

# Extensional superposition and its relation to compositionality in language and thought

*Chris Thornton*  
University of Sussex  
Brighton  
UK  
c.thornton@sussex.ac.uk

March 23, 2021

## **Abstract**

Semantic composition in language must be closely related to semantic composition in thought. But the way the two processes are explained differs considerably. Focusing primarily on propositional content, language theorists generally take semantic composition to be a truth-conditional process. Focusing more on extensional content, cognitive theorists take it to be a form of concept combination. But though deep, this disconnect is not irreconcilable. Both areas of theory assume that extensional (i.e., denotational) meanings must play a role. As this article demonstrates, they also have the potential to fulfil a mediative function. What is shown is that extensional meanings are themselves, inherently compositional. On this basis, it becomes possible to model semantic composition without assuming the existence of any specifically linguistic/conceptual apparatus. Examples are presented to demonstrate this direct style of modeling. Abstract connections between composition in thought and language can then be made, raising the prospect of a more unified, theoretical account of semantic composition.

Keywords:

compositionality; semantics; concepts; linguistic composition; conceptualization; truth-conditionality;

## **1 Introduction**

To determine the meaning of a previously unencountered word, we typically need to look it up in a dictionary. The situation for previously unencountered sentences is quite different. Provided a sentence uses known words, its meaning is generally apparent. What explains this contrast between words and sentences? Following the work of Frege (1879), theorists have generally seen composition as

explanatorily critical.<sup>1</sup> According to the compositionality principle, the meaning of a sentence derives from the meaning of its parts, and the way they are combined (Partee, 1984). It is argued that the meaning of previously unencountered sentences can be inferred on this basis (Jackendoff, 1990; Pelletier, 2012). This principle also provides a way of explaining the infinite generativity of language. As Katz (1966) notes, it seems that there is no other way to explain ‘a finite mechanism with an infinite output’ (Katz, 1966, p. 152; See also Szabó, 2012, pp. 75-6).

As to the validity of the principle, there has been debate for more than a century (Pagin and Westerstål, 2010). It is noted that there are many cases (e.g., idioms) in which meaning seems to be constructed in a non-compositional way (Searle, 1980), and others (e.g., use of indexicals) where contextual factors seem to play a role.<sup>2</sup> How such cases should be viewed is also controversial. For some, compositionality and contextuality are mutually exclusive. Fodor for example argues that ‘it can’t be both that are true. Something’s gotta give.’ (Fodor, 2003, p. 96-7). Others see the potential for compatibility. Arbib considers that ‘language gets great power from compositionality, but not every utterance exploits this to the same degree’ (Arbib, 2012, p. 482). Frege revised his position over time. Early in his career, he placed particular emphasis on contextuality, noting that ‘it is enough if the sentence as whole has meaning; thereby also its parts obtain their meanings’ (Frege, 1884/1953, section 60).<sup>3</sup> Later, he settled ‘... at the end of his career, close to compositionality.’ (Janssen, 2012, p. 45)

Further complicating matters is the question of how the principle should be interpreted. According to Partee’s (1984) definition, the meaning of an expression depends on the meanings of the parts, and the way they are *syntactically* combined. This begs several questions. If contextual factors potentially influence syntactic combination, does this mean the compositionality principle implicitly allows contextuality to play a role? Or does this, in the words of (Szabó, 2012, p. 65), make the principle ‘hopelessly vague’? Should we then insist on the so-called insulationist or ‘building-block’ interpretation (Cohen, 1986, p. 223), according to which contextual influences are ruled out explicitly? The need to clarify the relationship between compositionality and contextuality has long been acknowledged (Janssen, 1997).

This debate is referenced for completeness. How to reconcile compositionality and contextuality will not be addressed further in this article. Rather, the concern is with how composition in thought relates to composition in language. Attention will be given particularly to an over-arching question: What are meanings, such that they can be combined in a compositional way?

It is natural to begin by examining the basic distinction between referen-

---

<sup>1</sup>However, the roots of the idea can be traced back to the writings of Aristotle at least (Hodges, 2012).

<sup>2</sup>Recanati (2012, p. 177) notes that ‘... the meaning of an expression may well depend upon the meaning of the complex in which it occurs (top-down influence), and it may also depend upon the meaning of the other words that occur in the same complex (lateral influence).’

<sup>3</sup>Wittgenstein takes a similar line in arguing that ‘[o]nly the sentence has sense; a name has a meaning only in the context of a sentence’ (Wittgenstein, 1974/1921, 3.3).

tial and non-referential meaning. Particularly prominent is Frege’s illustration (1879, 1892), in which the phrase *the morning star* is contrasted with the phrase *the evening star*. The word *morning*, it seems, acquires a meaning of a referential kind, by referring to the state of affairs that occurs at the beginning of every day. Equally apparent is that the phrase *the morning star* must have a meaning that is more than purely referential. The referential content of the phrase appears to be the planet Venus. The same entity is also referenced by the phrase *the evening star*, which seems to have a different meaning, however. As Frege observes, the existence of meaning that is non-referential in character must be acknowledged.

The terminology used to characterise the distinction between referential and non-referential content may vary, however. For present purposes, no distinction will be made between Frege’s notion of referential content, and Carnap’s (1947) notion of extensional content. Carnap’s concept is more general, as it allows the referenced entity to be a class (Lewis, 1970). The extensional meaning—or more simply, *extension*—of the word *tree*, for example, is identified as the set of all trees. Frege and Carnap also differ in how they describe the residual form. Frege (1892) terms the non-referential content of an expression its *sense*, while Carnap (1947) calls it the *intensional content*. In what follows, Carnap’s terminology will be used primarily: referential (non-referential) content will generally be termed extensional (non-extensional).

Acknowledging the existence of non-extensional meaning leads on to recognizing that meanings can be constructed. Whereas a word like *morning* acquires its meaning from being mapped to a source of inherent content (e.g., a particular state of affairs), a phrase like *the morning star* acquires its meaning constructively—from the way its constituents are put together. How to explain this constructivity has long been an objective in semantics (Hinzen, 2012a). It is also a concern for psychology. Here, too, there is a desire to distinguish content that is mapping-based from content that is constructed. Differences of detail do exist, however. Semanticists focus on the mapping of words onto what is in the world, whereas psychologists focus on the mapping of words onto what is in the mind (cf. Zettersten and Lupyan, 2020). For the former group, it is worldly states of affairs that are the basic providers of content. For the latter, it is mental concepts. This difference notwithstanding, the same conclusion is reached: meanings are found to be established in either a referential or constructive way.

The thesis that words obtain their meanings from the way they are mapped to concepts is characterized by Lupyan and colleagues as the ‘words-as-mapping’ view (Lupyan and Lewis, 2019). A standard exposition of this is from Li and Gleitman (2002):

It is possible to suppose that these linguistic categories and structures are more-or-less straightforward mappings from a preexisting conceptual space, programmed into our biological nature: humans invent words that label their concepts. (Li and Gleitman, 2002, p. 266)

What Lupyan and colleagues have been able to demonstrate, however, is that

the words-as-mapping view is untenable in regard to human language (Lupyan and Bergen, 2015; Lupyan and Lewis, 2019; Zettersten and Lupyan, 2020). They show that ‘[r]ather than simply mapping onto pre-existing conceptual representations, words help construct these representations’ (Lupyan and Lewis, 2019, p. 2). Their conclusion is that words ‘... not only carve nature at its joints, but also carve joints into nature’ (Zettersten and Lupyan, 2020, p. 2). In Lupyan’s view, it is the capacity of words to ‘sculpt mental representations’ that underpins their constructive power (Lupyan, 2016, p. 516). The proposal that words can be used constructively in this way is termed the ‘words-as-cues’ view (Lupyan and Lewis, 2019).

In linguistics, there is a desire to provide this process of semantic construction with a formal model. The framing of the compositionality principle is an important step towards this goal. From the observation that semantic construction can be self-referential, it can be deduced that constructed meanings may contribute to the construction of further meanings in an ongoing way. The effect can be illustrated using the morning star example. The meaning of *the morning star* can form a constituent within (the meaning of) *the morning star is bright*, which can form a constituent within (the meaning of) *the morning star is bright on wednesdays*, and so on. Construction of meaning is found to be inherently recursive. This is, in essence, what the compositionality principle acknowledges.

Partee’s framing of the principle goes further, however, by adding a specifically syntactocentric element. According to Partee, the meaning of a compound expression is a function of the meanings of its parts and the way they are *syntactically* combined (Partee, 1984, p. 281). This paves the way for a detailed model of semantic construction (Winter, 2016). Consider utterance (1) below.

- (1) The tree has green leaves.
- (2) Does the tree have green leaves?

It can be argued that the two embedded phrases, *the tree* and *green leaves*, both have extensional meaning.<sup>4</sup> In addition, the verb *has* serves to express the proposition that the referenced tree is endowed with leaves which are green in colour. As propositions can be expressed in logic, it is natural to relate the way the meaning of the sentence is composed to the way an analogous proposition is constructed in logic. For example, we might consider the following to be a plausible analogue.

$$\exists x, y : \mathbf{tree}(x) \wedge \mathbf{leaves}(y) \wedge \mathbf{green}(y) \wedge \mathbf{has-attribute}(x, y)$$

Given a framework in which the extensional elements are made referentially unambiguous, and in which the verb encodes an entailment that is demonstrably true, a precise, logical model of the construction of the proposition is then

---

<sup>4</sup>As Winter notes, ‘According to [the compositionality principle], the denotation of a complex expression is determined by the denotations of its immediate parts and the ways they combine with each other.’ (Winter, 2016, p. 28).

obtained. This can be treated as a model of the way the sentence’s meaning is constructed.

This logic-based approach has become central to the study of the semantics of natural language (Winter, 2016). Stemming from the work of Montague (1973), Davidson (1967) and others, the approach is increasingly adhered to in the field (e.g. Partee, 1984; Szabó, 2000; Winter, 2016). A strong, syntactocentric commitment is implied, however. Composition of an expression’s meaning is assumed to be driven and directed by its syntactic construction. As the syntactic construction of the expression proceeds, it is assumed each application of a syntactic rule triggers application of a corresponding semantic rule. This then serves to realize and/or combine the relevant constituent meanings. Semantic composition is seen to be led by syntactic construction, in what Bach (1976) terms the ‘rule-to-rule’ regime.

This scheme does present some difficulties, however. With semantic composition enslaved to syntactic composition in this way, it is unclear how meaning could ever have an independent effect on linguistic output. As Arbib comments, ‘autonomous syntactic rules put words together in very general ways without regard for the meaning of the result’ (Arbib, 2012, p. 482). The idea that an utterance might be shaped by the meaning it is intended to express seems to be ruled out. Under the rule-to-rule arrangement, the meaning of an utterance stems from the way it is syntactically assembled.

