SUSSEX FLUIDS WORKSHOP 2017 – ABSTRACTS

I. Mini-Courses

Thierry Gallay
Axisymmetric vortex rings in viscous fluids: uniqueness and qualitative properties

A three-dimensional incompressible flow is called a vortex ring if the associated vorticity distribution is concentrated in a solid torus, so that the fluid particles spin around an imaginary line that forms a closed loop. Such flows are ubiquitous in nature, and appear to be very stable. For the Euler equations with axial symmetry, a large family of uniformly translating vortex rings can be constructed by variational techniques or by perturbation methods. In the singular limit where the cross section shrinks to zero while the total circulation is kept fixed, the vortex ring degenerates into a filament, and the translation speed blows up as predicted by the local induction approximation.

In the viscous case, it is possible to show that the Navier-Stokes equations have a unique axisymmetric solution without swirl if the initial vorticity is a circular vortex filament with arbitrarily large circulation Reynolds number. Solutions constructed in this way are archetypal examples of viscous vortex rings, and can be thought of as axisymmetric analogues of the self-similar Lamb-Oseen vortices in two-dimensional flows. The main difficulty in this approach is that we have to consider solutions with very singular initial data: even local existence is beyond the reach of usual perturbation methods, and the uniqueness proof ultimately relies on stability properties of two-dimensional vortices, but requires several new ideas.

In these lectures I shall first introduce a simple functional framework to study the axisymmetric Navier-Stokes equations without swirl. Then I will focus on the vortex ring problem, and explain in some detail the key steps of the uniqueness proof. If there is enough time, I shall also describe some qualitative properties of the viscous vortex rings for small times, or in the vanishing viscosity limit.

All the original results presented in these lectures have been obtained in collaboration with Vladimír Šverák (Minneapolis).

László Székelyhidi
The h-principle in fluid dynamics: non-uniqueness and anomalous dissipation

It is known since the pioneering work of V. Scheffer and A. Shnirelman in the 1990s that weak solutions of the incompressible Euler equations behave very differently from classical solutions, in a way that is very difficult to interpret from a physical point of view. In particular such solutions are highly non-unique and have several unphysical features such as arbitrary growth of energy. Nevertheless, weak solutions in three space dimensions have been studied in connection with a long-standing conjecture of L. Onsager from 1949 concerning anomalous dissipation and, more generally, because of their possible relevance to Kolmogorov's K41 theory of turbulence.

In a series of joint publications with Camillo De Lellis we established a connection between the theory of weak solutions of the Euler equations and the Nash-Kuiper theorem on rough isometric immersions. Through this connection one can interpret the wild behaviour of weak solutions of the Euler equations as an instance of Gromov's celebrated h-principle.

In these lectures I will explain this connection, outline the most recent progress concerning

- Onsager's conjecture
- Selection principles

and discuss some future directions.

References

[1] C. De Lellis and L. Székelyhidi Jr, *High dimensionality and h-principle in PDE* Bull. Amer. Math. Soc., vol. 54, no. 2, pp. 247-282, Apr. 2017.

[2] L. Székelyhidi Jr, *Weak solutions of the Euler equations: non-uniqueness and dissipation* Journées Équations aux Derivées Partielles, vol. 10, Available at `www.math.uni-leipzig.de/preprints/p1601.0020.pdf`

[3] L. Székelyhidi Jr, *From Isometric Embeddings to Turbulence* in HCDTE Lecture Notes. Available at `www.math.uni-leipzig.de/preprints/p1406.0020.pdf`

II. Workshop Lectures

Jean-Yves Chemin

About some possible blow-up for the incompressible 3D Navier-Stokes equations

In this talk, we first recall some basic blow-up conditions due essentially to Jean Leray. Then we recall the Escauriaza, Seregin and Šverák theorem for the blow-up of the $\dot{H}^{\frac{1}{2}}$ norm. The purpose of this talk is to investigate what happens for possible blow-up when we consider initial data which are more regular than the scaling, for instance $\dot{H}^1$. The results presented here are from to PhD dissertation of Eugénie Poulon.

