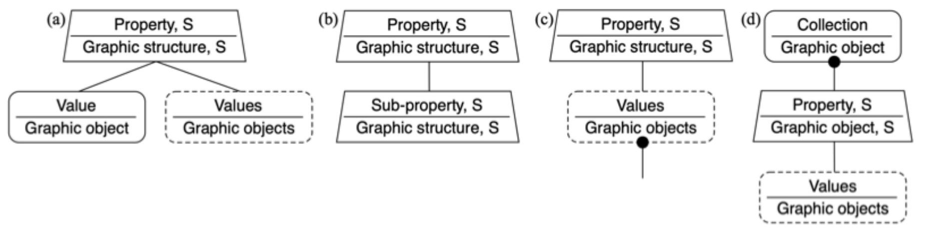

Fig. 5. Templates for (a) *pick*, (b) *filter*, (c) *for-each*, and (d) *reduce*.

3.1 Collections

We have found, frequently, that R-symbols are not just 'one-off' symbols within a representation: there are many points on a chart, many regions in an Euler diagram, and so forth. To capture this regularity, we allow for *class* R-symbols, Fig. 4e. However, we might want to discuss R-symbols as a group, or talk in general about the R-symbols without specifying an R-symbol in the class. We define four idioms on collections of representations: *pick*, *filter*, *for-each*, and *reduce*. Some readers might note that these names were inspired by functional programming, and draw helpful analogies [1].

The simplest collection idiom is *pick*: a single R-symbol is extracted from the class of R-symbols. This idiom can identify a single R-symbol as being of particular interest in an interpretation. We connect a new R-symbol(s) below the dimension and exclude it from the sibling class R-symbol, shown in Fig. 5a. An example in the Playfair's line graph model is shown by the purple shading in Fig. 3 (and Fig. 9).

When the model requires some subset of the R-symbol collection, we use the *filter* idiom. While all the R-symbols in a collection might belong to the same R-Dimension, that R-Dimension might be very general: sometimes, a specific subset is more useful in some context. In effect, this is a sub-R-Dimension, so is notated by introducing new sub-R-Dimensions below the original R-Dimension, Fig. 5b. The name of the *filter* idiom is inspired by the *filter* function common in programming languages: given a collection of values, extract just the values that match some predicate. For example, in the orange shading in Fig. 3, if a modeller wanted to just talk about the 'import data point' then only this schema would have been drawn, and thus considered as a *filter* idiom.

Often, some interpretation is true for all R-symbols in a class, regardless of which specific R-symbol is being considered. In RISN, we call this idiom *for-each*, Fig. 5c: any schemas under a class R-symbol in the model are true for all members of the class. We can draw analogy to the standard mathematical phrase 'without loss of generality': something true for every member of a set. For example, in the model of Playfair's line graph, Fig. 3 (left), the anchoring of the 'Export data' class R-symbol under the 'Year values' class R-symbol expresses the idea that each year has an export data value. In functional programming, this would be a *map*.

For the sake of clarity, class R-symbols merit further comment in the context of the *for-each* idiom. Class R-symbols are limited in how they connect to descendent schemas: like single R-symbols, they connect either to sub-R-symbols, or via anchoring. We discussed both types of connection in Section 2. In both cases, they apply to *each* *individual concept* included in the class R-symbol, not to the 'class' of R-symbols. For example, in Fig. 3, we have a class R-symbol 'Year values' under the 'Year' R-dimension plus individual R-symbols for year '1754' and four others. It would have been *incorrect* to make the '1754' R-symbol a child of the 'Year values' class R-symbol as it is not a sub-R-symbol of *every* R-symbol in the class 'Year values'.

Finally, when the individual R-symbols within the class are not specifically interesting, but the grouping of them is, we *reduce* them to a single R-symbol capturing the concept of the collection of R-symbols, Fig. 5d. The R-symbol for the concept of the collection is at the top, the class R-symbol for all the members of the collection is at the bottom of the structure, and in between we include an R-Dimension to identify the aspect common to the members that define the category.

