Analysis of Variance: repeated measures

Logic behind ANOVA:

ANOVA compares the amount of systematic variation (from our experimental manipulations) to the amount of random variation (from the participants themselves) to produce an *F*-ratio:

F = systematic variation random variation ("error") Tests for comparing three or more groups or conditions:

(a) Nonparametric tests: Independent measures: Kruskal-Wallis.

Repeated measures: Friedman's.

*(b) Parametric tests:* One-way independent-measures Analysis of Variance (ANOVA).

One-way repeated-measures ANOVA.

systematic variation

 $F = \frac{1}{random variation ("error")}$ 

Large value of F: a lot of the overall variation in scores is due to the experimental manipulation, rather than to random variation between participants.

Small value of *F*: the variation in scores produced by the experimental manipulation is small, compared to random variation between participants.

ANOVA is based on the *variance* of the scores. The variance is the standard deviation squared:

variance = 
$$\frac{\sum (X - \overline{X})^2}{N}$$

In practice, we use only the top line of the variance formula (the "Sum of Squares", or "SS"):

sum of squares = 
$$\sum (X - \overline{X})^2$$

We divide this by the appropriate "*degrees of freedom*" (usually the number of groups or participants minus 1).

**One-way Repeated-Measures ANOVA:** 

Use this where you have: (a) *one* independent variable (with 2 or more levels);

(b) one dependent variable;

(c) each participant participates in *every* condition in the experiment (repeated measures).

A one-way repeated-measures ANOVA is equivalent to a repeated-measures *t*-test, except that you have more than two conditions in the study.

Effects of sleep-deprivation on vigilance in air-traffic controllers:

No deprivation vs. 12 hours' deprivation: One Independent Variable, 2 levels – use repeated-measures *t*-test.



No deprivation vs. 12 hours vs. 24 hours:

One Independent Variable, 3 levels (differing quantitatively) – use one-way repeated-measures ANOVA.

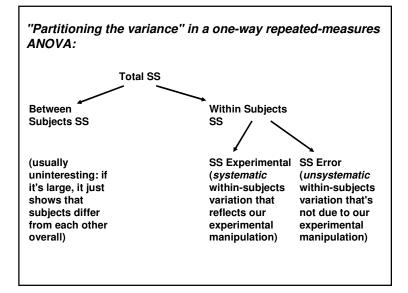


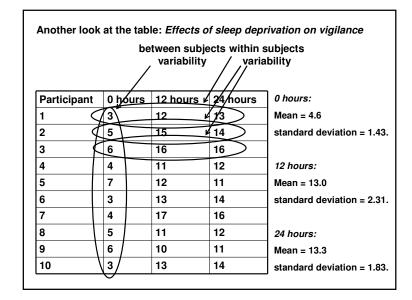
Effects of sleep deprivation on vigilance:

Independent Variable: length of sleep deprivation (0, 12 hours and 24 hours). Dependent Variable: 1 hour vigilance test (number of planes missed).

Each participant does all 3 conditions, in a random order.

| Participant | 0 hours | 12 hours | 24 hours | 0 hours:                   |
|-------------|---------|----------|----------|----------------------------|
| 1           | 3       | 12       | 13       | Mean = 4.6                 |
| 2           | 5       | 15       | 14       | standard deviation = 1.43. |
| 3           | 6       | 16       | 16       |                            |
| 4           | 4       | 11       | 12       | 12 hours:                  |
| 5           | 7       | 12       | 11       | Mean = 13.0                |
| 6           | 3       | 13       | 14       | standard deviation = 2.31. |
| 7           | 4       | 17       | 16       |                            |
| 8           | 5       | 11       | 12       | 24 hours:                  |
| 9           | 6       | 10       | 11       | Mean = 13.3                |
| 10          | 3       | 13       | 14       | standard deviation = 1.83. |





| The ANOVA summary table:                     |  |            |                   |                   |  |  |  |  |
|--|--|------------|-------------------|-------------------|--|--|--|--|
| Source:                                      | SS                                     | df         | MS                | F                 |  |  |  |  |
| Between subjects                             | 48.97                                  | 9          | 5.44              |                   |  |  |  |  |
| Within subjects                              | 534.53                                 | 20         |                   |                   |  |  |  |  |
| Experimental                                 | 487.00                                 | 2          | 243.90            | 92.36             |  |  |  |  |
| Error  | 47.53                                  | 18         | 2.64              |                   |  |  |  |  |
| Total  | 584.30                                 | 29         |                   |                   |  |  |  |  |
| Total SS: reflects the total                 | amount of varia                        | tion among | st all the scores |                   |  |  |  |  |
| <i>Between subjects SS:</i> a m<br>subjects. | easure of the am                       | ount of un | systematic varia  | tion between the  |  |  |  |  |
|  | : a measure of th<br>s due to our expe |            |                   | iation within the |  |  |  |  |