Sara Daneri

The Cauchy problem for dissipative solutions of the Euler equations

Our work is related to the recently proven Onsager's conjecture, according to which below Hölder regularity 1/3 there exist solutions of the incompressible Euler equations which dissipate the total kinetic energy. We deal with the Cauchy problem for such kind of solutions. We improve a joint work with L. Székelyhidi, where we proved the existence of infinitely many Hölder $1/5 - \epsilon$ initial data, each one admitting infinitely many Hölder $1/5 - \epsilon$ solutions with preassigned total kinetic energy, raising the exponent of this wild initial data and solutions to the optimal $1/3 - \epsilon$. This is a joint work with E. Runa.

Alberto Enciso

Vortex reconnection: creation and destruction of knotted vortex structures in 3D Navier-Stokes

An important property of the 3D Euler equations is that the topology of the vortex structures of the fluid does not change in time as long as the solutions do not develop any singularities. To put it differently, the set of (say) vortex tubes and vortex lines of the fluid at time t is diffeomorphic to that of the initial vorticity, provided that the solution remains smooth up to this time. Of course, numerical simulations and experiments with real fluids have shown that the situation is completely different in the case of viscous fluids. In this talk we will show how vortex tubes and vortex lines, of arbitrarily complex topologies, are created and destroyed in smooth solutions to the 3D Navier-Stokes equations. This is joint work with Renato Luca and Daniel Peralta-Salas.

Julien Guillod

On the nonuniqueness of the Navier-Stokes initial value problem

The best known local well-posedness results for the Navier-Stokes initial value problem are obtained by perturbation methods in scale-invariant spaces. In this talk we will show numerically that this problem is ill-posed outside the perturbation regime. More precisely, we numerically construct two different solutions having the same initial datum in borderline spaces. This is joint work with Vladimír Šverák.

Philip Isett

A Proof of Onsagers Conjecture for the Incompressible Euler Equations

In an effort to explain how anomalous dissipation of energy occurs in hydrodynamic turbulence, Onsager conjectured in 1949 that weak solutions to the incompressible Euler equations may fail to exhibit conservation of energy if their spatial regularity is below 1/3-Hölder. I will discuss a proof of this conjecture that shows that there are nonzero, $(1/3 - \epsilon)$-Hölder Euler flows in 3D that have compact support in time. The construction is based on a method known as "convex integration," which has its origins in the work of Nash on isometric embeddings with low codimension and low regularity. A version of this method was first developed for the incompressible Euler equations by De Lellis and Székelyhidi to build Hölder-continuous Euler flows that fail to conserve energy, and was later improved by Isett and by Buckmaster-De Lellis-Székelyhidi to obtain further partial results towards Onsager's conjecture. The proof of the full conjecture combines convex integration using the Mikado flows introduced by Daneri-Székelyhidi with a new "gluing approximation" technique. The latter technique exploits a special structure in the linearization of the incompressible Euler equations.

Christophe Lacave

Vanishing viscosity and rugosity limit

We study the high Reynolds number limit of a viscous fluid in the presence of a rough boundary. We consider the two-dimensional incompressible Navier-Stokes equations with Navier slip boundary condition, in a domain whose boundaries exhibit fast oscillations in the form $x_2 = \varepsilon^{1+\alpha}\eta(x_1/\varepsilon)$, $\alpha > 0$. Under suitable conditions on the oscillating parameter ε and the viscosity ν, we show that solutions of the Navier-Stokes system converge to solutions of the Euler system in the vanishing limit of both ν and ε. The main issue is that the curvature of the boundary is unbounded as $\varepsilon \to 0$, which precludes the use of standard methods to obtain the inviscid limit. Our approach is to first construct an accurate boundary layer approximation to the Euler solution in the rough domain, and then to derive stability estimates for this approximation under the Navier-Stokes evolution. This work is in collaboration with D. Gérard-Varet, T. Nguyen and F. Rousset.

Evelyne Miot

Uniqueness and stability issues for the Vlasov-Navier-Stokes system

In this talk I will present a uniqueness and stability result for the 2D Vlasov-Navier-Stokes system, holding for weak solutions with divergence-free velocity fields belonging to the natural energy space and bounded densities satisfying some specific assumptions on the moments. This is joint work with Daniel Han-Kwan, Ayman Moussa and Ivan Moyano.

