Representation Challenge

Example of ways to solve a problem

Problem:

What is the probability that the sum of two dice is more than 7?

Approach 1:

Step 1: Draw all possible pairs when 2 dice are thrown:

1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6	5,6	6,6

Step 2: Note it forms a square of 6x6, so there are 36 pairs

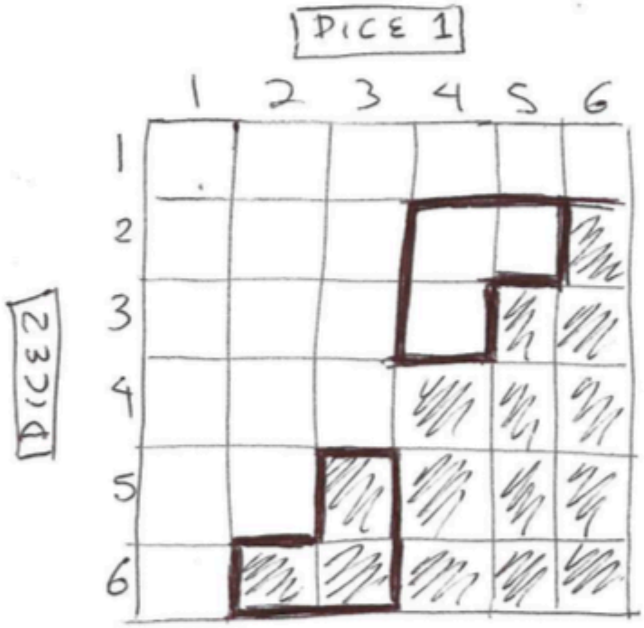
Step 3: Highlight those pairs that add up more than 7:

1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6	5,6	6,6

Step 4: The number of pairs that add 8 or more are 15

Step 5: That means that only 15 of 36 pairs add more than 7. The probability is 15/36

Approach 2:



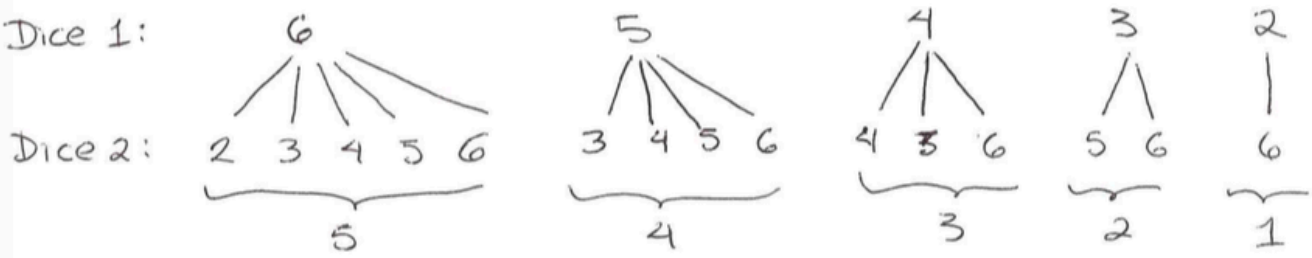
- Draw a grid with all possible combinations for both dice
- Fill in the boxes that add up 8 or more
- Notice that if you rearrange the filled boxes, you get a rectangle of dimensions