A second problem involves the character of the meanings obtained. The meaning of an element with extensional content (e.g., *the tree*) is modeled in terms of what is referenced, as we would expect. The meaning of an entity of a propositional kind is modeled as a truth-value. An expression such as *The tree has green leaves* is considered meaningful if it evaluates to a true proposition. It is on account of this ‘Truth-Conditionality Criterion’ (Winter, 2016, p. 17), that the approach is said to be truth-conditional in character (Napoletano, 2019).<sup>5</sup>

The extent to which the meanings of complex expressions can be seen as truth-values then becomes of interest.<sup>6</sup> The initial indications are not unfavourable. It is widely agreed that the meaning of an expression must derive ultimately from what it says *about* the world (Gilberman, 2016), and it is clear that a proposition which states something true about the world satisfies this criterion. It is also possible to see the approach as a natural continuation of the referential approach to meaning. The approach can be seen as having its roots in the work of Frege and Carnap: ‘Frege hypothesizes the assumption that the reference of a sentence is its truth value’ (Janssen, 2012, p. 28). Likewise Carnap ‘regards a sentence as a complex name (for a truth value)’ (Janssen, 2012, p. 35).

---

<sup>5</sup>Formally, truth-conditional semantics aims to determine ‘... a way of assigning truth conditions to sentences based on A) the extension of their constituents and B) their syntactic mode of combination’ (Rothschild and Segal, 2009). Those pursuing this approach are said to adopt a view ‘... according to which the meaning of a declarative sentence is the set of possible worlds where the sentence is true’ (Szabó, 2012, p. 74).

<sup>6</sup>The question is easily avoided within the truth-conditional paradigm, however. As Kracht notes, ‘Montague never asked what meanings *are*.’ (Kracht, 2012, p. 52).

The difficulty is that some utterances have a meaning which, it seems, cannot possibly be a truth value. Questions (interrogatives) are one such case. Consider utterance (2) above. The meaning of this question seems to be closely related to the meaning of utterance (1). But as the question makes no claim about the world, it cannot be considered either true or false. Similar difficulties are encountered with—to name a small selection—optatives (e.g., *let's dance*), irony, sarcasm, counter-factual sentences, sentences about probabilities, sentences about causal relations, belief sentences, concessives like *but* and *although*, adverbs and gradable adjectives (Iten, 2000).<sup>7</sup>

The problem of Travis cases should also be mentioned. On the face of it, utterance (1) is either true or false. Having a well-defined truth value, it would seem to accede to the truth-conditional approach. But as Travis (1994) points out, there are situations in which the truth of this utterance might differ depending on viewpoint. Travis considers a situation in which, as a decorative measure, naturally brown leaves are painted green. For a photographer, utterance (1) would then be true, whereas for a botanist, it would be false.<sup>8</sup>

From a purely cognitive perspective, scenarios of this kind may not be seen as problematic. Viewpoints are mental construals. As such, they can be considered akin to concepts, and hence legitimate carriers of meaning. It may be argued, accordingly, that what Travis cases really indicate is the need to understand how meanings can be conceptual. A proposal along these lines has been developed by Jackendoff (2002, 2007).<sup>9</sup> It would be a mistake to think that adopting a cognitive perspective will resolve all difficulties, however. How semantic composition operates in thought is no less vexed than the question of how it operates in language. One problem is that the process has been modeled in a very wide variety of ways (e.g. Estes and Glucksberg, 2000; Gagné and Shoben, 1997; Murphy, 1988; Wisniewski, 1997; Wisniewski and Wu, 2012). Hampton and Jonsson (2012) define conceptual composition much as Partee (1984) defines linguistic composition (cf. Prinz, 2002, p. 283). According to them, the ‘... content of a complex concept is completely determined by the contents of its parts and their mode of combination’. Effects of contextuality are explicitly accommodated, however, by allowing that content can also be determined by ‘general knowledge’ (Hampton and Jönsson, 2012, p. 386).

Although conceptual models are generally developed without reference to linguistic theories of composition, it is not unnatural to expect a correspondence (Hinzen, 2012b). Semantic composition in thought, it seems, cannot be unrelated to semantic composition in language. The idea that the mind might employ *two* systems for the construction of complex ideas—one operating in the conceptual system and one operating separately in the language system—strikes

---

<sup>7</sup>Ways of dealing with some of these difficulties within the truth-conditional paradigm are suggested (e.g. Montague, 1968, 1970). Hamblin (1973), for example, offers a way of dealing with questions, as do Groenendijk and Stokhof (1991).

<sup>8</sup>This is just one of many arguments that Travis has raised against the truth-conditional approach. As Hansen notes, ‘Since the 1970s, Charles Travis has been waging a guerrilla war against mainstream truth conditional theories of meaning’ (Hansen, Forthcoming, p. 1).

<sup>9</sup>I am grateful to an anonymous reviewer for helpful suggestions on this issue.

some theorists as implausible. For them, it makes more sense to assume that semantic composition is in the hands of a single system. Hinzen (2009), for example, argues that

thought is as generative and discretely infinite as language is ... such productivity is only possible if there is a generative system behind thought that powers it. Could that system really employ radically different generative principles than the ones we now think the computational system of language (syntax) exhibits? (Hinzen, 2009, p. 128)

Unfortunately, it is not obvious how integration of the two systems can be achieved. How could one generative system serve both conceptual and linguistic ends? For basic forms of extensional meaning, a simple sharing arrangement might be feasible. The extensional meaning of the word *chair* can be equated with the extensional meaning of the concept CHAIR; the extensional meaning of the word *dog* can be equated to the extensional meaning of the concept DOG, and so on. But as Lupyan and Lewis (2019) note, mapping words onto concepts in this 1-to-1 way fails to explain the constructive power that words have.

The arrangement also becomes increasingly problematic as more complex linguistic entities are taken into account. Consider the verb *has* in utterance (1). It appears that this relational word must activate a concept that has slots for the verb's roles. The concept should have one slot for the subject of the signified relationship, and one for its object. The activated concept looks to be dyadic rather than monadic or triadic. The implication seems to be that mental concepts must be *subcategorized* by adicity,<sup>10</sup> on which basis multiple variations of many concepts would be needed. Consider *give*. The fact that this word can be used in a non-transitive, transitive and ditransitive way seems to entail at least three versions of the concept GIVE: one monadic, one dyadic and one triadic. Given what Pietroski calls the 'remarkable combinability' of human concepts (Pietroski, 2012, p. 136), is it realistic to assume mental concepts are subcategorized in this way? The fact that we are able to combine concepts so freely seems to suggest otherwise.

Even with this problem set aside, a stumbling block remains. To connect semantic composition in thought to semantic composition in language, it is necessary to give an account of conceptual composition. It has been shown, however, that mental concepts are often represented in the form of prototypes (i.e., stereotypes), and as theorists have pointed out, it seems prototypes cannot be composed, even in principle (Fodor and Lepore, 1996; Fodor, 1998, 2008).

There are multiple obstacles to be confronted, then. The general result is that, as Hinzen et al. (2012) note,

---

<sup>10</sup>The assumed requirement for adicity/valency matching has roots in Frege's (1892) observation that 'complex concepts are seen to be formed by combining an unsaturated concept with one or more saturating concepts.' (Pietroski, 2010, p. 248). As Pietroski notes, '... abstracting away from the saturaters in a completed concept leaves a mental 'sentence frame' that can be described as an unsaturated concept whose adicity is the number of saturatable positions: arrived(x) is monadic, saw(x, y) is dyadic, give(x, y, z) is triadic, etc.' (Pietroski, 2010, p. 249).

... compositionality today is widely recognized as a key issue all over the cognitive sciences and remains a challenge for various models of cognition that are in apparent conflict with it. (Hinzen et al., 2012, p. 1)

In Pietroski's (2012) view, the key question that arises is this:

How are the meanings of expressions in a natural human language related to concepts? And how is semantic composition related to conceptual composition? (Pietroski, 2012, p. 129)

The aim of the present article is to set out some work that may shed light on this. The main contribution is a new result. It is shown that extensional meanings are themselves compositional. They possess this property by virtue of their capacity for superposition, and it is possessed inherently, whatever the semantic carrier may be. Whether meanings of this kind are represented by words or concepts makes no difference. The meanings themselves have the potential to be superposed. They are thus compositional in both a formal and cognitive/psychological sense.

What relevance does this result have for the relationship between conceptual and linguistic composition? It was seen above that theorists largely agree about the foundational character of extensional content. While the construction of new meaning may be seen as progressing in different ways, for theorists of both language and cognition it is extensional content that serves as the most basic ingredient. Demonstrating the inherent composability of this form of meaning fulfils a conciliative function, then. A bridge is built between the two areas of work, by means of which linguistic and conceptual proposals can be brought together. It is also demonstrated that semantic composition can proceed in a native and self-supporting way, without recourse to any conceptual/linguistic apparatus.

The remainder of the article sets out the result and explores its implications in more detail. There are three main sections. Sections 2 and 3 provide the analysis on which the result is based. Formal specifications are combined with examples so as to provide a concrete context in which the implications of the analysis can be clarified. No more than elementary, set-theoretic mathematics is used at any point. Section 2 focuses on the simplest case of extensional superposition. This is the 1-to-1 scenario, in which one extension is superposed over another. Section 3 goes on to examine the general, recursive case, in which compositional structures of arbitrarily many levels are developed. The implications of the result for human language and conceptualization are then considered in Section 4. Attention is given particularly to ways in which the result might be applied to the general issues noted above. The final section then presents a summary, and suggestions for future research.

One technical point should be made before proceeding. As noted, the result to be presented involves extensional meaning. No commitment is made to any particular medium, however. Ideally, the presentation from here on would make



no further reference to carriers of meaning, such as words or concepts. Unfortunately, adopting this approach leads to convoluted forms of expression that are hard to understand. Extensional meanings are more easily discussed if they are assumed to be carried in a particular way. For purposes of what follows, therefore, extensional meanings will be assumed to attach to *concepts* specifically. These will be named by uppercase terms in the conventional manner: DOG will name the concept of a dog, CHAIR the concept of a chair, and so on. It is only the extensional meanings (i.e., instance sets) of concepts that play any role in the argument, however. No cognitive assumptions are made about the concepts in question. Neither are any assumptions made about the status of instances.

## 2 Basic extensional superposition

It is useful to begin the analysis by examining the compositional opportunities that arise for just two extensional meanings. Let  $X$  and  $Y$  be two concepts, and  $\xi()$  be the extension function, such that  $\xi(X)$  denotes the extension of concept  $X$ , and  $\xi(Y)$  the extension of concept  $Y$ . While  $X$  and  $Y$  may be completely unrelated, it is possible they have instances in common. If  $X = \text{GIFT}$  and  $Y = \text{NECKLACE}$ , for example, it may be the case that some necklaces are also gifts. If so, a new concept can be composed by extensional superposition.

