

The h-principle in fluid mechanics: non-uniqueness and anomalous dissipation

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It is known since the pioneering work of V. Scheffer and A. Shnirelman in the 1990s that weak solutions of the incompressible Euler equations behave very differently from classical solutions, in a way that is very difficult to interpret from a physical point of view. In particular such solutions are highly non-unique and have several unphysical features such as arbitrary growth of energy. Nevertheless, weak solutions in three space dimensions have been studied in connection with a long-standing conjecture of L. Onsager from 1949 concerning anomalous dissipation and, more generally, because of their possible relevance to Kolmogorov's K41 theory of turbulence.

In a series of joint publications with Camillo De Lellis we established a connection between the theory of weak solutions of the Euler equations and the Nash-Kuiper theorem on rough isometric immersions. Through this connection one can interpret the wild behaviour of weak solutions of the Euler equations as an instance of Gromov's celebrated h-principle.

In these lectures I will explain this connection, outline the most recent progress concerning

- Onsager's conjecture
- Selection principles

and discuss some future directions.

References

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- [3] L. Székelyhidi Jr, *From Isometric Embeddings to Turbulence* in HCDTE Lecture Notes. Available at www.math.uni-leipzig.de/preprints/p1406.0020.pdf