This idiom is inverse to *for-each*: while *for-each* allows us to consider every member of a collection identically but individually, *reduce* allows us to consider the entire collection as a single unit. A common use for the *reduce* idiom is in plots of data, where there are emergent structures that exist only as collections of 'simpler' R-symbols. For example, in Fig. 9 below (grey shading), the 'Value of exports in a year' class R-symbol is reduced to the 'Line of exports' R-symbol via the 'Value of Exports' R-dimension.

Together, these collection idioms provide succinct, expressive modelling options for collections of R-symbols.

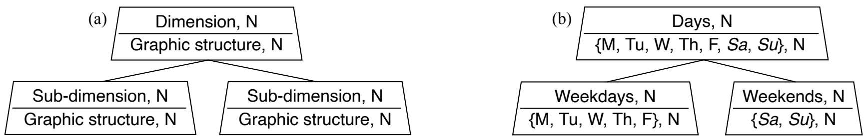

Fig. 6. (a) General model of sum R-dimensions. (b) Example using weekdays.

3.2 R-Dimension idioms

As mentioned earlier, we may think of an R-Dimension as a 'type' of R-symbols – all the R-symbols that are under the same R-Dimension in the hierarchy fill the same semantic role in the representation. Taking inspiration from this 'type' analogy, we present two idioms named after algebraic data types [6]: *sum* R-Dimensions, and *product* R-Dimensions.

A sum R-Dimension is an R-Dimension that has two or more sub-R-Dimensions. Just as a sum type is the union of its constituents, a sum R-Dimension is the union of the sub-R-Dimensions. We encode a sum R-Dimension in RISN in the obvious way: the sum R-Dimension is directly above its sub-R-Dimensions in the hierarchy. Fig. 6a presents the general idiom, while Fig. 6b is a diary example from a "week to a view" diary that differentiates weekday and weekend blocks. An example of sum R-dimensions in the Playfair line graph model, orange shading in Fig. 3, states that all datapoints are comprised of export plus import datapoints (see Fig. 10 for another example).

A product R-Dimension is an R-Dimension that combines two or more R-Dimensions. Just as a product type is the cartesian product of constituent types, the R-symbols of a product R-Dimension can be considered as some combination of the R-symbols of the constituent R-Dimensions. The direct analog in algebraic data types would be a tuple type. Product R-Dimensions are encoded in RISN as being directly under their constituent R-Dimensions in the hierarchy. The general idiom is shown in Fig. 7a, and an alternative shortcut of the idiom is in Fig. 7b for convenience. Fig. 7c is an example about citations that code the idea that combining author's surname, an ordinal quantity, with a year of publication, an interval quantity, produces a citation, which is an ordinal quantity. An example in the Playfair line graph model, green shading in Fig. 3, captures the idea that a datapoint for equal amounts of trade occurs when the data points for export and import data are identical.

For both sum and product R-Dimensions, the quantity scales of their resulting R-dimensions require careful consideration; the interaction in particular for a product R-dimension is complex, with no simple domain-independent rules governing the quantity scale of the resulting R-dimension.

Although R-Dimension idioms were presented in isolation, they can compose in powerful ways. With these R-Dimension structures for sums and products, we have a concise, powerful way to model rich interpretations by composing R-symbols or decomposing R-schemes.

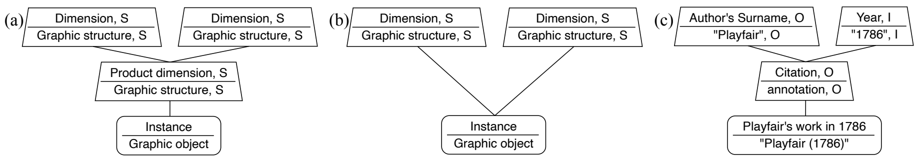

Fig. 7. (a) General model of product R-dimension. (b) Alternative shortcut for (a). (c) Example model of citations as product of author and year.