Say the extension of NECKLACE is superposed on the extension of GIFT. The former then serves as a bound on the latter, yielding the concept of a gift that is also a necklace. Vice versa, we obtain the concept of a necklace that is also a gift. The two forms are distinct, however, as they have different instantiation probabilities. The probability that a gift is also a necklace may (and generally will) differ to the probability that a necklace is also a gift.<sup>11</sup>

In what follows, ‘ $\dashv$ ’ will be used to denote composition by extensional superposition. The bounding (i.e., constraining) concept will be placed before the operator, and the bound (i.e., base) concept after it. The concept of  $Y$  bounded by  $X$  is thus expressed by

$$X \dashv Y$$

This can be read as ‘ $X$  over  $Y$ ’. The instances of this concept are all those that are common to  $X$  and  $Y$ :

$$\xi(X \dashv Y) = \xi(X) \cap \xi(Y) \tag{1}$$

Similarly, the concept of  $X$  bounded by  $Y$  (or ‘ $Y$  over  $X$ ’) is expressed by

$$Y \dashv X$$

The instance set is the same:

---

<sup>11</sup>It is specifically the formation of superpositional composites that is considered here, not the generic process of concept *combination*, as studied by, e.g., (Hampton, 1991; Thagard, 1997; Rips, 1995; Wisniewski, 1997; Costello and Keane, 2001; Hampton, 1997, 2011).

$$\xi(X \dashv Y) = \xi(Y \dashv X) = \xi(X) \cap \xi(Y) \quad (2)$$

The two constructions differ probabilistically, however. For each case, we can derive the probability that an instance of the base concept is also an instance of the constraining concept. The two probabilities have the potential to differ, and generally will do so. In the case of X over Y, the probability of instantiation<sup>12</sup> is the ratio

$$\frac{|\xi(X) \cap \xi(Y)|}{|\xi(Y)|}$$

The notation  $P(X \dashv Y)$  will be used to denote this probability:

$$P(X \dashv Y) = \frac{|\xi(X) \cap \xi(Y)|}{|\xi(Y)|} \quad (3)$$

In the case of Y over X, the probability of instantiation is

$$\frac{|\xi(X) \cap \xi(Y)|}{|\xi(X)|}$$

on which basis,

$$P(Y \dashv X) = \frac{|\xi(X) \cap \xi(Y)|}{|\xi(X)|} \quad (4)$$

Using the concepts GIFT and NECKLACE, it is possible to superpose NECKLACE over GIFT, or GIFT over NECKLACE. The latter yields

$$\xi(\text{GIFT} \dashv \text{NECKLACE}) = \text{necklaces that are also gifts}$$

The probabilities of the two variants are

$$P(\text{GIFT} \dashv \text{NECKLACE}) = \frac{|\xi(\text{GIFT}) \cap \xi(\text{NECKLACE})|}{|\xi(\text{NECKLACE})|}$$

and

$$P(\text{NECKLACE} \dashv \text{GIFT}) = \frac{|\xi(\text{GIFT}) \cap \xi(\text{NECKLACE})|}{|\xi(\text{GIFT})|}$$

If necklaces are more likely to be gifts than vice versa, then

$$P(\text{GIFT} \dashv \text{NECKLACE}) > P(\text{NECKLACE} \dashv \text{GIFT})$$

It is important to stress that these probabilities are intrinsic, relational quantities. A probability of this kind expresses the degree to which the bound concept's extension falls within that of the binding concept. The intrinsic probability of a construction may bear no relation to the probability of observing its instances in the world. For example, the intrinsic probability of a necklace being also a

---

<sup>12</sup>The term *instantiation* refers, in all cases, to realizing an instance of a concept.

gift may be high even while necklaces of this particular type are vanishingly rare in practice.

How to categorize intrinsic probability using standard terminology is not entirely straightforward. An intrinsic probability is the probability of instantiation that a composite concept (of this particular kind) acquires from its structural form. To some degree, it resembles a fuzzy set-membership measure (Zadeh, 1965, 1982). It quantifies the degree to which an instance of the base concept is within the bounding extension. This interpretation is not quite watertight, however, as, neither of the sets in question is fuzzy. An intrinsic probability might also be described as a *frequentist* measure (Hájek, 2002), as it quantifies the frequency with which instances of one concept are observed to instantiate another. No less justifiably, it might be viewed as a *subjectivist* or Bayesian measure (Griffiths et al., 2010), on the grounds that it derives from the construction of a concept, which is itself a subjective entity.

More clear-cut is the relationship between intrinsic probability and ontology. It can be shown that constructions with an intrinsic probability of 1 are implicitly ontological. Consider a composite of FRUIT and BERRY. With FRUIT placed in the bounding role, we obtain FRUIT  $\dashv$  BERRY. This is the concept of instances of BERRY that are also instances of FRUIT (i.e., berries that are also fruits). It will be seen that this is a special case. Since berries are fruits by definition, the extension of BERRY is *entirely* contained within the extension of FRUIT. This composite thus has an intrinsic probability of 1.

$$P(\text{FRUIT} \dashv \text{BERRY}) = \frac{|\xi(\text{FRUIT}) \cap \xi(\text{BERRY})|}{|\xi(\text{BERRY})|} = 1$$

A superpositional construction of maximum probability is implicitly an ontological assertion, then. Since FRUIT  $\dashv$  BERRY has an intrinsic probability of 1, it can be said to express the knowledge that BERRY is subsumed by FRUIT. It is, in effect, a class-containment relation or *is-a* link (Brachman, 1983).

With incomplete extensional containment, the situation changes. Consider SNACK  $\dashv$  BERRY. This is the concept of berries that are also snacks. Assuming instances of BERRY need not be instances of SNACK, the extension of the former is not wholly contained within that of the latter. With containment being on a partial basis, the construction cannot be interpreted as an *is-a* link. What it identifies, rather, is a composite concept that is new, in the sense of differing from both of its constituents.

Assuming snacks are rarely made up of single berries, the extensional containment of SNACK within BERRY is minimal, with the same applying to the composite's intrinsic probability. Variations of the composite may rate more highly, however. Consider SNACK  $\dashv$  APPLE. Assuming an apple is a more likely snack than a berry, this concept has a correspondingly higher intrinsic probability. The same would seem to apply to SNACK  $\dashv$  SANDWICH. On the other hand, consider SNACK  $\dashv$  LOAF. Assuming a loaf is a less probable snack than a berry, then

$$P(\text{SNACK} \dashv \text{LOAF}) < P(\text{SNACK} \dashv \text{BERRY})$$

There is also the potential for a composite to have an intrinsic probability of zero. Such cases represent the opposite extreme of the implicitly ontological constructions described above. A composite of this kind has the property that *no* instance of the base concept is also an instance of the bounding concept. The two concepts are unrelated in this sense. Illustrations may seem bizarre. Consider SNACK  $\dashv$  LORRY. This is the concept of a lorry that is also a snack. Assuming there is no containment of the base within the bounding extension, the composite has an intrinsic probability of zero:

$$P(\text{SNACK } \dashv \text{ LORRY}) = 0$$

Alternatively, we could borrow one of the ideas from Chomsky’s sentence ‘colorless green ideas sleep furiously’ (Chomsky, 1957). The composite COLORLESS  $\dashv$  GREEN expresses the concept of instances of COLORLESS that are also instances of GREEN (i.e., things that are both colorless and green). Given there is no extensional containment of GREEN within COLORLESS, the concept is found to have an intrinsic probability of zero.

It may be worth emphasizing again the empirical neutrality of these evaluations. While the intrinsic probability of a construction such as SNACK  $\dashv$  LORRY may be zero, this does not mean the concept cannot be formed in thought. With a suitable degree of imagination, the idea of a snack that is also a lorry can certainly be mentally entertained. An intrinsic probability is a formal property of a superpositional construction. It may give no indication of the frequency with which instances are observed.

Also worth restating is the connection between extensional superposition, ontology and conceptual creativity. It is because superpositional constructions have intrinsic probabilities that they form a continuum; and it is at one extreme of this continuum that constructions express the prototypes of ontological theory. A composite with an intrinsic probability of 1 (e.g., FRUIT  $\dashv$  BERRY) is an implicitly ontological construct. It can be seen to state that the construction’s constituents are related by subsumption. A construction with an intrinsic probability of zero (e.g., BIRD  $\dashv$  BERRY) can be seen as stating that the constituents have entirely unrelated instances. All other cases (e.g., SNACK  $\dashv$  APPLE) lie between these two extremes. Neither implicitly ontological nor intrinsically improbable, they are inherently innovative in character. Each defines a new, hybrid concept that differs from the constituents utilized in the construction.

### 3 Complex composites

Up to this point, the focus has been on basic, 1-to-1 composites. Constructions of a more complex kind can be formed, however. Recursion is also a possibility. As a composite expresses a concept itself, it can serve as a constituent in a further act of composition. Complex structures can be formed in this way. Consider the relationship between SNACK and EDIBLE  $\dashv$  BERRY. If some instances of SNACK are also instances of EDIBLE  $\dashv$  BERRY there is an implied concept: the concept of snacks that are also edible berries. This can be expressed as

SNACK  $\dashv$  EDIBLE  $\dashv$  BERRY. (In a similar way, the concept of ‘colorless green ideas’ could be expressed as COLORLESS  $\dashv$  GREEN  $\dashv$  IDEA.)

Hierarchical constructions are also possible. Say, we have the three concepts, X, Y and Z. It may be that instances of Y and Z taken in combination form an instance of X. If so, it is possible to construct the concept of instances of X that are also *combinations* of (instances of) Y and Z. Generalizing the present notation, this can be expressed as

$$X \dashv \begin{array}{c} Y \\ Z \end{array}$$

The concept on the left is the bounding element as before. What is bound, however, is now a combination of concepts. As the bounding effect is one of hierarchical accommodation, constructions of this kind can also be described in terms of accommodating/accommodated concepts. In the construction above, X can be considered the *accommodating* concept, for the *accommodated* combination of Y and Z. This terminology is used in some of the cases described below.

The intrinsic probability of a construction of this kind is derived in the same way as before, by Eq. 3. But there is a need to redefine the extension of the base entity, as this is now a combination of concepts. The set of all instances of a combination of concepts contains all ways the instances of the concepts can combine. Mathematically, it is the unordered,  $n$ -ary Cartesian product of their extensions (i.e., the Cartesian product expressed in sets rather than  $n$ -tuples). Given  $C$  denotes a combination of  $n$  concepts, the notation

$$\prod_{X \in C} \xi(X)$$

will be used to identify the unordered,  $n$ -ary Cartesian product of  $C$ . This is the set of combinations that can be formed by including one instance from each concept in  $C$ .<sup>13</sup>

Defining  $\xi^*(\cdot)$  as a variant of the extension function that accepts either a single concept or a combination of concepts, it can then be asserted that

$$\xi^*(C) = \begin{cases} \xi(C) & \text{if } C \text{ is a concept} \\ \prod_{X \in C} \xi(X) & \text{if } C \text{ is a set of concepts} \end{cases} \quad (5)$$

This allows the extension of a composite bounding  $C$ , where  $C$  is either a single concept or a combination of concepts, to be defined as

$$\xi(X \dashv C) = \xi(X) \cap \xi^*(C) \quad (6)$$

Eq. 1 is then generalized in a way that allows for hierarchical construction.

