Machine Learning - Lecture 12
Perceptrons

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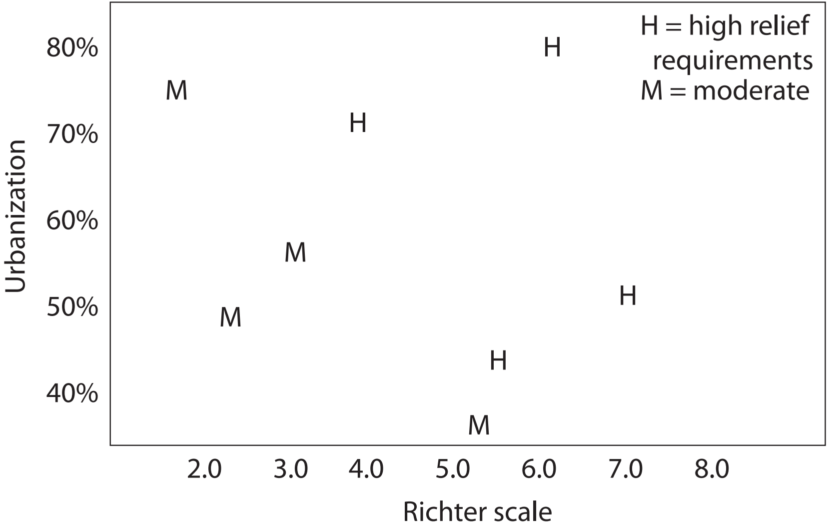
Sample problem

```
  X         Y          CLASS
(0-100)  (0.0-9)

  44        5.5     ->   H
  49        2.4     ->   M
  51        7.0     ->   H
  75        0.9     ->   M
  71        3.8     ->   H
  56        3.1     ->   M
  80        6.1     ->   H
  36        5.3     ->   M
```

Datapoint plot



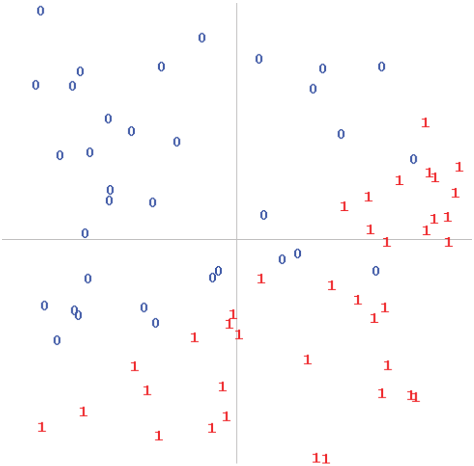
Financial prediction example

In this domain, data are financial quantities, e.g., daily prices of commodities.

The aim is to predict future prices.

The goal is (usually) to maximize trading profits.

FTSE-100 movement classifications



X is % increase in price of gold; Y is % increase in FTSE-100

Val=1: market rise sustained on the following day; Val=0: rise not sustained

Linear separation of classes

Linear separation is the third of the simpler forms of patterning.

Normally only seen with numeric data, i.e., continuous variables.

From statistics, we have a simple and robust method for modeling and predicting patterning of this form.

A little maths involved but the process can be visualised as geometry.

Inner products

An easy way to define a linear boundary involves using **inner products**.

Assuming datapoints are fully numeric, we can calculate the inner product of any two by multiplying together their corresponding values (and adding up the results).

So if x and y are two datapoints, their inner product is calculated as

$$\sum_i x_i y_i$$

Boundaries from thresholds

If we look at how datapoints compare with some fixed reference point, we find a nice relationship between inner products and *lines*.

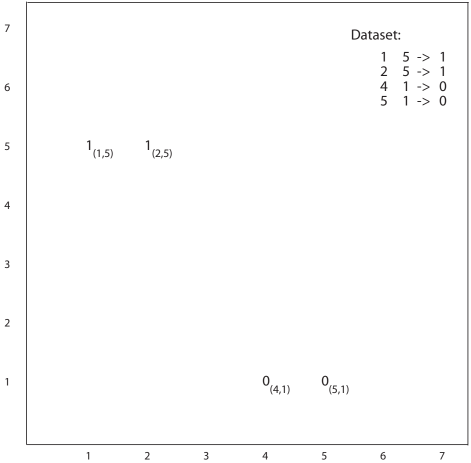
All datapoints for which the inner product with the fixed reference point exceeds some given threshold turn out to be one side of a line.

