Introduction to Logic

Assessed Exercise 2

February 2005

Read the text carefully and then attempt each of the problems given below. Your solutions to the problems should be submitted by 4pm on Thursday 10th March 2005. Please note that this deadline is absolute and late submissions may be penalized.

Problems

1. Consider carefully the argument shown below.

   > If people are entirely rational, then either all of a person's actions can be predicted or the universe is essentially deterministic. Not all of a person's actions can be predicted in advance. Therefore, if the universe is not essentially deterministic, then people are not entirely rational.

   Now do the following:

   (a) identify the premises and conclusion of the argument;

   (b) formalise the argument by representing the premises and conclusion as statements of propositional logic;

   (c) test the validity of the argument using the method of semantic tableau.

2. Given two statements of propositional logic A and B, explain how it is possible to use the method of semantic tableaux to test whether or not the statements are logically equivalent.

3. The police have been called to the magnificent residence of Lord and Lady Muck to investigate the strange disappearance of Lady Muck's priceless pearl necklace. After interviewing Lord and Lady Muck and their staff, the police are able to establish the following facts:

   - The crime took place in the afternoon and the disappearance of the necklace is a mystery.
   - If Lady Muck had seen the thief, then the disappearance of the necklace would not be a mystery.
   - If Lord Muck did not commit the crime, then either the Butler or the Maid must have done it;
   - Either the Butler did not commit the crime, or it was not committed in the afternoon.
   - If the the Maid took the necklace, then her Ladyship would have seen the thief.

   Carefully consider the facts of the case, and then do the following:

   (a) formalize the facts as statements of propositional logic.

   (b) construct a semantic tableau to show that the set of facts is consistent.

   (c) by inspection of the tableau, try to deduce who stole the necklace.

4. Prove the following statements using the system of Natural Deduction:

   (a) $\{(p \rightarrow r)\} \vdash (p \wedge q) \rightarrow r$

   (b) $\{(p \rightarrow q), (p \rightarrow r)\} \vdash p \rightarrow (q \wedge r)$

   (c) $\{(q \rightarrow r)\} \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$

   (d) $\{(\neg p \rightarrow q), (q \rightarrow \neg r)\} \vdash (r \rightarrow p)$