$$5 \times \left(\frac{6}{2}\right) = 15$$

- But there are 6×6 total options, so the probability is

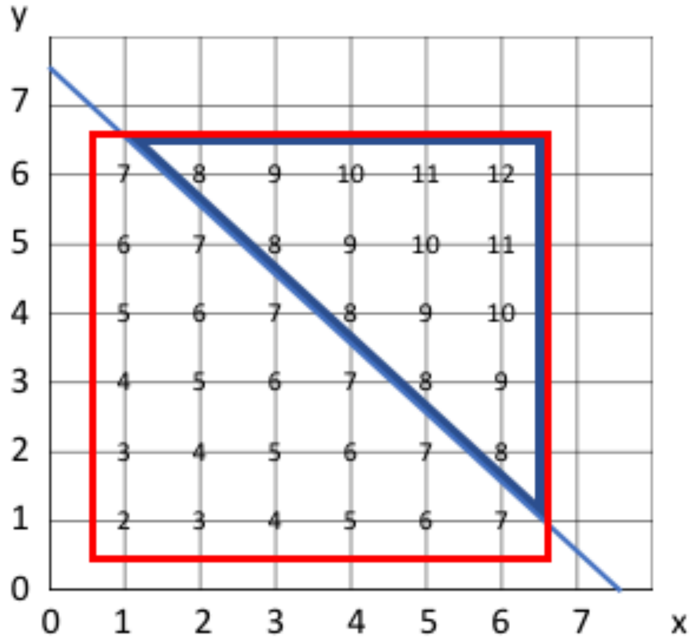
$$\frac{15}{36} = \frac{5}{12}$$

Approach 3:



* Show all possible pairs that add more than 7
* There are $5+4+3+2+1 = 15$ possibilities.
* The total number of pairs are $6 \times 6 = 36$
* The probability is $\frac{15}{36} = 0.416$

Approach 4:



Solution method for someone who knows integral calculus.

Step 1) Draw graph of two dice with sums as points

Step 2) Divide the area in to numbers >7; blue triangle

Step 3) Formula for the dividing line is:

$$y = 7.5 - x$$

Step 4) Formula for the distance, d, between the y=6.5 line and the dividing line is:

$$d = 6.5 - (7.5 - x) = x - 1$$

Step 5) Find the area, *A*, of the triangle by integrating d with respect to x

$$A = \int_1^{6.5} x - 1 \, dx = \left[\frac{x^2}{2} - x\right]_1^{6.5} = \left[\frac{6.5^2}{2} - 6.5\right] - \left[\frac{1}{2} - 1\right] = 14.625 + 0.5 = 15.125$$

Step 6) Area, A', for all sums = 36 unit^2

Step 7) Chance of selecting area A at random = A/A' = 15.125/36 =0.420

Approach 5:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**Step 1) Generate a set of 100 dice throws**							**Step 2) Generate another set of 100 dice throws**						
2	100 random integers between 1 and 6 (Randbetwen function)							100 random integers between 1 and 6 (Randbetwen function)						
3	5	3	3	2	6			5	6	5	6	6		
4	6	4	6	1	1			3	1	2	1	5		
5	1	1	4	5	2			3	1	1	6	4		
6	4	4	3	1	6			2	4	2	4	4		
7	4	5	1	1	2			3	2	4	3	5		
8	6	3	1	4	2			2	1	3	6	3		
9	6	1	6	2	3			1	6	6	1	5		
10	5	2	6	2	3			5	3	5	1	4		
11	4	1	6	3	4			3	5	6	2	5		
12	6	2	1	2	6			4	6	6	6	1		
13	6	5	3	6	1			2	3	3	4	6		
14	2	3	4	1	4			6	1	5	3	3		
15	6	4	3	1	1			2	4	5	6	1		
16	4	1	6	3	2			3	6	2	2	2		
17	4	2	3	2	2			1	6	4	5	1		
18	1	4	4	3	4			3	1	2	4	4		
19	2	2	4	2	6			3	6	4	3	4		
20	5	1	6	4	5			2	6	4	4	1		
21	4	3	1	1	1			2	6	6	1	1		
22	6	6	6	4	6			6	6	3	5	3		
23														
24														
25				**Step 3) Find the sum of pairs of throws**										
26				Add values for each matching pair of throws										
27			E.g.	10	9	8	8	12						
28				9	5	8	2	6						
29				4	2	5	11	6						
30				6	8	5	5	10						
31				7	7	5	4	7						
32				8	4	4	10	5						
33				7	7	12	3	8						
34				10	5	11	3	7						
35				7	6	12	5	9						
36				10	8	7	8	7						
37				8	8	6	10	7						
38				8	4	9	4	7						
39				8	8	8	7	2						
40				7	7	8	5	4						
41				5	8	7	7	3						
42				4	5	6	7	8						
43				5	8	8	5	10						
44				7	7	10	8	6						
45				6	9	7	2	2						
46				12	12	9	9	9						
47														
48														
49	**Step 4) Find the freqeuncy of each sum**													
50	Sum	2	3	4	5	6	7	8	9	10	11	12	Total	
51	Count	5	3	8	13	8	20	20	8	8	2	5	100	
52														
53														
54		**Step 5) Calculate number and proportion >7**					sum > 7	43						
55							Prob(sum>7)	0.43						
56														

Approach 6:

Solution method for someone who know about permutations and combinations and is familiar throwing pairs of dice.