*Error SS:* a measure of the amount of unsystematic variation within each participant's set of scores.

Total SS = Between subjects SS + Within subjects SS

Assessing the significance of the F-ratio (by hand): The bigger the F-ratio, the less likely it is to have arisen merely by chance.

Use the between-subjects and within-subjects degrees of freedom to find the critical value of F.

Your F is significant if it is *equal to or larger* than the critical value in the table.

| Here, look up the critical <i>F</i> -           |    |         |       |         |
|---|----|---------|-------|---------|
| value for 2 and 18 degrees of                   |    | 1       | 2     | 3       |
| freedom   | 1  | 161.448 | 199.5 | 215.707 |
|   | 2  | 18.513  | 19    | 19.164  |
|   | 3  | 10.128  | 9.552 | 9.277   |
| Columns correspond to                           | 4  | 7.709   | 6.944 | 6.591   |
| EXPERIMENTAL degrees of                         | 5  | 6.608   | 5.786 | 5.409   |
| freedom   | 6  | 5.987   | 5.143 | 4.757   |
|   | 7  | 5.591   | 4.737 | 4.347   |
| Rows correspond to ERROR                        | 8  | 5.318   | 4.459 | 4.066   |
| degrees of freedom                              | 9  | 5.117   | 4.256 | 3.863   |
|   | 10 | 4.965   | 4.103 | 3.708   |
|   | 11 | 4.844   | 3.982 | 3.587   |
| Here, go <i>along</i> 2 and <i>down</i> 18:     | 12 | 4.747   | 3.885 | 3.49    |
| critical F is at the intersection               | 13 | 4.667   | 3.806 | 3.411   |
|   | 14 | 4.6     | 3.739 | 3.344   |
|   | 15 | 4.543   | 3.682 | 3.287   |
| Our obtained <i>F</i> , 92.36, is <i>bigger</i> | 16 | 4.494   | 3.634 | 3.239   |
| than 3.55; it is therefore                      | 17 | 4.451   | 3.592 | 3.197   |
| significant at <i>p</i> <.05. (Actually         | 18 | 4.414   | 3.555 | ) 3.16  |
| it's bigger than the critical                   | 19 | 4.381   | 3.522 | 3.127   |
| value for a <i>p</i> of 0.0001)                 | 20 | 4.351   | 3.493 | 3.098   |

Interpreting the Results:

A significant F-ratio merely tells us that there is a statistically-significant difference between our experimental conditions; it does not say *where* the difference comes from.

In our example, it tells us that sleep deprivation affects vigilance performance.

To pinpoint the source of the difference:

(a) *planned comparisons* - comparisons between groups which you decide to make *in advance* of collecting the data.

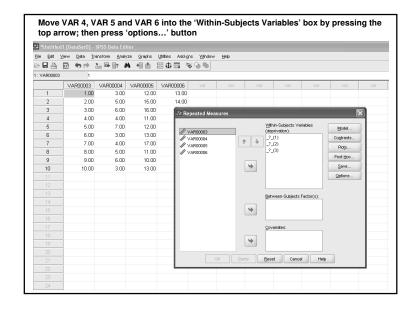
(b) *post hoc tests* - comparisons between groups which you decide to make *after* collecting the data: Many different types - e.g. Newman-Keuls, Scheffé, Bonferroni.