---

<sup>13</sup>The definition is purely set-theoretic and thus differs from that of a ‘mereological sum’ (Blackburn, 2008). Also of note is that this notation deviates from the standard approach, in which the product is considered to be *ordered*, i.e., expressed in terms of  $n$ -tuples rather than sets.

Elements of  $\xi^*(C)$  may be concept combinations or even—given the potential for recursion—multi-level structures. This increase in generality creates two difficulties. To determine the intersection of Eq. 6, it is necessary to compare elements for equality. How is this to be done if elements in one case are atomic entities, and structures in the other? An assumption is required to resolve this. For comparative purposes, an extensional element is considered to be the instance it represents. On this basis, an entity is considered to be equal to a structure just in case both represent the same instance.

A second difficulty involves intrinsic probability. The definition of Eq. 6 allows that an extensional element may be a structure of concepts, each of which has its own intrinsic probability. How is the intrinsic probability of the over-arching entity to be calculated? This is resolved by means of a least-commitment interpretation. The intrinsic probability of a compositional structure is taken to be the *minimum* observed. This ensures that a composite with an intrinsically impossible element is itself classified as intrinsically impossible.

Letting  $K^*$  represent the set of  $K$ 's constituents (where  $K$  represents a composite), the intrinsic probability of  $K$  is defined as

$$P(K) = \text{MIN}_{X \in K^*} P(X) \quad (7)$$

To see how hierarchical composition by extensional superposition works in practice, consider the concepts LAWN, PATH and GARDEN. On the assumption that the combination of a lawn and path can constitute a garden, there is a potential construction expressing the concept of a garden constituted in this way. Its designation is

$$\text{GARDEN} \dashv \begin{array}{l} \text{LAWN} \\ \text{PATH} \end{array}$$

The generative implications of Eqs. 5 and 6 can then be explored. Let the extensions of GARDEN, LAWN and PATH be as follows:

$$\begin{aligned} \xi(\text{GARDEN}) &= \{\text{residential garden, market garden}\} \\ \xi(\text{LAWN}) &= \{\text{front lawn, croquet lawn}\} \\ \xi(\text{PATH}) &= \{\text{gravel path, tarmac path}\} \end{aligned}$$

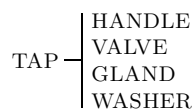
Assuming a residential garden can be constituted of a front lawn and a gravel or tarmac path, but that all other constitutive arrangements are illegitimate (e.g., a market garden cannot have *any* type of lawn as a constituent) the extension of the composite then contains just two elements:

$$\xi \left( \text{GARDEN} \dashv \begin{array}{l} \text{LAWN} \\ \text{PATH} \end{array} \right) = \left\{ \begin{array}{l} \text{residential garden}[\text{front lawn, gravel path}], \\ \text{residential garden}[\text{front lawn, tarmac path}] \end{array} \right\}$$

Of note here is the way the requirement for constitutive legitimacy affects the result. Composition using GARDEN has an effect akin to applying a garden() function. Illegitimate combinations of constituents are effectively filtered out, leaving only two possibilities. In the formula shown, square-brackets are used to show where a combination of concepts has been equated

with (i.e., deemed constitutive of) an accommodating concept. The description ‘residential garden[front lawn, gravel path]’ identifies the constitutively legitimate case in which a front lawn and gravel path are considered to constitute a residential garden. In contrast, ‘market garden[front lawn, gravel path]’ identifies the constitutively *illegitimate* case of a market garden constituted of a front lawn and gravel path. Eqs. 5 and 6 in combination produce both a generative effect (structures are assembled), and a functional one (these represent only constitutively legitimate cases).

Like their non-hierarchical counterparts, hierarchical composites form a continuum, with implicitly ontological entities at one extreme, and intrinsically implausible constructions at the other. The degree of extensional containment can vary from complete to non-existent. Consider the situation of complete containment. In this case, every instance of the accommodating concept is a combination of the described form. The GARDEN construction above is clearly *not* of this type, as not every garden is made up of a lawn and path. But consider



This expresses the concept of a tap constituted of a handle, valve, gland and washer. For the sake of argument, let it be assumed that every tap has these four parts. Complete containment of the accommodated combination’s extension is then assured, and ontological status is conferred. As an assertion that a tap has a handle, valve, gland and washer as parts, it can be classified as a *part-whole hierarchy* or *meronymy* (Tversky, 1989). It expresses the ‘partonomic’ style of knowledge representation, in which a hierarchical structure is given to a particular entity in order to represent its ‘subdivision into parts’ (Tversky, 1989, p. 983; see also Winston and Herrmann, 1987; Gerstl and Pribbenow, 1995; Tversky, 2005).

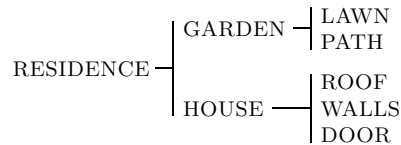
In a part-whole hierarchy, it is typical to have more than one level of structure. The subdivision into parts is generally continued recursively, with parts at one level being further subdivided. This is illustrated by the following, two-level construction:



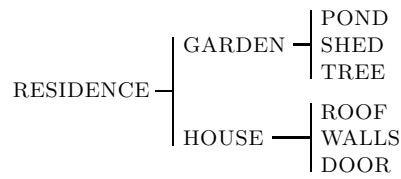
This expresses the concept of a car comprised of body, wheels and engine, where the engine consists of a carburetor, block and ignition. On the assumption that every car has this make-up, complete extensional containment is achieved at all levels. The structure is implicitly ontological. It can be classified as a representation of knowledge about cars, expressed in the form of a part-whole hierarchy.

More than one level of structure is the norm for part-whole hierarchies. But

this is also possible in the general case, where extensional containment is on a partial rather than complete basis. Entirely new concepts are then obtained from each construction, and these are combined in a hierarchical way. One constructed concept comes to serve as a constituent within the construction of another. Consider this two-level structure:



This expresses the concept of a residence made up of a house and garden, where the latter combines a lawn and path, and the former combines a roof, walls, and door. It will be seen that the construction combines composites at two levels. The concept of a garden combining a lawn and path comes to serve as a constituent within the higher-level RESIDENCE construction. Replacing this lower-level entity changes the idea ultimately obtained. For example,



expresses the subtly different concept of a residence comprising a house and garden, where the latter combines a pond, shed and tree (rather than a lawn and path). This cannot be considered a part-whole hierarchy, as residences, houses and gardens can all be put together in different ways. Rather, the intrinsic probability of the construction derives from the degree of extensional containment in the usual way.

Part-whole hierarchies often feature concrete concepts. But incorporation of abstract concepts is possible, and this is also feasible in hierarchical composites. Consider, for example



This expresses the concept of a wedding that combines a reception and a service, where the latter comprises a betrothal and a blessing. This illustrates integration of abstract concepts in a compositional construction.

As in the non-hierarchical scenario, intrinsically impossible constructions can easily be produced. A construction can be placed into this category simply by incorporating an intrinsically impossible element (see above). Consider this variant of the wedding concept:





On the grounds that a service cannot have an election as a constituent, the BETROTHAL + ELECTION combination is not subsumed by the extension of SERVICE. The composite has an intrinsic probability of zero. The intrinsic impossibility of a single element ensures that the enclosing construction is also considered intrinsically impossible.

Intrinsic impossibility of *all* accommodated elements is equally feasible. Consider

$$\text{MEAL} \left\{ \begin{array}{l} \text{ENGINE} \\ \text{BRICK} \end{array} \right.$$

This expresses the rather implausible concept of a meal constituted of an engine and a brick. On the assumption that no such meal exists, the extension of the composite is empty. It has an intrinsic probability of zero.

It was noted earlier that 1-to-1 composites are generally productive. They usually have the effect of producing concepts that are inherently new, in the sense of differing from the constituents used. This innovativity is multiplied level-by-level in a hierarchical construction. The more complex the construction, the more creative the end-result. This can be illustrated by assembling a multi-level structure in a bottom-up way. Say we begin with

$$\text{MEAL} \left\{ \begin{array}{l} \text{PASTA} \\ \text{SALAD} \\ \text{FRUIT} \end{array} \right.$$

This expresses the reasonably plausible concept of a meal of pasta, salad and fruit. Also defined is the concept of a toolkit consisting of hammer, screwdriver and pliers:

$$\text{TOOLKIT} \left\{ \begin{array}{l} \text{HAMMER} \\ \text{SCREWDRIVER} \\ \text{PLIERS} \end{array} \right.$$

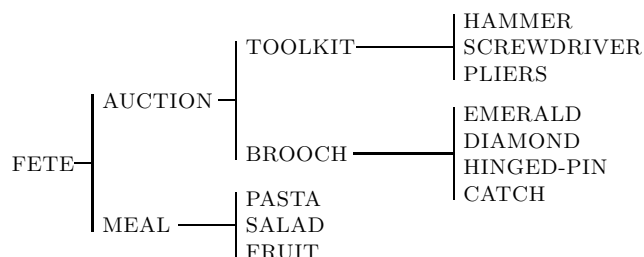
A further addition is the concept of a brooch whose parts are an emerald, a diamond, a hinged pin and a catch:

$$\text{BROOCH} \left\{ \begin{array}{l} \text{EMERALD} \\ \text{DIAMOND} \\ \text{HINGED-PIN} \\ \text{CATCH} \end{array} \right.$$

We can then place AUCTION (the concept of an auction) as the accommodating element for the latter two constructions. This yields the concept of an auction of a (certain type of) toolkit and a (certain type of) brooch.

$$\text{AUCTION} \left\{ \begin{array}{l} \text{TOOLKIT} \left\{ \begin{array}{l} \text{HAMMER} \\ \text{SCREWDRIVER} \\ \text{PLIERS} \end{array} \right. \\ \text{BROOCH} \left\{ \begin{array}{l} \text{EMERALD} \\ \text{DIAMOND} \\ \text{HINGED-PIN} \\ \text{CATCH} \end{array} \right. \end{array} \right.$$

An additional layer in the construction can then be superimposed by citing FETE as accommodation for the AUCTION concept just derived, and the MEAL concept derived previously. The final result is the concept of a fete combining a certain kind of meal with a certain type of auction:



This example highlights the constructive capacity of extensional meanings. Their ability to act as ‘stepping stones’ in the construction of a new meaning becomes apparent. The formation of composites out of pre-existing constructs has the effect of bringing new, combinational meanings into existence. Structures built up in this way have the surface appearance of a part-whole hierarchy, but normally cannot be classified as such. There may be less than complete extensional containment at each node (as in the example above). The structure above has the appearance of a part-whole hierarchy. It is, in fact, the construction of a multi-level concept—in this case, the concept of a fete incorporating a meal and auction, where the former consists of pasta, salad and fruit, and the latter consists of a toolkit and brooch. It is possible this idea has never been articulated before. The possibility of constructing unlimited variations on the same theme reveals the infinite generativity of composition by extensional superposition.