All other datapoints are on the other side.

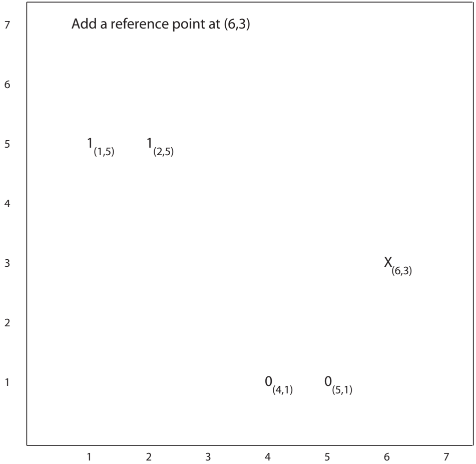
This gives us an easy way of representing linear boundaries.

We can define them in terms of a fixed reference point and an inner-product threshold.

Example



Reference point



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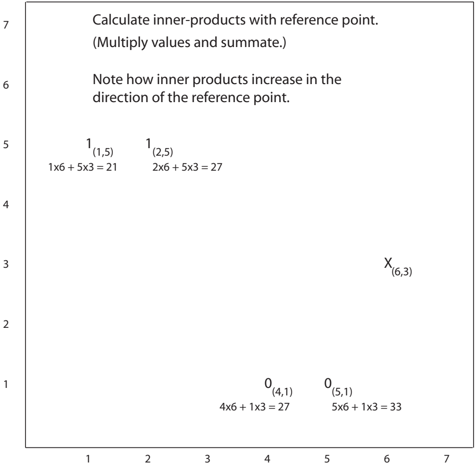
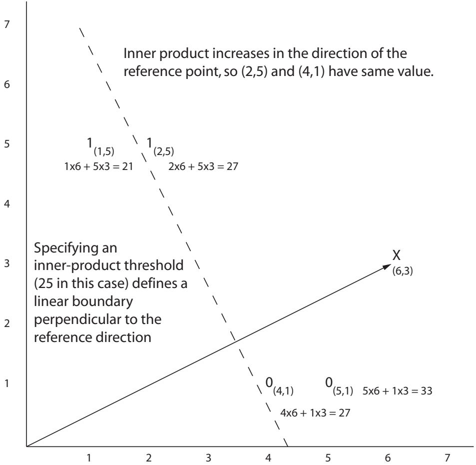


An inner-product threshold defines a line



Finding the boundary by error correction

The position of the linear boundary is a function of the reference point.

Moving the reference point closer to the origin moves the line in the same direction.

Also vice versa.

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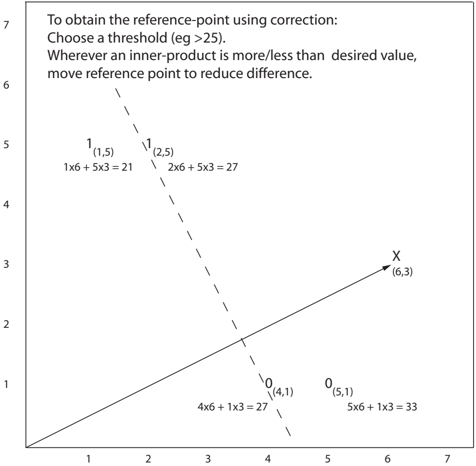
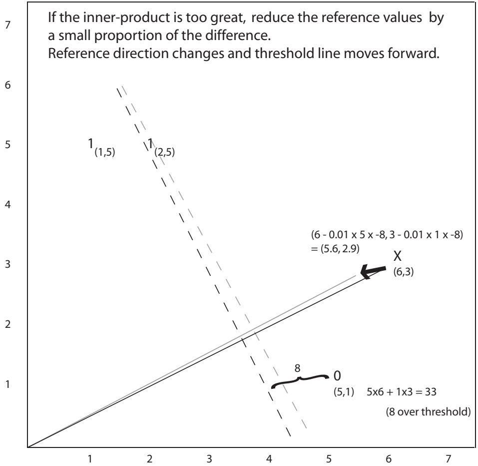
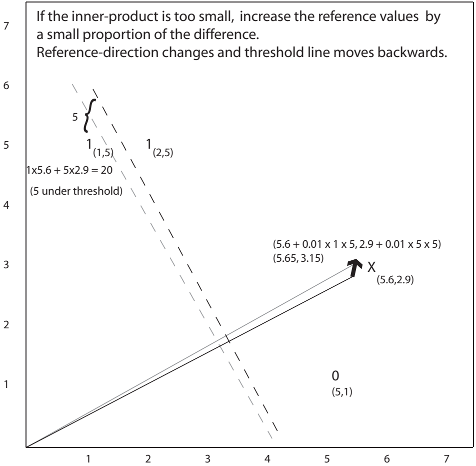
Using error correction