Using SPSS for a one-way repeated-measures ANOVA on effects of fatigue on vigilance

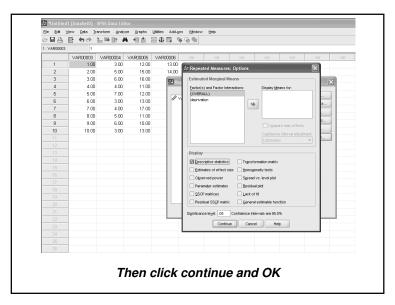
| r <b>v</b> | <u>File E</u> dit <u>)</u> | <u>√</u> iew <u>D</u> ata <u>T</u> i | ransform <u>A</u> nal | yze <u>G</u> raphs | Utilities Add- | ons <u>W</u> indow | Help |
|------------|----------------------------|--------------------------------------|-----------------------|--------------------|----------------|--------------------|------|
| ,          | 6 🛛 🗛                      | 🖽 🔶 🔿                                | <u>* 🖬 ]?</u> 6       | M + 1              |                | <b>₩ % ●</b>       |      |
|            | 11 :                       |                                      |                       |                    |                |                    |      |
|            |                            | VAR00003                             | VAR00004              | VAR00005           | VAR00006       | var                | var  |
|            | 1                          | 1.00                                 | 3.00                  | 12.00              | 13.00          | 1                  |      |
|            | 2                          | 2.00                                 | 5.00                  | 15.00              | 14.00          | 1                  |      |
|            | 3                          | 3.00                                 | 6.00                  | 16.00              | 16.00          | 1                  |      |
|            | 4                          | 4.00                                 | 4.00                  | 11.00              | 12.00          | 1                  |      |
|            | 5                          | 5.00                                 | 7.00                  | 12.00              | 11.00          | 1                  |      |
|            | 6                          | 6.00                                 | 3.00                  | 13.00              | 14.00          | 1                  |      |
|            | 7                          | 7.00                                 | 4.00                  | 17.00              | 16.00          | 1                  |      |
|            | 8                          | 8.00                                 | 5.00                  | 11.00              | 12.00          | 1                  |      |
|            | 9                          | 9.00                                 | 6.00                  | 10.00              | 11.00          | 1                  |      |
|            | 10                         | 10.00                                | 3.00                  | 13.00              | 14.00          | I                  |      |
|            | 11                         | ]                                    |                       |                    |                |                    |      |
|            | 12                         |                                      |                       |                    |                |                    |      |
|            | 13                         |                                      |                       |                    |                |                    |      |
|            | 14                         |                                      |                       |                    |                |                    |      |
|            | 15                         |                                      |                       |                    |                |                    |      |
|            | 16                         |                                      |                       |                    |                |                    |      |
|            | 17                         |                                      |                       |                    |                |                    |      |
|            | 18                         |                                      |                       |                    |                |                    |      |

| 🗟 *Untitled | 1 [DataSet0] - SPSS D              | ata Editor                                   |
|-------------|------------------------------------|--|
| Eile Edit ⊻ | jew <u>D</u> ata <u>T</u> ra⊓sform | Analyze Graphs Utilities Add-ons Window Help |
| 62 🔒 🖨      | 📴 🕈 🖻 🕌 🖬                          | Regorts 🔸 😽 🔕 🐿                              |
| 11 :        |                                    | Descriptive Statistics                       |
|             | VAR00003 VAR0                      | Tables                                       |
| 1           | 1.00                               | Compare Means                                |
| 2           | 2.00                               | General Linear Model                         |
| 3           | 3.00                               | Generalized Linear Models 🕨 👫 Multivariate   |
| 4           | 4.00                               | Mixed Models                                 |
| 5           | 5.00                               | Correlate Variance Components                |
| 6           | 6.00                               | Regression +                                 |
| 7           | 7.00                               | L <u>og</u> linear                           |
| 8           | 8.00                               | Classi <u>t</u> y                            |
| 9           | 9.00                               | Data Reduction                               |
| 10          | 10.00                              | Sc <u>a</u> le                               |
| 11          |                                    | Nonparametric Tests                          |
| 12          |                                    | Time Series                                  |
| 13          |                                    | Survival                                     |
| 14          |                                    | 53 Missing Value Analysis                    |
| 15          |                                    | Multiple Response                            |
| 16          |                                    | Complex Samples                              |
| 17          |                                    | Quality Control                              |
| 18          |                                    | C ROC Curve                                  |