Broadly speaking, then, the same situation arises for both hierarchical and non-hierarchical composites. In both cases, there is a continuum, with intrinsically impossible, extensionally empty constructions at one extreme, and implicitly ontological constructions at the other. A construction at either extreme may be regarded as sterile in the sense that it fails to produce a normal concept. In one case, the concept expressed has an empty extension. In the other, it can be reduced to a subsumption relation. In the latter case, ontological terminology becomes appropriate. If all accommodated entities are combinations of object concepts, the entity can be classified as a meronymy (part-whole hierarchy). In the case of the accommodated entity being a single concept, the construction can be classified as a class-containment relation or *is-a* link.

## 4 Semantic composition in thought and language

The analysis set out above refers to concepts; but as stated previously, extensional meanings are the real focus. It is these that are shown to be compositional. Attending to meanings rather than the bearers of meaning (i.e. words or concepts) has the advantage of maintaining a level of generality that embraces

both thought and language. It is the extensional meanings of both linguistic and conceptual entities that are shown to have the capacity to stack together. The result potentially applies, therefore, to the way new meanings are produced in both language and thought.

It is conjectured by theorists of both language and cognition that composition must involve processing of extensional meaning. Both fields take this form of content to be foundational. At the same time, both envisage the process of composition to be stage-managed by a constructive apparatus. Language theorists tend to see this apparatus as a truth-conditional mechanism, whereas cognition theorists tend to see it as concept-combinatorial. What the present result suggests, however, is that no dedicated apparatus may be required. As extensional meanings are inherently capable of composition, it is conceivable that superposition is the sole medium. Whether this is the arrangement that exists in the mind is, of course, an empirical question requiring further investigation of both a theoretical and empirical kind.

Pending that work, no firm conclusions can be drawn. It is possible, however, to identify cases that illustrate the potential reach of the approach. How extensional specifications ‘work’ as bearers of meaning has long been understood. What has now been seen is that they also work well as building blocks. It has been shown that composition by extensional superposition provides a path that leads away from extensional meaning, towards non-extensional meaning. As this latter form encapsulates extensional elements, and is arguably quasi-extensional in its capacity for reference to imaginary entities (see below), a better description for it is *composed extensional content*. Adopting this term, the path provided by extensional superposition can be said to lead away from purely extensional content, towards *composed* extensional content.

The degree to which this path takes in constructions of thought (i.e. conceptual composites) has been illustrated to some degree in the previous two sections. Various archetypes of conceptual representation have been seen. These include constructs akin to classical definitions (e.g. an edible berry), classifications (a berry classified as a snack), conceptual complexes (a garden comprising a path and lawn) and hierarchical schemas (a fete comprising a meal of pasta and salad, combined with an auction of tools and jewelry). The path also takes in archetypes of ontology. Constructs of maximum intrinsic probability are found to express ontological facts. Non-hierarchical instances express *is-a* links; hierarchical instances express part-whole hierarchies. Conversely, constructs with an intrinsic probability of zero are found to express conceptualizations of a fantastical or counter-intuitive nature (e.g. a lorry that is also a snack).

The path can also be shown to pick out certain forms of language. As noted in the introduction, it is the constructivity of words that particularly draws attention towards semantic composition in language. Frege’s morning/evening star example serves as the familiar illustration. As Frege notes, if the phrases *the morning star* and *the evening star* have different meanings while also referring to the same object, it follows that they must have more than merely referential content.

This example can be recreated in compositional terms. Consider the super-

positional construct

ENTITY-SEEN-AT-DAWN  $\dashv$  STAR

This expresses the concept of a star that is also an ‘entity seen at dawn’. The evening counterpart is

ENTITY-SEEN-AT-DUSK  $\dashv$  STAR

This expresses the concept of a star that is also an ‘entity seen at dusk’. Notice that the two constructions correspond to Frege’s phrases. Both use constituents with extensional content in the building of semantically distinct references to the same object.

References to imaginary entities can be assembled in a similar way. Consider the case of unicorns. As these animals do not exist in reality, it seems a purely extensional specification is ruled out. Here, too, it is possible to devise a specification in which the meaning is rendered in the form of composed extensional content. Consider, for example

HORNED  $\dashv$  HORSE

This expresses the concept of a horse that is also horned. The construction has referential content, in the form of the identification of a horned horse (i.e. a unicorn). The fact that unicorns do not exist in reality makes no difference to the integrity of the specification. Its meaning does not stem from real animals. It stems from the way the extensional meanings of the concepts HORSE and HORNED are superposed. What is obtained is an extensionally precise reference to an imaginary entity.

Certain linguistic constructions can also be modeled in this way. Recall utterance (1) from the introduction: this was the sentence *The tree has green leaves*. The meaning of the phrase *green leaves* can be modeled by a construction expressing the concept of leaves that are (also) green:

GREEN  $\dashv$  LEAVES

Likewise, the meaning of *the tree* can be modeled by a construction in which an arbitrary tree is also made a definite referent (a ‘DEF-REF’).

DEF-REF  $\dashv$  TREE

Using these two constituents, the following hierarchical construction can then be assembled.

HAS-ATTRIBUTE  $\dashv$   $\left\{ \begin{array}{l} \text{SUBJECT} \dashv \text{DEF-REF} \dashv \text{TREE} \\ \text{OBJECT} \dashv \text{GREEN} \dashv \text{LEAVES} \end{array} \right.$

At the root of the structure is HAS-ATTRIBUTE (which is assumed to name the concept of ‘having an attribute’). Notice also the use of SUBJECT and OBJECT to capture roles. The meaning of *green leaves* is conceptualized as

SUBJECT within a having-an-attribute instance, while the meaning of *the tree* is conceptualized as the corresponding OBJECT. Taken in full, the construction expresses the idea of a definite tree that has green leaves as an attribute. To a reasonable degree, the meaning of the construction models the meaning of the utterance.

Of note is the way this approach addresses the issue of adicity. As discussed in the introduction, the potential for a single word to have multiple valencies seems to create a need for multiple representations of the same concept. What is required, it appears, is a mapping that connects the various valencies that each word has to corresponding adicities of the same concept (Pietroski, 2012). The problem is that this arrangement seems implausibly convoluted, given ‘humans have concepts that combine so rapidly and easily’ (Pietroski, 2012, p. 148).

Modeling composition as extensional superposition overcomes this difficulty to some degree. Rather than concepts having preordained adicities, extensional meanings are taken to be represented in a way that accommodates hierarchical constitution. By virtue of Eqs. 5 and 6, any one instance may be made up of several others; the need to assume predesignated adicities is then avoided. Whether a compositional construction can be formed is determined by extensional intersection. The adicity/valency matching problem is eliminated at the cost of allowing hierarchically structured meanings.

This particular example can also serve to illustrate what Lupyán and Lewis call the ‘words-as-cues’ view (Lupyán and Lewis, 2019; and see above). Each word in the utterance *The tree has green leaves* maps to a single concept, and each concept references a pre-existing meaning. It would be wrong, however, to assume this implies application of the ‘words-as-mapping’ view. The meaning of the whole is constructed compositionally, in a process that uses the words to shape (i.e. sculpt) a conceptual representation. There is a correspondence with the process Lupyán (2016) envisages. To the extent that words serve as cues within it, the model can be seen to express the ‘words-as-cues’ view.

## 4.1 Non-truth-conditional content

A key advantage of the superpositional approach is that it gets around some of the problems associated with truth-conditional semantics. In this approach, recall, the meaning of a compositionally constructed expression is taken to be a truth value, and ongoing composition is assumed to be achieved by combining such values in a logical way (Davidson, 1967; Recanati, 2001). As noted previously, meanings that are not truth values then present a difficulty, and the signs are that these are not uncommon. As Napoletano (2019) observes, ‘the fact that expressions have the particular truthconditional contents (extensions or intensions) they have does not even *partly* explain facts about semantic phenomena’ (Napoletano, 2019, p. 541, emphasis added).

The present approach abandons the core assumption of the truth-conditional approach (that meanings are truth values), while remaining broadly compatible with the truth-conditionality criterion. A truth value can be given to a meaning constructed by extensional superposition. The claim that the leaves of a certain

tree are green could be shown to be true or false within a certain model. But, importantly, this would not entail a reduction of the meaning to a truth value. Viewed as a superpositional construction, the utterance would retain a well-defined content of the composed extensional type. The meaning of the claim would then be established *prior* to its truth, in accordance with the principle that ‘reference is ontologically prior to truth’ (Fodor, 2008, p.215).

With semantic composition modeled in this way, untruthful sentences also pose less of an obstacle. Declarative assertions can be meaningful even if false. This bears on the issue of Travis cases. As seen above, Travis (1994) observed that sentences can vary in their truth value depending on the viewpoint adopted (Travis, 1994, 2008; Gilberman, 2016; Hansen, Forthcoming). Representing the meaning of declarative sentences in a way that makes truth a derivable property overcomes the difficulty. This can also be seen as applying the perspective of Jackendoff (2002, 2003, 2007). It was shown above that composed extensional content is implicitly cognitive. Modeling a meaning in this way is akin to modeling it as a cognitive construal. What this example can be seen to show, accordingly, is that Travis cases can be resolved by recognizing that meanings can be cognitive construals.

Non-declarative forms of meaning, such as interrogatives and imperatives, also become more tractable. This can be illustrated using utterance (2). The interrogative counterpart of utterance (1), this is the question *Does the tree have green leaves?* The meaning of this question can also be modeled compositionally. Its content is that of the original, declarative assertion, but reconceptualized as a question. A natural approach is thus to model its meaning as the original construction with the concept QUESTION superposed.

$$\text{QUESTION} \multimap \text{HAS-ATTRIBUTE} \quad \left| \begin{array}{l} \text{SUBJECT} \multimap \text{DEF-REF} \multimap \text{TREE} \\ \text{OBJECT} \multimap \text{GREEN} \multimap \text{LEAVES} \end{array} \right.$$

Here, the content of the original sentence is made an instance of QUESTION, in a construction that (literally) reconceptualizes the original content as a question.