3.3 Coordinate systems

Representations are often structured around *coordinate systems*: literally, systems that coordinate information. In addition to the obvious cases – such as tables, and the Cartesian axes of graphs – coordinate systems occur when one or more R-Dimensions provide an indexing system for one, or more, R-Dimensions for sets of data. Coordinate systems setup linked conceptual and graphical spaces within which individuals are located. In practice, we find two idioms for modelling coordinate systems; *explicit* and *implicit*. In the case of explicit coordinate systems, the modeller specifically identifies a fixed set of R-dimensions that constitute the coordinate system that are distinct from the R-dimension(s) that categorises the dataset(s). A template for this case is shown in Fig. 8a. Information visualisations with graphical objects that define quantities, such as axes with scales or legends setting up categories, are typically interpreted as explicit coordinate systems. Alphanumerical index systems, such as book classification schemes, are explicit coordinate systems. Books in an unorder collection are indexed by R-dimensions for subject areas, sub-topics, author, year and the like.

In contrast, in an *implicit* coordinate system the distinction between what is an indexing R-dimension and a data R-dimension is not taken by the interpreter to be fixed but interchangeable. What counts as data depends on the user's current context. Fig. 8b shows the template for this idiom; the nested R-Scheme has gone, so the R-Dimensions all occur at the same level. The particular interpretation for Playfair's line graph in Fig. 3 includes an *implicit* coordinate system (yellow rectangle), because the modeller did not wish to single out points in the graph as the only dataset. Rather, the 'Data point' R-dimension is used as an index along with the 'Money' R-dimension to make a coordinate system dealing with 'Trade directions' and 'Trade differences' (centre left).

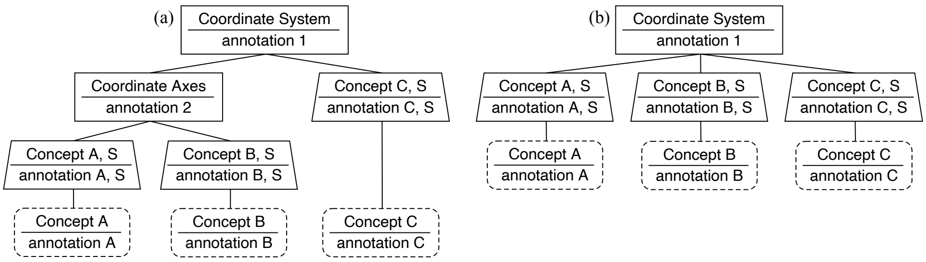

Fig. 8. (a) Template for a nested (explicit) coordinate system for a 2D representation. (b) Template of flat (implicit) coordinate system for a 2D representation.

Summary. Idioms, common sub-network structures of RIST schemas, have been discovered and each possess distinctive interpretive functions. This provides some reassurance about the potential validity, or at least utility, of schemas and relations proposed by RIST. Idioms introduce a new layer of interpretations between the elementary schemas and whole networks, which imposes theoretically desirable constrains on the space of possible network structures for modelling. In turn, this suggests that attempts to model the interpretations of representations could profitably focus on interpretive functions of idioms, an idea that is to be outlined in the last section.

4 Diversity of interpretations

So far, we have presented refinements to RIST's schema relations and introduced idioms to encode particular interpretive functions, both of which improve the adequacy of the theory. This section considers our first, albeit small-scale, empirical test of RIST and the capabilities of RISN. In particular, we wish to show that the theory and modelling notation are able to capture the alternative interpretations of a representation made by different readers, as mentioned in the Introduction. In the test, three of the authors ("reviewers"), who are experienced users of representational systems, independently created RISN models for 3 different representations. The representations were Playfair's line graph (Fig. 1), the Home tab from Microsoft PowerPoint's toolbar, and a chart about monetary flows in an economy depicted as a hydraulic model[5]. They were selected due to their diversity in both their form and function. Here, just the model for

[5] 'The Round Flow of Money Income and Expenditure, 1922': https://commons.wikimedia.org/wiki/File:The_Round_Flow_of_Money_Income_and_Expenditure,_1922.jpg

Playfair's line graph will be examined in detail, see Fig. 3, Fig. 9 and Fig. 10, but we summarize the outcomes of the other two representations.