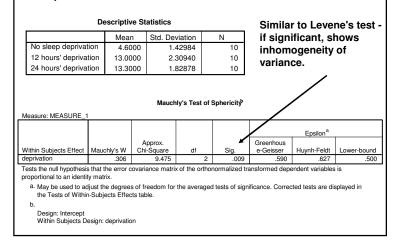
| > 54 A II 47 A A 4 A A A A A A A A A A A A A A A  | 🚮 *Untitleo  |          |                 |            |          |       |                          |             |               |      |  |
|---|--------------|----------|-----------------|------------|----------|-------|--------------------------|-------------|---------------|------|--|
| VAR0003      t      VAR0005      VAR0005      VAR0005      VAR0005      VAR0005      Var      Var </th <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th>w ∏eib</th> <th></th> <th></th> <th></th> <th></th> |              |          |                 |            |          |       | w ∏eib                   |             |               |      |  |
| VAR0003      VAR0004      VAR0005      VAR0006      var  | 6 🛛 🗛        | 匠 🕈 🖻    | <u>}</u> ₩ ]? 4 | M 11 ml ml | ≣ ∰ ≣ *  | ð 🕑 🛑 |                          |             |               |      |  |
| 1  1.00  3.00  12.00  13.00    2  2.00  5.00  15.00  14.00    3  3.00  6.00  16.00    4  4.00  4.00  11.00  12.00    5  5.00  7.00  12.00  11.00    6  6.00  3.00  16.00    7  7.00  4.00  17.00    9  9.00  6.00  10.00    10  10.00  3.00  14.00    11  10.00  3.00  14.00    12  | 1 : VAR00003 | 1        |                 |            |          |       |                          |             |               |      |  |
| 2    2.00    5.00    15.00    14.00      3    3.00    6.00    16.00    16.00      4    4.00    4.00    11.00    12.00      5    6.00    7.00    12.00    11.00      6    6.00    3.00    13.00    14.00      7    7.00    4.00    17.00    16.00      8    8.00    5.00    11.00    12.00      9    9.00    6.00    10.00    11.00      10    10.00    3.00    13.00    14.00      11   |              | VAR00003 | VAR00004        | VAR00005   | VAR00006 | var   | var                      |             | var           | var  |  |
| 3    3.00    6.00    16.00    16.00      4    4.00    4.00    11.00    12.00      5    6.00    7.00    12.00    11.00      6    6.00    3.00    13.00    14.00      7    7.00    4.00    17.00    16.00      8    8.00    5.00    11.00    12.00      9    9.00    6.00    11.00    12.00      10    10.00    3.00    13.00    14.00      111   | 1            | 1.00     | 3.00            | 12.00      | 13.00    |       |                          |             |               |      |  |
| 4  4.00  4.00  11.00  12.00  Sign=Scipt Acta Name:    5  6.00  3.00  13.00  14.00    7  7.00  4.00  17.00  16.00    8  8.00  5.00  11.00  10.00    9  9.00  6.00  10.00  11.00    10  10.00  3.00  13.00  14.00    11  10.00  3.00  13.00  14.00    12  | 2            | 2.00     | 5.00            | 15.00      | 14.00    |       |                          |             |               |      |  |
| 5      5.00      7.00      12.00      11.00      Yethn-Subject Factor Name:      Genvition        7      7.00      4.00      17.00      16.00      Namber of Levits 3      Genvition        8      8.00      5.00      11.00      12.00      11.00      Subject Factor Name:      Genvition        9      9.00      6.00      10.00      11.00      12.00      Add        10      10.00      3.00      13.00      14.00      Factor Name:      Add        11  | 3            | 3.00     | 6.00            | 16.00      | 16.00    |       |                          |             |               |      |  |
| 6      6 00      3 00      13 00      14 00      Vghrs-Super Factor Name        7      7 00      4.00      17 7.00      16 00      Namber of Levels: 3        8      8 00      50.00      11 00      12 00      Namber of Levels: 3        9      9.00      6.00      10 00      11 00      20 mg etc.        10      10 0.00      3.00      13 00      14 00      Program        12      -      -      -      Program      Program        12      -      -      -      Program      Program      Program        13      -      -      -      -      Program   | 4            | 4.00     | 4.00            | 11.00      | 12.00    |       | 👫 Repeated i             | Aeasures De | fine Factor(s | ) 🗙  |  |
| 6      6      0.00      3.00      14.00      generation        7      7.00      4.00      17.00      16.00      Namer of users 3        8      8.00      5.00      11.00      12.00      Add        9      9.00      6.00      10.00      11.00      200        10      10.00      3.00      13.00      14.00      Page        11      10.00      3.00      13.00      14.00      Page        11      10.00      3.00      13.00      14.00      Page        13      1      14      14      14      14      14      14        15      1      1      1      14   | 5            | 5.00     | 7.00            | 12.00      | 11.00    |       | Mithin Subject           | Factor Name |               |      |  |
| 8      8.00      5.00      11.00      12.00        9      9.00      6.00      11.00      11.00        10      10.00      3.00      13.00      14.00        11      10      3.00      13.00      14.00        13        Measure igner        13        Agat        16        Agat        16        Agat        17         Benoge        18             19              19                                   <  | 6            | 6.00     | 3.00            | 13.00      | 14.00    |       | <u>vy</u> u in r-Subject |             |               |      |  |
| 9      9.00      6.00      10.00      11.00      Owner        10      10.00      3.00      13.00      14.00      Progree      Progree        11      10.00      3.00      13.00      14.00      Progree      Progree        12      10      10.00      3.00      14.00      Progree      Progree        12      10      10.00      3.00      14.00      Progree      Progree        13      10      10.00      10.00      14.00      Progree      Progree        14      10      10.00      10.00      14.00      Progree      Progree        15      10      10.00      10.00      Progree      Progree      Progree        16      10.00      10.00      Progree      Progree      Progree      Progree        18      10.00      10.00      Progree      Progree      Progree      Progree        20      10.00      10.00      Progree      Progree      Progree      Progree   | 7            | 7.00     | 4.00            | 17.00      | 16.00    |       | Number of Leve           | its: 3      |               |      |  |
| 9      9.00      6.00      10.00      11.00        10      10.00      3.00      13.00      14.00        11      10      3.00      13.00      14.00        12      10      10      10.00      3.00        13      10      10.00      3.00      14.00        14      10      10.00      4.00      Measure blane        15      10      10.00      2.00      Renzge        16      10      10.00      2.00      Renzge        19      10      10.00      10.00      10.00        20      10      10.00      10.00      10.00   | 8            | 8.00     | 5.00            | 11.00      | 12.00    |       | Add                      |             |               |      |  |
| 10      10.00      3.00      13.00      14.00        11      11      Regore      Regore        12      Add      Add        15      Add      Carrot        16      Carrot      Renge        17      Renge      Renge        18      Carrot      Renge        19      Corot      Hebp   | 9            | 9.00     | 6.00            | 10.00      | 11.00    |       |                          |             |               |      |  |
| 11      Measure lighter        13      Measure lighter        14      Add        15      Add        16      Clarge        17      Renoge        18      Clarge        19      Cortett        20      Crepter  | 10           | 10.00    | 3.00            | 13.00      | 14.00    |       |                          |             |               |      |  |
| 13      Measure Igene        14   |              |          |                 |            |          |       |                          |             |               |      |  |
| 14<br>14<br>15<br>16<br>17<br>17<br>18<br>19<br>20<br>Cepte Best Conct Heb  |              |          |                 |            |          |       |                          |             |               |      |  |
| 15<br>16<br>17<br>17<br>18<br>19<br>20<br>Ceptre<br>Best Concel Heb   |              |          |                 |            |          |       | Measure Name:            |             |               |      |  |
| 16 Cgarge<br>17 Renoge<br>19 Capro Renoge<br>20 Capro Renoge  |              |          |                 |            |          |       |                          |             |               |      |  |
| 17 Renoge<br>19<br>20 Cepte Beset Cancel Help   |              |          |                 |            |          |       |                          |             |               |      |  |
| 10<br>19<br>20 Cetter Best Cancel Heb   |              |          |                 |            |          |       | Change                   |             |               |      |  |
| 19<br>20 Define <u>Best</u> Cancel Heb  |              |          |                 |            |          |       | Remoye                   |             |               |      |  |
| 20 Define Reset Cancel Help   |              | _        |                 |            |          |       |                          |             |               |      |  |
|   |              |          |                 |            |          |       |                          |             |               |      |  |
| 21  |              |          |                 |            |          |       | Detine                   | Reset       | Cancel        | Help |  |
|   |              |          |                 |            |          |       |                          |             |               | _    |  |
|   |              |          |                 |            |          |       |                          |             |               |      |  |