## 4.2 Related work

It is natural to ask whether modeling semantic composition in the envisaged way offers a new approach, or just a re-hash of something gone before. It should be emphasized straight away that the approach is not, in any sense, a new exposition of traditional *extensionalism*. This advances the idea that meanings are always referential in character.<sup>14</sup> The present approach differs fundamentally

<sup>14</sup>It is widely accepted that words often do not have meanings of an extensional kind (Evans and Levinson, 2009). Relative adjectives such as ‘big’ and ‘sweet’ appear to defy any kind of extensional definition. Assuming the word ‘big’ implies the existence of concept BIG, it has to be supposed that this concept lacks extensional definition, for the same reason the word ‘big’ does. A potential way around this is to assume that BIG = COMPARATIVELY BIG, i.e. big relative to members of the same class.

in that it emphasizes the existence of non-referential meaning, or *composed extensional content* as it is here termed. Using traditional terminology (Frege, 1879; Carnap, 1947), a composed extensional content can be considered either a Fregean *sense* or a Carnapian *intensional content*. Rather than committing to any form of traditional extensionalism, then, the present approach is largely concerned with what Carnap calls intensional content.

#### 4.2.1 Generative semantics

A potential precursor of the present framework is Generative Semantics (Lakoff, 1971; Jackendoff, 1990). In this approach, utterances are assumed to derive from meaning structures, and these are hierarchical forms defined in terms of upper-cased concept names—structures not unlike the composites of the present approach. There is a kinship between the two approaches, in the sense that both attribute importance to compositional structure of both a semantic and conceptual kind.

Generative semantics neither adopts nor proceeds from a model of extensional superposition, however. It is also noncommittal in regard to concept constituency. As Kracht says, ‘Generative Semantics never bothered to elucidate the meanings of the upper-cased expressions in any detail; it was more concerned with lexical decompositions and capturing semantic regularities (active-passive and so on)’ (Kracht, 2012, p. 55).<sup>15</sup>

The main difference between the two approaches is more basic, however. A key concern in generative semantics is to explain how the syntactic forms of language come to be shaped by semantic forces. It is envisaged that this must involve multiple phases of representation, with later stages being more language-specific, and only the initial stage being purely conceptual. In the approach of two-level semantics (Lang and Maienborn, 2011), an initial purely conceptual stage of representation (termed conceptual structure) is followed by an intermediate stage termed semantic form.<sup>16</sup> The present approach differs in that it is concerned with semantic composition *alone*. It aims to maintain a level of abstraction at which the semantic medium can vary. The question of how purely conceptual representations come to be expressed in syntactic form

---

<sup>15</sup>Hinzen frames the demise of generative semantics slightly differently: ‘... what centrally brought down generative semantics was the syntax it posited: ever more unmotivated “transformations” between hypothesized underlying “semantic representations” and surface forms, without seeing these syntactic operations systematically accompanied by semantic effects.’ (Hinzen, 2012a, p. 354).

<sup>16</sup>This is a departure from mainstream practice. As Bierwisch (2007) says, it must ‘... be noted that most semantic theories simply do not acknowledge a representational difference between SF [semantic form] and C/I [conceptual/intentional structure]. Therefore the problem of characterizing representations of C/I simply does not arise e.g. in standard versions of formal semantics.’ (Bierwisch, 2007, p. 13) On the other hand, Bierwisch shows no opposition to the kind of representational structure that is produced by extensional superposition. He comments that it ‘... ‘seems to me fairly safe to assume that semantic representations are based on an abstract, transparent, combinatorial structure. As a matter of fact, all reasonable approaches to semantics are based on this assumption in one way or the other’ (Bierwisch, 2007, p. 23).

is deliberately set aside.<sup>17</sup>

A related connection is with the predicate/argument model of meaning. In this, a verb (or any entity functioning as a verb) is seen to define a predicate, which when applied to arguments defined by the verb's dependents, yields the meaning of the sentence (Kroeger, 2004). To the extent that predicates and arguments can be modeled as accommodating and accommodated concepts, an approach focusing on composed extensional content can be likened to one that focuses on predicates and arguments. Again there are important differences. In particular, the predicate/argument approach relates meaning to truth, whereas truth plays no role in composed extensional content. Furthermore, the predicate/argument conception it is not based on an extensional model of conceptual composition.

#### 4.2.2 Concept theory

The analysis can also be related to certain areas of concept theory. Much experimentation has been carried out in an attempt to establish what mental concepts are (Murphy, 2002; Machery, 2009). The classical assumption, that concepts are definitions, has largely been abandoned following the discovery of typicality effects (e.g. Rosch and Mervis, 1975). The fact that people are often willing to rate instances according to their typicality (for a particular concept) suggests concepts must be represented, not in the form of all-or-nothing definitions, but in the form of prototypes (Osherson and Smith, 1981; Hampton, 1995). The evidence is not quite conclusive, however. It is found that in classifying potential cases of a concept, people may take account not only of the relevant prototype, but also of actual instances. This leads to the so-called exemplar theory, in which it is assumed concepts are represented in terms of exemplar sets.

This debate is relevant here partly because of its interaction with the issue of compositionality. It has been argued that concepts cannot be prototypes because—it is claimed—prototypes cannot compose. On this assumption, they fail to satisfy what many consider a non-negotiable requirement. This problem has been particularly emphasized by Fodor and colleagues (Fodor and Lepore, 1996; Fodor, 1998). As Fodor sees it, concepts must compose since ‘how else could one explain why our concepts are productive and systematic?’ (Fodor, 2008, p. 45). The consequence for Fodor is that concepts cannot possibly be prototypes (Fodor, 2008).<sup>18</sup>

The case against prototypes is strengthened by the phenomenon of emergent features (Carey, 2009). It is found that when two concepts are combined, features may be generated that neither concept possesses independently. In the familiar example, the phrase *pet fish* is assumed to combine PET and FISH. It is then found that instances of the combined concept are likely to be seen as ‘living in bowls’, even though this is not a characteristic of either pets or fish (Wisniewski and Wu, 2012; Gleitman et al., 2012). One way to explain this

---

<sup>17</sup>But see (Thornton, 2016) for a preliminary proposal.

<sup>18</sup>As Connolly et al. see it, ‘(1) concepts must be compositional, (2) stereotypes are not compositional, (3) therefore, concepts are not stereotypes.’ (Connolly et al., 2007, p. 2).



is to assume that concepts are represented in the form of explanatory theories. Concept combination can then be held to involve processes of reasoning, with emergent features considered to arise inferentially. This leads to the so called ‘theory theory’, which holds that concepts are explanatory theories (Murphy and Medin, 1985).

Does the compositionality of extensional meanings have any bearing on whether concepts are definitions, prototypes, exemplar sets or theories? Strictly speaking, no claims can be made. Concerned with meaning itself, the present analysis avoids the question of what concepts are. As they are vehicles of meaning, however, and as composed extensional content is one form of meaning that must be carried, it is not unreasonable to ask which of the four concept theories is most consistent with the present analysis.

It will be seen immediately that the definition theory is fully accommodated. A classical definition specifies a combination of features (which are just concepts), and as such can be expressed in a superpositional way. The standard definition for the concept of a bachelor, for example, specifies the combination MALE + UNMARRIED. The effect can also be achieved using the construction MALE  $\neg$  UNMARRIED. That the exemplar theory is accommodated may also be apparent. An exemplar set is simply an enumeration of an extensional meaning. The main claim of the exemplar theory is thus wholly consonant with the present approach.

Consistency with the definition and exemplar theory does not imply inconsistency with the prototype theory, however. A superpositional construction is an arbitrarily deep structure that may embody multiple levels of representation. In this context, determining the degree of correspondence between two concepts involves the matching of tree structures. This is a process in which correspondence might be established only to a limited depth or degree. The graded classifications that prototype theory accounts for are predicted, therefore. Neither is there any inconsistency with the ‘theory theory’. On the assumption that matching conceptual structures is an implicitly inferential process, as in Gentner’s model of analogical structure mapping (Gentner, 1983, 1987), emergent features are potentially obtained.

Insofar as it is possible to predict what concepts are by working backwards from the present analysis, the conclusion has to be that they potentially take all four of the posited forms. They have the potential to express definitions, prototypes, exemplar sets and theories. This is in conformity with what at least one theorist has proposed (Machery, 2009). Another way to state the result is to say that the four posited forms do not differ in any fundamental way.

What does this neutrality imply for the claim that prototypes cannot compose? This has generally been upheld even in the face of considerable uncertainty as to how composition is achieved (Carey, 2009; Prinz, 2012; Gleitman et al., 2012; Rey, 2018). The claim that prototypes lack the ability to compose stems largely from the work of Fodor and colleagues (e.g. Fodor and Lepore, 1996). As an operational model of concept composition, Fodor favours something along the lines of the classical definition theory, due to the difficulty of imagining ‘... a solution to the productivity problem that doesn’t share form

with the form of the classical [theory]’ (Fodor, 1981, p. 296). It will be seen, however, that the definitional process Fodor refers to is not productive in the way a compositional process is required to be. A classical definition cites a combination of features, and these are themselves just concepts. Such constructs cannot be built up hierarchically. At the first level of construction, one obtains a combination of concepts. At the second, a combination of combinations of concepts. Unfortunately, a combination of combinations is itself a combination. The process cannot get beyond the finite set of possible combinations, therefore. It is not genuinely recursive and, in result, not infinitely productive.<sup>19</sup> While it is true that classical definitions cannot compose, and that prototypes represented as classical definitions have the same limitation, this does not apply to prototypes in general, or to any vehicle of composed extensional content.

### 4.2.3 Sam’s theme

Also relevant to the present proposal is the issue of semantic autonomy. It is a basic intuition that we are able to use language to express our thoughts. We have the capacity, it seems, to compose ideas in thought, and then to express them by means of language. In mainstream (e.g., Montegovian) semantic theory, however, semantic composition is entirely entrained by syntactic construction. Under the rule-to-rule regime, syntactic and semantic composition are said to operate ‘in tandem’. The arrangement is, in fact, one in which conceptual composition is wholly led by syntactic processing (Winter, 2016). The meaning of an utterance is assumed to be determined as a consequence of its syntactic construction. Under this arrangement, speakers would appear to lack the means of expressing structurally complex ideas, since the structure of any expression must derive from the application of syntactic rules.

The superpositional model corroborates the claim that complex ideas are potentially composed purely in thought. The supposition that the syntactic system is *required* to assist is seen to be untenable. This need not mean that conceptual composition is independent, however. Given the fluidity with which we combine the two processes, they must be, at least, closely coordinated. It is conceivable that they are partially isomorphic. In extensional superposition, an accommodating concept is combined with one or more accommodated concepts. Likewise, in syntactic composition, a ‘head’ element is combined with one or more ‘dependent’ elements (Greenberg, 1963; Croft, 2003; Miller, 2011). This makes it tempting to relate accommodating concepts to syntactic heads, and accommodated concepts to syntactic dependents. Something of this kind seems to be what Frege has in mind in saying that ‘corresponding to the whole-part relation of a thought and its parts we have, by and large, the same relation for the sentence and its parts.’ (Frege, 1919/1979, p. 255) Pietroski also seems to envisage something similar in his comment that ‘[o]ne can hypothesize that all cases of combining expressions are instructions to saturate one concept with another’ (Pietroski, 2010, p. 253).