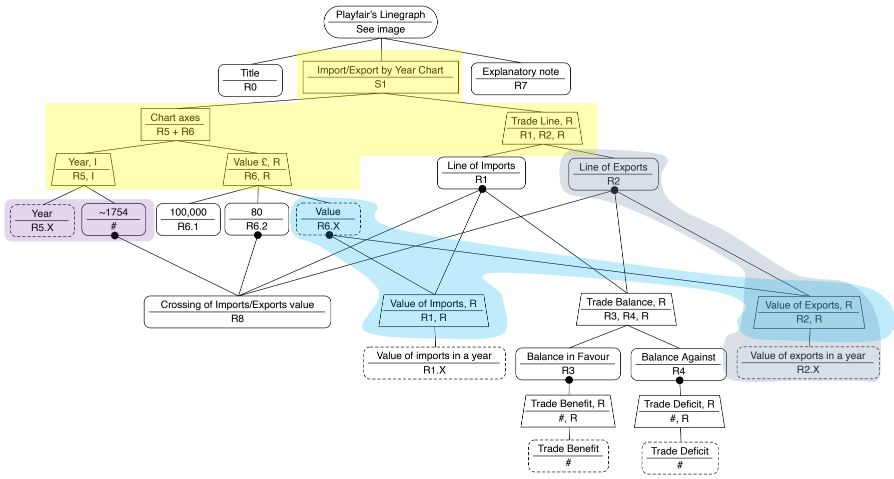

Fig. 9. Interpretation of Playfair's line graph by R2.

All reviewers had experience creating RISN models. They reviewed the guidelines for RIST/RISN before starting the task. They were instructed to model their own interpretation of the content of the representations. R1, R2 & R3, started by annotating the original line graph, Fig.1: R1's annotations are shown in Fig. 2, where *T*, *D*, and *S* labels stand for R-symbols, R-Dimensions, and R-schemes, respectively. The reviewers' RISN models for the line graph are shown in Figs. 3, 9 & 10. For reference, we highlighted parts of the models with coloured shadings. After finishing their individual models, the reviewers discussed the models and made edits that just corrected the invalid schema relations, which were few in number. We wished to determine if the models revealed meaningful differences in the reviewers' interpretations, and what the principal differences were.

R1's overall interpretation treats that representation as a complex coordinate system with five R-dimensions (Fig. 3, yellow shading). The concept of trade balance, 'Equal trade' R-symbol, depends on four of the R-dimensions, directly or indirectly, so is central to the network of schemas conceptually and happens to be positioned centrally in the diagram. Derived quantities, such as 'Trade volume (over a period of time)' and 'Rate (of change of trade)', are defined within the overarching coordinate system as a sub-R-dimension anchored on an R-symbol of some other R-dimension.

R2's interpretation has global coordinate system which incorporates the two graph axes as sub-system alongside an R-dimension for the lines in the graph (Fig. 9). Other R-dimensions, which were primary for R1, are derived concepts in R2's interpretation, defined relative to the context of particular values of the overarching coordinate system.

R3's model (Fig. 10) contrasts to R1 and R2 in terms of its overall interpretation. It gives the concepts of trade 'Balance' and 'Region' primacy and uses them to examine the relation of imports and exports relative to England. The coordinate system for the graph axes is seen as subservient to those ideas and is providing specific values as required.