| _           | [DataSet0] - |               |          |                 |             |       | _   |                               |            |           |     |
|-------------|--------------|---------------|----------|-----------------|-------------|-------|-----|-------------------------------|------------|-----------|-----|
|             |              | ansform Analy |          | Utilities Add-g |             | Help  |     |                               |            |           |     |
| .: VAR00003 | 1            |               |          |                 |             |       |     |                               |            |           |     |
|             | VAR00003     | VAR00004      | VAR00005 | VAR00006        | var         | var   | var | var                           | var        | var       | var |
| 1           | 1.00         | 3.00          | 12.00    | 13.00           | Vdi         | Vdi   | Vai | Vai                           | Vai        | Val       | Val |
| 2           | 2.00         | 5.00          | 15.00    | 14.00           |             |       |     |                               |            |           |     |
| 3           | 3.00         | 6.00          | 16.00    | _               |             |       |     |                               |            |           |     |
| 4           | 4.00         | 4.00          | 11.00    | M Re            | epeated Mea | sures |     |                               |            | ×         |     |
| 5           | 5.00         | 7.00          | 12.00    |                 |             |       |     | Within-Subjects Vi            | ariables   | Model     |     |
| 6           | 6.00         | 3.00          | 13.00    | 59              | VAR00003    | _     |     | (deprivation):<br>VAR00004(1) |            | Contrasts |     |
| 7           | 7.00         | 4.00          | 17.00    |                 |             | 1     | + + | VAR00005(2)                   |            |           |     |
| 8           | 8.00         | 5.00          | 11.00    |                 |             |       |     | VAR00006(3)                   |            | Plots     |     |
| 9           | 9.00         | 6.00          | 10.00    |                 |             |       |     |                               |            | Post Hoc  |     |
| 10          | 10.00        | 3.00          | 13.00    |                 |             |       | *   |                               |            | Save      |     |
|             |              |               |          |                 |             |       |     |                               |            | Options   |     |
|             |              |               |          |                 |             |       |     |                               |            |           |     |
|             |              |               |          |                 |             |       |     | Between-Subjects              | Factor(s): |           |     |
|             |              |               |          |                 |             |       |     |                               |            |           |     |
|             |              |               |          |                 |             |       | +   |                               |            |           | L   |
|             |              |               |          |                 |             |       |     |                               |            |           |     |
| 17          |              |               |          |                 |             |       |     | <u>C</u> ovariates:           |            |           |     |
| 18          |              |               |          |                 |             |       | •   |                               |            |           |     |
|             |              |               |          |                 |             |       | -   |                               |            |           | L   |
|             |              |               |          |                 |             |       |     |                               |            | _         | L   |
| 21          |              |               |          |                 |             | DK Ba | ste | Reset Canc                    | el Help    |           |     |
|             |              |               |          |                 |             |       |     |                               |            |           |     |
|             |              |               |          |                 |             |       |     |                               |            |           |     |