---

<sup>19</sup>It may be this that lies behind Fodor and Lepore’s concession that ‘nobody knows what makes concepts compositional’ (Fodor and Lepore, 1996, p. 270).

But if structural commonalities of this kind are the means of coordinating the two systems, it might as well be said that there is really just one system. Proposals to this effect have often been made. There is a well-established tradition in which it is argued that ‘deep down, syntax is really or ideally should be semantics’ (Hinzen, 2012a, p. 355). This thesis, dubbed *Sam’s Theme* by Fodor and Lepore (2002), has far-reaching roots; and as the comment from Frege (1919/1979) makes clear, these stretch far into the past.

There is also modern work of relevance. Baker (1997) proposes a scheme in which verb-related syntax is governed by conceptual structure of a type related to the lexical semantics of Jackendoff (2002). Löbner argues that the ‘syntactic structure of complex expressions is mirrored by the way in which their meanings are calculated’ (Löbner, 2012, p. 225). Thornton (2016) argues that conceptual composition produces a skeletal core to which syntactic postprocessing then adds. In a series of publications, Hinzen has argued for the view that syntactic and conceptual structure are essentially *identical* (Hinzen, 2006, 2008, 2009, 2011, 2012b, 2012a).<sup>20</sup> See also (Thornton, 2019, 2009) and the proposal of Hale and Keyser (1993).

The difficulty faced by proposals of this kind is to explain why syntax is so diverse in practice. If syntactic and semantic composition boil down to the same thing, why should the former show such variation? Among the more than 5000 languages that remain in use, there are few syntactic features that all languages have in common (Evans and Levinson, 2009). Syntactic systems are also seen to change in complex ways over time (Croft, 2000; Deutscher, 2006). Such variability and divergence would seem hard to explain if syntax is really semantics. On the other hand, is there any other way to reconcile the evidence? The autonomy of conceptual composition cannot be acknowledged without arriving, eventually, at the conclusion that this must shape linguistic composition in some way. If this is the case, it seems that some version of Sam’s theme must be correct.

With respect to extensionalism, generative semantics, concept theory and Sam’s theme, the implications of the present analysis vary considerably, then. In regard to extensionalism, there is firm opposition. In regard to generative semantics, there is a degree of kinship. In regard to concept theory, there is broad accommodation, and in regard to Sam’s theme there is renewed support. While the compositionality of extensional meaning has not been previously demonstrated, the result can be placed in relation to several fields in this way.

## 5 General Summary

Theorists of language seek to understand how new meanings are produced by linguistic composition. Theorists of cognition seek to understand how the effect

---

<sup>20</sup>Hinzen advocates a new direction in cognitive science, ‘where we use syntax as a long-missing *cognitive theory*, a theory that tells us what our concepts are like, and why: for universal syntactic constraints will apply to their formation’ (Hinzen, 2012a, p. 353, original emphasis).

is achieved by conceptual composition. It seems implausible to suppose these are independent processes, operating separately in two mental systems. Under such an arrangement, it is not clear how a conceptually constructed idea could ever shape an expression. Ways in which the two systems may be combined are of interest.

The present article suggests the inherent compositionality of extensional meanings may be significant. Theorists of both language and cognition assume extensional meanings play a foundational role in compositional construction. What has been shown is that these can be superposed, and are thus themselves inherently compositional. It becomes possible, on this basis, to cast extensional superposition as a compositional mechanism that is held in common, and exploited jointly in both language and thought.

On this basis, the work presented can be seen to suggest a new, linking hypothesis. This would assert the unity of composition in thought and language, and identify extensional superposition as the underlying mechanism. Such a hypothesis goes beyond the objective of the present article, however, which is merely to identify relationships between linguistic and conceptual composition. The attempt to evaluate it is, accordingly, left to future research.

## 6 Acknowledgement

The author would like thank multiple reviewers for helpful criticisms, comments and suggestions.

## References

- Arbib, M. A. (2012). Compositionality and Beyond: Embodied meaning in language and protolanguage. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 475-492), Oxford: Oxford University Press.
- Bach, E. (1976). An extension of classical transformational grammar. *Problems in Linguistic Metatheory (Proceedings of the 1976 conference)* (pp. 183-224), East Lansing, Michigan: Michigan State University.
- Baker, M. (1997). Thematic roles and grammatical categories. In Haegeman (Ed.), *Elements of Grammar* (pp. 73-137), Dordrecht: Kluwer.
- Bierwisch, M. (2007). Semantic form as interface. In Späth (Ed.), *Interfaces and interface conditions* (pp. 1-32), Berlin: de Gruyter.
- Blackburn, S. (2008). *The Oxford Dictionary of Philosophy (2 rev. ed.)*, Oxford: Oxford University Press.
- Brachman, R. J. (1983). What IS-A is and isn't: An Analysis of Taxonomic Links in Semantic Networks. *IEEE Computer*, 16, No. 10 (pp. 30-36).

- Carey, S. (2009). *The Origin of Concepts*, Oxford: Oxford University Press.
- Carnap, R. (1947). *Meaning and Necessity: a study in semantics and modal logic*, Chicago, IL: University of Chicago Press.
- Chomsky, N. (1957). *Syntactic Structures*, The Hague: Mouton.
- Cohen, L. J. (1986). How is conceptual innovation possible? *Erkenntnis*, 25 (pp. 221-238).
- Connolly, A. C., Fodor, J. A., Gleitman, L. R. and Gleitman, H. (2007). Why stereotypes don't even make good defaults. *Cognition*, 103 (pp. 1-22).
- Costello, F. J. and Keane, M. T. (2001). Testing two theories of conceptual combination: Alignment versus diagnosticity in the comprehension and production of combined concepts. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 27, No. 1 (pp. 255-271).
- Croft, W. (2000). *Explaining Language Change*, Harlow, UK: Longman.
- Croft, W. (2003). *Typology and Universals (2nd edn)*, Cambridge: Cambridge University Press.
- Davidson, D. (1967). Truth and Meaning. *Synthese*, 17 (pp. 304-323).
- Deutscher, G. (2006). *The Unfolding of Language: The Evolution of Mankind's Greatest Discovery*, London: Arrow Books.
- Estes, Z. and Glucksberg, S. (2000). Interactive Property Attribution in Concept Combination. *Memory and Cognition*, 28 (pp. 28-34).
- Evans, N. and Levinson, S. C. (2009). The myth of language universals: Language diversity and its importance for cognitive science. *Behavioral and Brain Sciences*, 32 (pp. 429-492).
- Fodor, J. and Lepore, E. (1996). The Red Herring and the Pet Fish: Why Concepts still can't be Prototypes. *Cognition*, 58 (pp. 253-270).
- Fodor, J. A. and Lepore, E. (2002). *The Compositionality Papers*, Oxford: Clarendon Press.
- Fodor, J. A. (1981). The present status of the innateness controversy. In Fodor (Ed.), *Representations: Philosophical Essays on the Foundations of Cognitive Science* (pp. 257-316), MIT Press.
- Fodor, J. A. (1998). *Concepts: Where Cognitive Science Went Wrong*, Oxford: Oxford University Press.
- Fodor, J. (2003). *Hume Variations*, Oxford: Oxford University Press.
- Fodor, J. A. (2008). *LOT 2: The Language of Thought Revisited*, Oxford: Oxford University Press.

- Frege, G. (1879). *Begriffsschrift: eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*, Halle: Nebert.
- Frege, G. (1884/1953). *The Foundations of Arithmetic. A logico-mathematical enquiry into the concept of number* (trans. J L Austin), Oxford: Basic Blackwell.
- Frege, G. (1892). Uber Sinn und Bedeutung (On Sense and Reference). *Zeitschrift für Philosophie und philosophische Kritik*, 100 (pp. 25-50).
- Frege, G. (1919/1979). Notes for Ludwig Darmstaedter. In Hermes (Ed.), *Posthumous Writings* (pp. 253-57), Chicago: University of Chicago Press.
- Gagné, C. L. and Shoben, E. (1997). Influence of Thematic Relations on the Comprehension of Modifier-noun Combinations. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 23 (pp. 71-87).
- Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science*, 7 (pp. 155-170).
- Gentner, D. (1987). Analogical Inference and Analogical Access. In Prieditis (Ed.), *Analogica: Proceedings of the First Workshop in Analogical Reasoning*, London: Pitman.
- Gerstl, P. and Pribbenow, S. (1995). Midwinters, end games, and body parts: a classification of part-whole relations. *International Journal of Human-Computer Studies*, 43 (pp. 865-889).
- Gilberman, D. (2016). Moving parts: a new indexical treatment of context-shifting predication. *Synthese*, 193, No. 1 (pp. 95-124).
- Gleitman, L. R., Connolly, A. C. and Armstrong, S. L. (2012). Can prototype representations support composition and decomposition? In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 418-436), Oxford: Oxford University Press.
- Greenberg, J. H. (1963). Some Universals of Grammar with particular reference to the Order of Meaningful Elements. In Greenberg (Ed.), *Universals of Language: Report of a Conference held at Dobbs Ferry, New York, April 13-15 1961* (pp. 73-113), Cambridge, Massachusetts: The MIT Press.
- Griffiths, T., Chater, N., Kemp, C., Perfors, A. and Tenenbaum, J. B. (2010). Probabilistic Models of Cognition: Exploring Representations and Inductive Biases. *Trends in Cognitive Sciences*, 14, No. 8 (pp. 357-364).
- Groenendijk, J. and Stokhof, M. (1991). Dynamic predicate logic. *Linguistics and Philosophy*, 14 (pp. 391-400).
- Hájek, A. (2002). Interpretations of probability. In Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*.