Step 1) The number of permutations of two dice is 36.

Step 2) There are maximum of six ways to obtain a sum of 7, because there are only six numbers and the minimum and maximum add to 7. (We could enumerate permutations to prove this.)

Step 3) From steps 1 and 2, the number of sum that don't add to 7 is 30 (i.e., 36-6).

Step 4) For each sum above 7 there is exactly one below. (It is presumed we know that know that 1 mirrors 6, 2 mirrors 5, and 3 mirrors 4: i.e., 6+6⇔1+1, 6+5⇔1+2, 6+4⇔1+3, 6+3⇔1+4, 6+2⇔1+5, and so forth.)

Step 5) Hence, the number of sums above (or below) 7 are equal, therefore the number of sums >7 is 30/2=15.

Step 6) Prob(sum>7)=15/36.

Approach 7:

A solution method with a simple process to generate sums and a count of each sum.

Step 1) Put the numbers on two cards in opposite orders.

Step 2) Arrange the cards with first two numbers aligned. Add the paired digits. The number of overlapping digits gives the count.

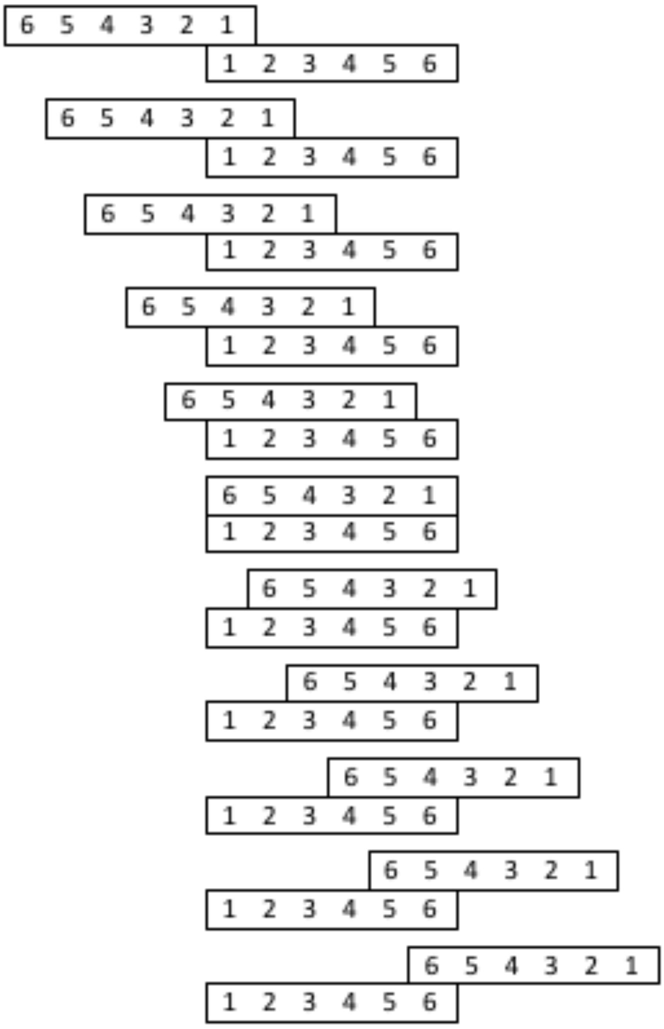
Step 3) Shift top card along 1 digit. Add and count again.

Step 4) Repeat step 3 until last pair of digits reached.

Step 5) Add counts of sums that are >7. $N_{sum>7}$=15.

Step 6) Total sum of all Ns=36

Step 7) Prob(sum >7)=15/36



Sum=2, N=1

Sum=3, N=2

Sum=4, N=3

Sum=5, N=4

Sum=6, N=5

Sum=7, N=6

Sum=8, N=5

Sum=9, N=4

Sum=10, N=3

Sum=11, N=2

Sum=12, N=1