

Testing for homogeneity of variance with Hartley's F_{max} test:

In order to use a parametric statistical test, your data should show *homogeneity of variance*: in other words, the spread of scores in each condition should be roughly similar. (The spread of scores is reflected in the variance, which is simply the standard deviation squared). Sometimes, it's quite obvious that the variances are very dissimilar; you just need to look at them. In other cases, it's less obvious, and a more formal test is required. There are various ways to test for homogeneity of variance.

When performing some statistical tests, SPSS routinely tests for homogeneity of variance. For example, if you perform an independent-measures t-test, SPSS will also show the results of a Levene's test on the data. If the Levene's test result is statistically significant (the result has a $p \leq .05$), it means that the data do not show homogeneity of variance. If the Levene's test is not significant ($p > .05$) then you can assume that the data show homogeneity of variance.

If you are performing the statistical tests by hand, then it's easier to use Hartley's F_{max} test than Levene's.

(a) Divide the larger variance by the smaller one. This gives you an F-ratio (NB: this should not be confused with the F-ratio that's produced in ANOVA, this F-ratio is quite different). If the variances are similar to each other, then the F-ratio will be close to 1: the more the variances differ, the larger the F-ratio will be.

$$F_{max} = \frac{\text{larger variance}}{\text{smaller variance}}$$

(b) If the F-ratio is very close to 1, you are safe in concluding that the data probably show homogeneity of variance. If the F-ratio is quite a bit larger than 1, then to decide how likely it is to get your obtained F-ratio by chance, you need to use a table of F-max values. To use the table you need to know the **d.f.** (the number of participants in a group, minus 1) and **k** (the number of groups or conditions). Note that Hartley's test assumes that there are equal numbers of participants in each group. The table shows values of F_{max} that are so large that they would occur by chance with a probability of $p = .05$. If your obtained value of F_{max} is larger than the relevant value in the table, then by implication the difference between the variances that you have obtained is even *less* likely to have occurred by chance.

(c) So: compare your obtained F-ratio to the appropriate one in the table. If your obtained F-ratio is equal to or larger than the table value, then your data lack homogeneity of variance, and you should not be performing a parametric test on them.

d.f. (number of participants per group - 1)	k (number of groups)					
	2	3	4	5	6	7
2	39	87.5	142	202	266	333
3	15.4	27.8	39.2	50.7	62	72.9
4	9.6	15.5	20.6	25.2	29.5	33.6
5	7.15	10.8	13.7	16.3	18.7	20.8
6	5.82	8.38	10.4	12.1	13.7	15
7	4.99	6.94	8.44	9.7	10.8	11.8
8	4.43	6.00	7.18	8.12	9.03	9.8
9	4.03	5.34	6.31	7.11	7.8	8.41
10	3.72	4.85	5.67	6.34	6.92	7.42
12	3.28	4.16	4.79	5.3	5.72	6.09
15	2.86	3.54	4.01	4.37	4.68	4.95
20	2.46	2.95	3.29	3.54	3.76	3.94
30	2.07	2.4	2.61	2.78	2.91	3.02
60	1.67	1.85	1.96	2.04	2.11	2.17

A worked example:

Suppose we do an experiment that has two conditions, A and B. Here are the data.

score:	condition:
6	condition A
7	condition A
5	condition A
6	condition A
7	condition A
8	condition A
11	condition B
12	condition B
13	condition B
11	condition B
4	condition B
16	condition B

For condition A, the mean and s.d. are 6.50 and 1.05.

For condition B, the mean and s.d. are 11.17 and 3.97.

We want to run a *t*-test on these data, but before doing so, we want to check for homogeneity of variance.

The variance for condition A is 1.05^2 , which is 1.1025.
 The variance for condition B is 3.97^2 , which is 15.7609.

It looks like the variance for condition B is quite a bit bigger than the variance for condition A.

Using the F_{\max} test:

$$F_{\max} = 15.7609 / 1.1025 = 14.2956$$

We have 6 participants per group, and 2 groups, so $d.f. = 5$ and $k = 2$. We compare 14.2956 to the relevant value in the table, which is 7.15. Our obtained F-ratio is *larger* than the value in the table, and so we would conclude that the variances are significantly different from each other - i.e. that the data do not show homogeneity of variance, and therefore we should avoid using a parametric test like the t -test.

	number of groups					
number of participants per group - 1	2	3	4	5	6	7
2	39	87.5	142	202	266	333
3	15.4	27.8	39.2	50.7	62	72.9
4	9.6	15.5	20.6	25.2	29.5	33.6
5	7.15	10.8	13.7	16.3	18.7	20.8
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