- Hale, K. and Keyser, S. J. (1993). On argument structure and the lexical expression of syntactic relations. In Hale and Keyser (Eds.), *The View from Building 20: Essays in Linguistics in Honor of Sylvain Bromberger* (pp. 53-109), Cambridge, MA: MIT Press.
- Hamblin, C. L. (1973). Questions in Montague English. *Foundations of Language*, 10, No. 1 (pp. 4153).
- Hampton, J. A. and Jönsson, M. L. (2012). Typicality and compositionality: the logic of combining vague concepts. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 385-402), Oxford: Oxford University Press.
- Hampton, J. A. (1991). The combination of prototype concepts. In Schwanenflugel (Ed.), *The Psychology of Word Meanings* (pp. 91-116), Hillsdale, N.J: Lawrence Erlbaum Associates.
- Hampton, J. A. (1995). Testing the Prototype Theory of Concepts. *Journal of Memory and Language*, 34 (pp. 686-708).
- Hampton, J. (1997). Conceptual combination: Conjunction and negation of natural concepts. *Memory and Cognition*, 25 (pp. 888-909).
- Hampton, J. A. (2011). Conceptual Combinations and Fuzzy Logic. In R.Belohlavek and G.J.Klir (Eds.), *Concepts and Fuzzy Logic* (pp. 209-231), Cambridge: MIT Press.
- Hansen, N. (Forthcoming). Just What Is It That Makes Travis's Examples So Different, So Appealing? In Collins, Davies and Dobler (Eds.), *Themes from Charles Travis: On Language, Thought and Perception*, Oxford: Oxford University Press.
- Hinzen, W., Werning, M. and Machery, E. (2012). Introduction. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 1-18), Oxford: Oxford University Press.
- Hinzen, W. (2006)., Oxford: Oxford University Press.
- Hinzen, W. (2008). Prospects for an explanatory theory of semantics. *Biolinguistics*, 2, No. 4 (pp. 348-363).
- Hinzen, W. (2009). Hierarchy, merge, and truth. In Piattelli-Palmarini, Uriagereka and Salaburu (Eds.), *Of Minds and Language: A dialogue with Noam Chomsky in the Basque Country* (pp. 124-141), Oxford: Oxford University Press.
- Hinzen, W. (2011). Emergence of a systematic semantics through minimal and underspecified codes. In Boeckx and Sciallo (Eds.), *The Biolinguistic Enterprise* (pp. 417-439), Oxford: Oxford University Press.

- Hinzen, W. (2012a). Syntax in the atom. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 351-370), Oxford: Oxford University Press.
- Hinzen, W. (2012b). The philosophical significance of Universal Grammar. *Language Sciences*, 34 (pp. 635-649).
- Hodges, W. (2012). Formalizing the relationship between meaning and syntax. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 245-261), Oxford: Oxford University Press.
- Iten, C. B. (2000). *'Non-truth-conditional' Meaning, Relevance and Concessives (PhD thesis)*, London: Department of Phonetics and Linguistics, University College London.
- Jackendoff, R. (1990). *Semantic Structures*, Cambridge, MA: MIT Press.
- Jackendoff, R. (2002). *Foundations of Language: Brain, Meaning, Grammar, Evolution*, Oxford: Oxford University Press.
- Jackendoff, R. (2003). Précis to Foundations of Language: Brain, Meaning, Grammar, Evolution. *Behavioral and Brain Sciences*, 26, No. 6 (pp. 651-65).
- Jackendoff, R. (2007). A Parallel Architecture perspective on language processing. *Brain Research*, 1146 (pp. 2-22).
- Janssen, T. M. V. (1997). Compositionality. In van Benthem and Meulen (Eds.), *Handbook of Logic and Language* (pp. 417-473), Oxford: Elsevier B.V.
- Janssen, T. M. V. (2012). Compositionality: Its Historic Context. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 19-46), Oxford: Oxford University Press.
- Katz, J. (1966). *The Philosophy of Language*, London: Harper and Row.
- Kracht, M. (2012). Compositionality in Montague Grammar. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 47-63), Oxford: Oxford University Press.
- Kroeger, P. (2004). *Analyzing syntax: A lexical-functional approach*, Cambridge, UK: Cambridge University Press.
- Löbner, S. (2012). Sub-compositionality. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 220-244), Oxford: Oxford University Press.
- Lakoff, G. (1971). On generative semantics. In Steinberg and Jakobovits (Eds.), *Semantics*, Cambridge Cambridge: Cambridge University Press.



- Lang, E. and Maienborn, C. (2011). In Maienborn, von Stechow and Portner (Eds.), *Semantics (HSK 33.1)*, (pp. 709-740), Berlin: de Gruyter.
- Lewis, D. (1970). General Semantics. *Synthese*, 22, No. 12 (pp. 18-67).
- Li, P. and Gleitman, L. R. (2002). Turning the Tables: Language and Spatial Reasoning. *Cognition*, 83, No. 3 (pp. 265-294).
- Lupyan, G. and Bergen, B. (2015). How Language Programs the Mind. *Topics in Cognitive Science*, 8, No. 2 (pp. 408-24), New Jersey:.
- Lupyan, G. and Lewis, M. (2019). From words-as-mappings to words-as-cues: the role of language in semantic knowledge. *Language, Cognition and Neuroscience*, 34, No. 10 (pp. 1319-1337).
- Lupyan, G. (2016). *Language Learning*, 66, No. 3 (pp. 516-553).
- Machery, E. (2009). *Doing without Concepts*, Oxford: Oxford University Press.
- Miller, J. (2011). *A critical introduction to syntax*, London: Continuum.
- Montague, R. (1968). Pragmatics. In Klibansky (Ed.), *Contemporary Philosophy. A Survey* (pp. 102-122), Florence: La Nuova Italia Editrice.
- Montague, R. (1970). Universal grammar. *Theoria*, 36 (pp. 373-298).
- Montague, R. (1973). The proper treatment of quantification in ordinary English. In Hintikka, Moravcsik and Suppes (Eds.), *Approaches to Natural Language* (pp. 221-242), Dordrecht: Reidel.
- Murphy, G. L. and Medin, D. L. (1985). The role of theories in conceptual coherence. *Psychological Review*, 92 (pp. 289-316).
- Murphy, G. L. (1988). Comprehending Complex Concepts. *Cognitive Science*, 12 (pp. 529-562).
- Murphy, G. L. (2002). *The Big Book of Concepts*, London, England: The MIT Press.
- Napoletano, T. (2019). How important are truth-conditions for truth-conditional semantics? *Linguistics and Philosophy*, 42 (pp. 541-575).
- Osherson, D. N. and Smith, E. E. (1981). On the Adequacy of Prototype Theory as a Theory of Concepts. *Cognition*, 9 (pp. 35-58).
- Pagin, P. and Westerstål, D. (2010). Compositionality II: Arguments and Problems. *Philosophy Compass*, 5, No. 3 (pp. 265-282).
- Partee, B. H. (1984). Compositionality. In Landman and veltman (Eds.), *Varieties of Formal Semantics (no. 3 in GRASS)* (pp. 281-311), Dordrecht: Foris.

- Pelletier, F. J. (2012). Holism and Compositionality. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 149-174), Oxford: Oxford University Press.
- Pietroski, P. M. (2010). Concepts, Meanings and Truth: First Nature, Second Nature and Hard Work. *Mind & Language*, 25, No. 3 (pp. 247-278).
- Pietroski, P. M. (2012). Semantic Monadicity with Conceptual Polyadicity. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 129-148), Oxford: Oxford University Press.
- Prinz, J. (2002). *Furnishing the Mind: Concepts and their Perceptual Basis*, Cambridge, MA: MIT Press.
- Prinz, J. (2012). Regaining composure: a defence of prototype compositionality. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 437-453), Oxford: Oxford University Press.
- Recanati, F. (2001). What is said. *Synthese*, 128, No. 1/2 (pp. 75-91).
- Recanati, F. (2012). Compositionality, flexibility, and context dependence. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 175-190), Oxford: Oxford University Press.
- Rey, G. (2018). Remembering Jerry Fodor and his work. *Mind & Language*, 33 (pp. 321-341).
- Rips, L. J. (1995). The current status of research on concept combination. *Mind and Language*, 10 (pp. 72-104).
- Rosch, E. and Mervis, C. B. (1975). Family Resemblances: Studies in the Internal Structure of Concepts. *Cognitive Psychology*, 7 (pp. 573-605).
- Rothschild, D. and Segal, G. (2009). Indexical Predicates. *Mind and Language*, 24 (pp. 467-93).
- Searle, J. (1980). The background of meaning. In Searle, Kiefer and Mierwisch (Eds.), *Speech Act Theory and Pragmatics* (pp. 221-32), Dordrecht: Reidel.
- Szabó, Z. G. (2000). *Problems of Compositionality*, London: Routledge.
- Szabó, Z. G. (2012). The Case for Compositionality. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 64-80), Oxford: Oxford University Press.
- Thagard, P. (1997). Coherent and creative conceptual combination. In Ward, Smith and Viad (Eds.), *Creative Thought: an Investigation of Conceptual Structures and Processes* (pp. 129-141), Washington, DC: American Psychological Association.

- Thornton, C. J. (2009). Representation Recovers Information. *Cognitive Science*, 33, No. 8 (pp. 1-30).
- Thornton, C. (2016). Three ways to Link Merge with Hierarchical Concept-Combination. *Biolinguistics*, 10 (pp. 78-106).
- Thornton, C. (2019). Does Conceptual Compositionality Affect Language Complexity? Comment on Lou-Magnuson and Onnis. *Cognitive Science*, 43, No. 8 (pp. 1-3).
- Travis, C. (1994). On Constraints of Generality. *Proceedings of the Aristotelian Society, New Series*, 94 (pp. 165-188), Oxford University Press on behalf of The Aristotelian Society.
- Travis, C. (2008). *Occasion-Sensitivity: Selected Essays (Introduction)* (pp. 1-16), Oxford: Oxford University Press.
- Tversky, B. (1989). *Developmental Psychology*, 25, No. 6 (pp. 983-995).
- Tversky, B. (2005). On Exploring Parts and Wholes. In Gero and Maher (Eds.), *Computational and Cognitive Models of Creative Design VI* (pp. 3-16), Key Centre of Design Computing and Cognition, University of Sydney, Australia.
- Winston, M. E. and Herrmann, D. (1987). A taxonomy of part-whole relations. In Chaffin (Ed.), *Cognitive Science*, 11, No. 4 (pp. 417-444).
- Winter, Y. (2016). *Elements of Formal Semantics: An Introduction to the Mathematical Theory of Meaning in Natural Language*, Edinburgh: Edinburgh University Press.
- Wisniewski, E. J. and Wu, J. (2012). Emergency!!! Challenges to a compositional understanding of noun-noun combinations. In Werning, Hinzen and Machery (Eds.), *The Oxford Handbook of Compositionality* (pp. 403-417), Oxford: Oxford University Press.
- Wisniewski, E. J. (1997). When Concepts Combine. *Psychonomic Bulletin and Review*, 4 (pp. 167-183).
- Wittgenstein, L. (1974/1921). *Tractatus Logico-Philosophicus (first pub. 1921)*, London: Routledge & Kegan Paul.
- Zadeh, L. (1965). Fuzzy sets. *Information and Control*, 8 (pp. 338-353).
- Zadeh, L. (1982). A note on prototype theory and fuzzy sets. *Cognition*, 12, No. 3 (pp. 291-298).
- Zettersten, M. and Lupyan, G. (2020). Finding categories through words: More nameable features improve category learning. *Cognition*, 196, No. 104135.