The SPSS output (ignore everything except what's shown here!):



|                    | Tes                | ts of Within-Sub           | jects Effect | s           |        |      |
|--------------------|--------------------|----------------------------|--------------|-------------|--------|------|
| Measure: MEASUR    | RE_1               |                            |              |             |        |      |
| Source             |                    | Type III Sum<br>of Squares | df           | Mean Square | F      | Sig. |
| deprivation        | Sphericity Assumed | 487.800                    | 2            | 243.900     | 92.360 | .00  |
|                    | Greenhouse-Geisser | 487.800                    | 1.181        | 413.186     | 92.360 | .00  |
|                    | Huynh-Feldt        | 487.800                    | 1.254        | 388.985     | 92.360 | .00  |
|                    | Lower-bound        | 487.800                    | 1.000        | 487.800     | 92.360 | .00  |
| Error(deprivation) | Sphericity Assumed | 47.533                     | 18           | 2.641       |        |      |
|                    | Greenhouse-Geisser | 47.533                     | 10.625       | 4.474       |        |      |
|                    | Huynh-Feldt        | 47.533                     | 11.286       | 4.212       |        |      |
|                    | Lower-bound        | 47.533                     | 9.000        | 5.281       |        |      |

Significant effect of sleep deprivation (F 2, 18 = 92.36, p<.0001)

OR, (if Mauchly's test was significant) use Greenhouse-Geisser (F 1.18, 10.63 = 92.36, p<.0001).