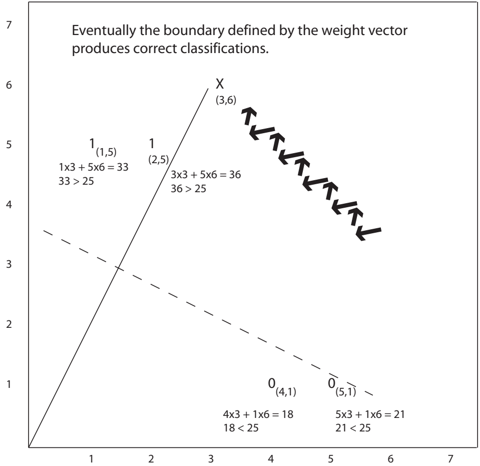
Inner product too high



Inner product too low



What happens in the end



Using an explicit error value

Instead of working in terms of overshooting and undershooting, it is easier to use an **error** measure.

The coordinates of the reference point are termed **weights**.

The reference point is called the **weight vector**.

The error for a datapoint is defined in terms of a **target value** for that datapoint (e.g., 1 or 0).

$$e_i = t_i - p_i$$

Here t_i is the target value for the i'th datapoint, and p_i is the inner product for that datapoint.

Using this definition we can get correction simply by *adding* a proportion of the error.

This takes care of both over and undershoots.

Delta rule

Assuming the error e_i for datapoint i defined as above, the new value for the i'th weight is

$$w_{i,t+1} = w_{i,t} + e_i v_i r$$

where v_i is the i'th value from the datapoint and $w_{t,i}$ is the current value of the i'th weight.

Here, we also have a scaling parameter r, known as the **learning rate**.

This rule for finding a linear boundary is called the **delta rule**.

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Demo

Demo using stockMarket data

The neural network connection

Error-correction is interesting partly due to the connection it makes between machine learning and neural networks.

Reference weights can be viewed as modeling the synaptic weights of neural cells in brains.

The algorithm becomes a way of simulating learning in neural networks.

In fact, this was one of the main ideas lying behind innovation of the method.

Perceptron Convergence Theorem

In the 1950s, Frank Rosenblatt demonstrated that a version of the error-correction algorithm is guaranteed to succeed if a satisfactory set of weights exist.

> *If there is a set of weights that correctly classify the (linearly seperable) training datapoints, then the learning algorithm will find one such weight set in a finite number of iterations*

The main proof was developed in

Rosenblatt, F. (1958). Two theorems of statistical separability in the perceptron. Mechanisation of Thought Processes: Proceedings of a Symposium held at the National Physical Laboratory, 1. London: HM Stationary Office.

Mark 1 Perceptron

Rosenblatt built a machine called the Mark 1 Perceptron, which was essentially an assembly of weight-vector representations for linear discriminations.

Noting the machine's ability to learn classification behaviours (through error-correction), Rosenblatt went on to make ambitious claims for the machine's 'true originality'.

Minsky and Papert

Some while later, Rosenblatt's claims were strongly questioned by Minsky and Papert, in their book 'Perceptrons'.

Machines based on linear-discriminant representations were noted to be incapable of learning boolean functions such as XOR.

```
1 1 -> 0
0 1 -> 1
1 0 -> 1
0 0 -> 0
```

This led to the so-called 'winter of connectionism'.

Minsky, M. L. and Papert, S. A. (1988). Perceptrons: An Introduction to Computational Geometry (expanded edn). Cambridge, Mass: MIT Press.

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