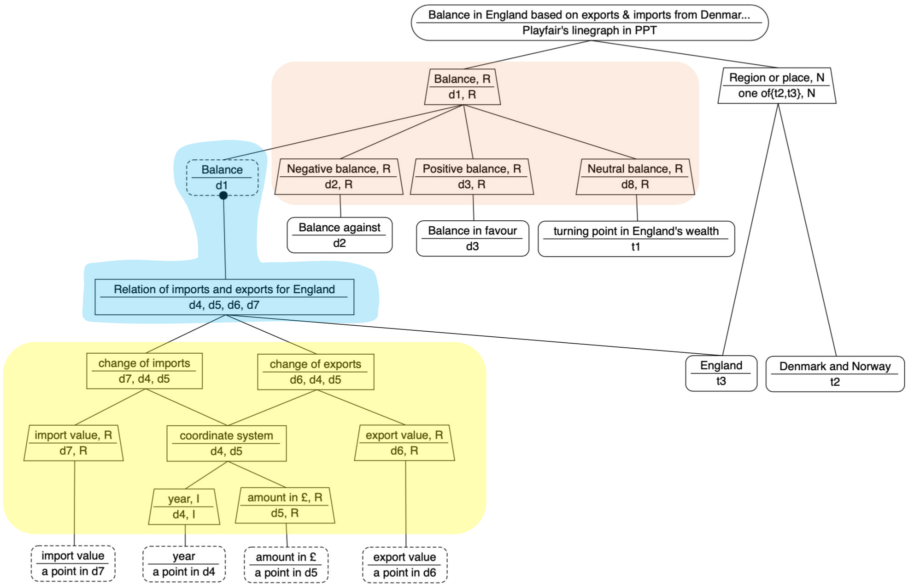

Fig. 10. Interpretation of Playfair's line graph by R3.

Comparing the topology of the models, all three models have approximately similar depth, but R1's model has greater breadth, which reflects concepts not in R2 and R3's interpretation. Examining the range and priority of concepts, R3's interpretation focuses on the topic's conceptual content – *what* is represented – whereas R1 and R2 are oriented more towards the means by which the line graph conveys the information – *how* the content is represented – using a global, high-level, coordinate system.

The idioms introduced in Section 3 provide a useful level of abstraction for our analysis of the models; like molecules being understood through their functional groups, we can understand our RISN models through their idioms. The coloured areas in Figs. 3, 9 and 10 exemplify some of them. The coordinate system idiom (in yellow) appears across all models, as described in the summaries above, but at different levels. The sum R-dimension is present in two of models: examples are shown Fig. 3 and Fig. 10 (orange shading). R1 splits the 'Data points' global R-dimension into exclusive sub-R-dimensions for 'Export' and 'Import' data. R3 divides trade 'Balance' into the three categories of 'Negative', 'Positive' and 'Neutral'. R1 and R2 also make equivalent distinctions related to trade balance, but a lower level.

There are also differences among how reviewers use idioms. All three use coordinate systems (Figs. 3, 9 & 10, yellow shading) and the 'for-each' idiom (blue shading), but their primacy in the interpretations varies. For R1 and R2, the coordinate takes precedence, with the 'for-each' idiom serving a narrower role. In contrast, R3 gives the 'for-each' idiom priority and hangs a coordinate system under that idiom. Another case is that of important "trade balance" concept, which is encoded in different ways by all three reviewers: R1 uses a product R-dimension idiom (Fig. 3, light green); R2 has a single R-symbol for a concept anchored on other R-symbols (Fig. 9, 'Crossing of Imports/Export values'); and for R3 it is an R-symbol of a sub-R-dimension of the primary 'Balance' R-dimension.

Similar observations apply to the PowerPoint toolbar and the economic flowchart modelling. For example, for the PowerPoint toolbar, R2's model includes the use of R-schemes for concepts extensively, whereas R1 and R3 tend to categorize and group concepts with R-dimensions. In spite of this, there is little variation in terms of the depth of the models across reviewers. The models for the economic flow are also diverse across reviewers. R3's model focusses on the topic, R2's model focusses more on the structure of the diagram, and R1's model is a mixture.

The modelling activities were followed by a session of reflection by the reviewers. From instances of ambiguity among the reviewer interpretations, it was apparent that there are some specific limitations to RISN expressiveness that need to be addressed. In particular, the semantics of the relation links between R-dimension and class R-symbol schemas needs clarifying, and when R-dimensions and class R-symbols have "common elements" or are disjoint.

