KR-IST: Lecture 7b
First-order Logic

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From reasoning to knowledge representation

With any set of implication rules (e.g., the celebrity rulebase), search-like processes can be used to determine implied facts and conclusions.

The combination of rulebase and inference method can be viewed as a *representation of knowledge* for the domain, i.e., a **knowledge base** (**KB**).

A system which packages up knowledge represented this way is a **knowledge-based** or **expert system**.

From knowledge representation to logic

Rule-based methods of knowledge representation are also known as **logics**.

With a history stretching back 2000 years, the study of logic and formal reasoning was a kind of pre-computation AI.

AI methods of knowledge representation (**KR**) are generally based on adapted systems of formal logic.

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Truth tables

The meaning of logical relationships is defined using **truth tables**.

A truth table shows how truth values combine under the relevant relationship.

\`AND'

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

\`OR'

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
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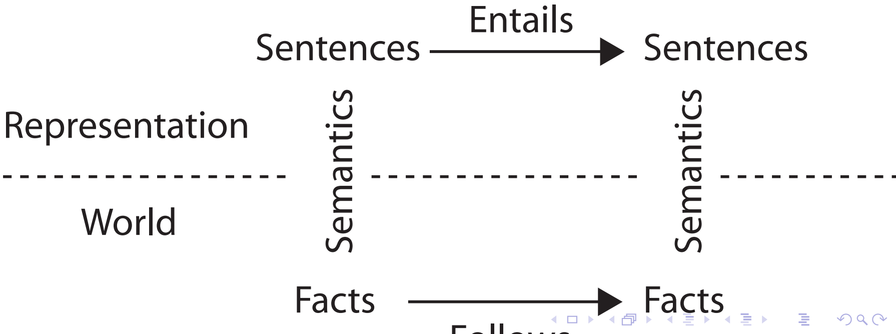
Semantics

A semantics maps sentences to facts in the world; e.g., the mapping determines which objects in the world are referenced by which objects in the language. This is called a **referential semantics**.

The way one fact follows another should be mirrored by the way one sentence is *entailed* by another.



Problems with propositional logic

In Propositional logic, we have no way to represent properties of objects.

We cannot represent property-based generalisations.

For example, it is impossible to represent this **categorical syllogism** in Propositional logic:

> Every person is mortal
> Tony Blair is a person
> Therefore Tony Blair is mortal

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- ‘There are exactly two purple mushrooms.’ $\exists$ x $\exists$ y: mushroom(x) $\wedge$ purple(x) $\wedge$ mushroom(y) $\wedge$ purple(y) $\wedge$ $\neg$ (x=y) $\wedge$ $\forall$ z: (mushroom(z) $\wedge$ purple(z)) $\Rightarrow$ ((x=z) $\vee$ (y=z))

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- 'Deb is not tall.' $\neg$ tall(Deb)
- 'X is above Y if X is on top of Y or else there is a pile of one or more other objects directly on top of one another starting with X and ending with Y.' $\forall$ x $\forall$ y: above(x,y) $\Leftrightarrow$ (on(x,y) $\vee$ $\exists$ z (on(x,z) $\wedge$ above(z,y)))

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Problems with FOL

FOL is a powerful language for representing knowledge.

But its expressiveness complicates the derivation of inferences. (It gets easier if we exliminate existential quantification and assume 'negation by failure'.)

Also, in FOL you cannot construct sentences which make assertions about other sentences. For example, you cannot say things like 'there exists a property such that...'

For this task, you need a **higher-order logic**.

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The Frame problem

A fundamental difficulty for sentential representation is the **frame problem**.

This affects all varieties of knowledge representation but is particularly apparent where evaluation is in terms of truth, and rules are used to define the results of *actions*.

Frame problem example

Suppose we have

$$\text{paint}(X, C) \Rightarrow \text{color}(X, C)$$
$$\text{move}(X, P) \Rightarrow \text{position}(X, P)$$

and it is known that

paint(tony, blue).
move(tony, garden).

We should then be able to infer that

$$\text{colour}(\text{tony}, \text{blue}) \wedge \text{position}(\text{tony}, \text{garden})$$

But the inference is, in fact, logically *unsound*

There is the possibility that the colour of tony gets changed by the move action.

Addressing the frame problem

The most obvious way to protect against the frame problem is to add rules which capture the *non-effects* of actions.

Such rules are known as **frame axioms**.

For example

$$\text{move(X, P)} \wedge \text{color-before-move(X, C)} \Rightarrow \text{color(X, C)}.$$

asserts the fact that moving an object will not affect its colour.

However, this is not satisfactory.

Since *most* actions do not affect *most* properties of a situation, in a domain comprising m actions and n properties, we are going to need approximately $m \times n$ frame axioms.

The Epistemological Frame Problem

The underlying puzzle is how a cognitive creature with many beliefs about the world can update those beliefs when it performs an act so that they remain roughly faithful to the word.

Imagine being the designer of a robot that has to carry out an everyday task, such as making a cup of tea. Now, suppose the robot has to take a tea-cup from the cupboard. The present location of the cup is represented as a sentence in its database of facts alongside those representing innumerable other features of the ongoing situation, such as the ambient temperature, the configuration of its arms, the current date, the colour of the tea-pot, and so on. Having grasped the cup and withdrawn it from the cupboard, the robot needs to update this database. The location of the cup has clearly changed, so that's one fact that demands revision. But which other sentences require modification?

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- Produce the most accurate propositional representation of this assertion you can think of.
- Produce the best predicate logic representation you can think of.

Exercises cont.

- Consider this categorical syllogism.

  Every knowledge representation is formal
  Propositional logic is a knowledge representation
  Therefore propositional logic is formal

- Produce the most accurate propositional representation of this assertion you can think of.
- Produce the best predicate logic representation you can think of.

Exercises cont.

Exercises cont.

- Represent the first three verses of the song ‘House of the Rising Sun’ in FOL.

Exercises cont.

- Represent the first three verses of the song 'House of the Rising Sun' in FOL.

  There is a house in New Orleans
  They call the Rising Sun
  And it's been the ruin of many a poor boy
  And God I know I'm one

  My mother was a tailor
  She sewed my new bluejeans
  My father was a gamblin' man
  Down in New Orleans

  Now the only thing a gambler needs
  Is a suitcase and trunk
  And the only time he's satisfied

Exercises cont.

Exercises cont.

- Using first-order logic, represent as accurately as possible the information contained in these comments on the availability of mortgages during the credit crunch.

Exercises cont.

- Using first-order logic, represent as accurately as possible the information contained in these comments on the availability of mortgages during the credit crunch.

To be considered for the best mortgage deals during the current
difficult conditions, you must borrow substantially less than the
full purchase price, have a perfect credit record and be able to act fast.

Only people who have built up savings over
several years and have shown their ability to live on less than their salary are able to get a mortgage.

It is first-time buyers who are hardest hit by the need to

Exercises cont.

Exercises cont.

- Use FOL to represent the information contained in the lyric to ‘When a man loves a woman’.

Exercises cont.

- Use FOL to represent the information contained in the lyric to 'When a man loves a woman'.

  When a man loves a woman
  Can't keep his mind on nothing else
  He'll trade the world
  For the good things he's found
  If she's bad, he can't see it
  She can do no wrong
  Turn his back on his best friend
  If he put her down

  When a man loves a woman
  Spend his very last time
  Tryin' to hold on to what he needs
  He'd give up all his comfort