James Robinson

Good approximation using spaces of eigenfunctions and the energy equality for the CBF equation

(Joint with C. Fefferman and K. Hajduk) We present a method that allows for approximation of functions in spaces of eigenfunctions simultaneously in L^p and in Sobolev spaces. The general method relies on the theory of semigroups and provides approximations that converge in domains of fractional powers of some linear operator A. By identifying these domains explicitly for the Laplacian and Stokes operators we are able to approximate using functions that are smooth and satisfy other desirable properties (e.g. are divergence free or vanish on the boundary). As an application we show that weak solutions of the Convective Brinkamn-Forchheimer equations (the Navier-Stokes equations with an additional damping term) automatically satisfy an energy equality.

III. Contributed Posters, Session 1

Michal Bathory

Outflow boundary condition leading to minimal energy dissipation for an incompressible flow

We present a certain type of implicit boundary conditions for artificial boundaries such as the inlet or outlet. These conditions require that the resulting flow minimizes some given functional such as the energy dissipation. It can be then shown that the final system always has a solution. Moreover, in some situations we are able to deduce some explicit boundary conditions. It is then interesting to compare these with the boundary conditions which are commonly used.

Tomasz Dębiec

Energy conservation for the Euler-Korteweg equations

We provide sufficient regularity conditions on the solutions to the Euler-Korteweg and the related Quantum Hydrodynamics systems in order for the energy to be conserved. This is achieved by means of commutator estimates similar to those used by Constantin et al. for the homogeneous incompressible Euler equations. Joint work with P. Gwiazda, A. Swierczewska-Gwiazda and A. Tzavaras.

Romeo Mensah

Singular limits of stochastic compressible fluids

We present some results on the stochastic compressible Navier-Stokes system of equations. We review some of the existence results and also explore its relationship – via singular limit arguments – with other fluid dynamic models like the randomly driven incompressible Navier-Stokes and Euler equations.

Wojciech Ożański

Partial regularity for a surface growth model

(joint work with J. Robinson) The surface growth model,

$$u_t = -u_{xxxx} - \partial_{xx}(u_x)^2,$$

shares a number of striking similarities to the three-dimensional incompressible Navier-Stokes equations, which include the results regarding existence and uniqueness of solutions. We consider the surface growth model on the one-dimensional torus $\mathbb{T}^1$ and we show two partial regularity results, which parallel those available for the Navier-Stokes equations. Consequently we show that the space-time singular set has the upper box-counting dimension no larger than 7/6 and that it has one-dimensional parabolic Hausdorff measure zero.

IV. Contributed Posters, Session 2

Tobias Barker
Uniqueness results for weak Leray-Hopf solutions of the Navier-Stokes system with initial values in critical spaces

We provide new classes of initial data that ensure that the associated weak Leray-Hopf solutions of the three-dimensional Navier-Stokes equations (in the whole space) coincide on some time interval.

Stefano Modena
Lagrangian representation for 1D systems of conservation laws

One of the key observations in Fluid Dynamics is that the fluid flow can be described from two different (and in some sense complementary) points of view: the Lagrangian point of view (in which the trajectory in space-time of each single fluid particle is tracked), and the Eulerian point of view (in which one looks at fluid motion focusing on fixed locations in the space through which the fluid flows as time passes). Such key observation has been successfully applied to the analysis of some particular partial differential equations (among all, the transport equation), leading to important theoretical results.

In a joint work with Stefano Bianchini, we show that also a general hyperbolic system of conservation laws in one space variable $u_t + F(u)_x = 0$ can be analyzed from a Lagrangian point of view. Here $F : \mathbb{R}^N \to \mathbb{R}^N$ is any given smooth function, which is only assumed to be strictly hyperbolic, i.e. its differential $DF(u)$ has N distinct real eigenvalues in each point of its domain and the unknown is the function $u(t,x) \in \mathbb{R}^N$, $t \geq 0$, $x \in \mathbb{R}$.

Stefano Scrobogna
On the dynamic of stratified fluids

We prove that the equations describing non-homogeneous, inviscid, three-dimensional fluids are globally well-posed when the Froude number is sufficiently small both in space-periodic and whole space setting. No smallness assumption is assumed on the initial data.

Chuong Tran
A geometrical approach to the problem of Navier-Stokes regularity

A new kind of regularity criteria for the global well-posedness problem of the three-dimensional Navier–Stokes equations in the whole space is presented. If the region $\Omega = \{(x,t) \mid |u(x,t)| > C(q)\|u\|_{L^{3q-6}}\}$, where $q \geq 3$ and $C(q)$ appropriately defined, shrinks fast enough as $q \nearrow \infty$, then regularity persists.