5 Discussion

We presented Representational Interpretive Structure Theory, RIST. It proposes that interpretation of representations is cognitively grounded in four schemas whose primary function is to associate (a) concepts from the to-be represented target domain with (b) graphical objects in the representation that stand for those concepts. RIST specifies a small number of relations that link these schemas. RIST contends that an interpretation of a representation consists of a network of schemas that are linked by the relations. Different interpretations have alternative network structures. By examining numerous networks that model diverse representations, idioms were discovered that are common to representations with distinct formats. Idioms appear to perform specific interpretive functions and operate at an intermediate level between the elementary schemas and complete networks for whole representations.

RISN is a modelling notation for RIST, which possess distinct modelling symbols for each class of schemas. The symbols are connected together with lines that stand for relations between the schemas. RIST schema networks are modelled as networks of RISN symbols.

A small-scale experiment was conducted in which three reviewers produced models of their own interpretations of three heterogenous representations. The RISN networks produced across the different representations were varied and the networks produced by different reviewers, of the same representation, were also distinctive. The models varied both in the content and in their topology. Further, close examination of the models reveals that the overall interpretations are readily explicable in terms of the idioms. In other words, a reviewer could use the idioms to guide their understanding of the meaning of a RISN model produced by another reviewer. Some idioms were shared across all the reviewer's models for a given representation and in other cases different idioms were deployed in the interpretations of alternative reviews on the same representation. Thus, this small study provides some tentative preliminary evidence of the acceptability of RIST and the utility of RISN. However, further studies are needed in order to make more definite claims. Such studies are planned.

The Introduction proposed three desiderata for a cognitive theory of the interpretation of representations. The first concerns the facility to model alterative interpretations made by different individuals. The present study begins to demonstrate that RIST/RISN has this capability. Further, although anecdotal, the authors recognise that R1 has particular expertise with Cartesian plots, so it is no surprise that R1's model of the line graph had a greater breadth than the models of R2 and R3, as it included a greater range of concepts. Also, R3 was the least familiar with PowerPoint, so it is also not unexpected that the network models of R1 and R2 were broader. All this suggests that RIST/RISN could be used in an approach to model differences in the interpretative structure of learners with different level of experience of target representations.

The outcome of the small study also suggests that it may be feasible to model the different interpretive structures of alternative representations of the same subject matter. RIST/RISN might provide a useful method for the evaluation of alternative representations for particular topics. This would satisfy the second and third requirements described in the Introduction.

Finally, we note that this research was conducted as part of a wider project that is developing automated systems for the selection of representations for individual problem solvers with varying levels of competence on different classes of problems [13], [14], [15]. One aspect of the project is to devise a measure of the cognitive cost of representations [5], which can be used to assess the relative difficulty a user will likely experience with alternative representations. We note that RIST/RISN may provide an addition route to such assessments though the analysis of the contents of the schemas and the nature of their networks.

Acknowledgements. This work was supported by the EPSRC grants EP/R030642/1, EP/T019603/1, EP/T019034/1 and EP/R030650/1.

References

1. Bird, R., & Wadler, P. (1988). *Introduction to Functional Programming*. Hemel Hempstead: Prentice Hall International (UK).
2. Cheng, P. C.-H. (2002). Electrifying diagrams for learning: principles for effective representational systems. *Cognitive Science, 26*(6), 685-736.
3. Cheng, P. C.-H. (2011). Probably good diagrams for learning: Representational epistemic recodification of probability theory. *Topics in Cognitive Science, 3*(3), 475-498.
4. Cheng, P. C.-H. (2020). A sketch of a theory and modelling notation for elucidating the structure of representations. In A. Pietarinen, P. Chapman, L. Bosveld-de Smet, V. Giardino, J. Corter, & S. Linker (Eds.), *Diagrammatic Representation and Inference. Diagrams 2020. Lecture Notes in Computer Science, vol 12169*. Cham: Springer.

5. Cheng, P. C. H., Garcia Garcia, G., Raggi, D., Stockdill, A., & Jamnik, M. (2021). Cognitive Properties of Representations: A Framework. In A. Basu, G. Stapleton, S. Linker, C. Legg, E. Manalo, & P. Viana (Eds.), *Diagrammatic Representation and Inference* (pp. 415-430). Cham: Springer International Publishing.
6. Gordon, M., Milner, R., & Wadsworth, C.P. (1979). *Edinburgh LCF: A Mechanised Logic of Computation*. Lecture Notes in Computer Science. Berlin: Springer.
7. Gurr, C. A. (1998). On the isomorphism, or lack of it, of representations In K. Marriott & B. Meyer (Eds.), *Visual Language Theory* (pp. 293-306). New York, NY: Springer-Verlag.
8. Jamnik, M., & Cheng, P. C.-H. (2021). Endowing machines with the expert human ability to select representations: why and how. In S. Muggleton & N. Chater (Eds.), *Human-Like Machine Intelligence* (pp. 355-378). Oxford: Oxford University Press.
9. Kintsch, W. (1998). *Comprehension: A paradigm for cognition*. New York, NY: Cambridge University Press.
10. Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science, 11*, 65-99.
11. Peebles, D. J., & Cheng, P. C.-H. (2003). Modelling the effect of task and graphical representations on response latencies in a graph-reading task. *Human factors, 45*(1), 28-45.
12. Pinker, S. (1990). A theory of graph comprehension. In R. Freedle (Ed.), *Artificial Intelligence and the Future of Testing* (pp. 73-126). Hillsdale, NJ: Lawrence Erlbaum.
13. Raggi, D., Stockdill, A., Jamnik, M., Garcia Garcia, G., Sutherland, H. E. A., & Cheng, P. C. H. (2020). Dissecting Representations. In A.-V. Pietarinen, P. Chapman, L. Bosveld-de Smet, V. Giardino, J. Corter, & S. Linker (Eds.), *Diagrammatic Representation and Inference* (pp. 144-152). Cham: Springer.
14. Raggi, D., Stapleton, G., Stockdill, A., Jamik, M., Garcia Garcia, G., & Cheng, P. C.-H. (2020). How to (re)represent it? In S. Pan (Ed.), *32nd International Conference on Tools with Artificial Intelligence*: IEEE.
15. Raggi, D., Stockdill, A., Jamnik, M., Garcia Garcia, G., Sutherland, H. E. A., & Cheng, P. C.-H. (2019). Inspection and selection of representations. In C. Kaliszyk, E. Brady, A. Kohlhase, & C. Sacerdoti Coen (Eds.), *Intelligent Computer Mathematics - CICM 2019, Lecture Notes in Computer Science, vol 11617*. (pp. 227-242). Berlin: Springer.
16. Schank, R. C., & Abelson, R. P. (1977). Scripts, plans, goals, and understanding: an enquiry into human knowledge structures. Mahwah, NJ: Erlbaum.
17. Shimojima, A. (2015). Semantic properties of diagrams and their cognitive potentials. Stanford, CA: CSLI Press.
18. Stenning, K., & Oberlander, J. (1995). A cognitive theory of graphical and linguistic reasoning: logic and implementation. *Cognitive Science, 19*(1), 97-140.
19. Stevens, S. S. (1946). On the Theory of Scales of Measurement. *Science, 103*(2684), 677–680.
20. Stockdill, A., Raggi, D., Jamnik, M., Garcia Garcia, G., & Cheng, P. C. H. (2021). Considerations in Representation Selection for Problem Solving: A Review. In A. Basu, G. Stapleton, S. Linker, C. Legg, E. Manalo, & P. Viana (Eds.), *Diagrammatic Representation and Inference* (pp. 35-51). Cham: Springer International Publishing.
21. Zhang, J. (1996). A representational analysis of relational information displays. *International Journal of Human Computer Studies, 45*, 59-74.
22. Zhang, J. (1997). The nature of external representations in problem solving. *Cognitive Science, 21*(2), 179-217.