# Introduction to Logic 1

- What is Logic?
- Why Study Logic?
- Object-language/Meta-language.
- Propositions, Beliefs and Contradictions.
- Formalisation

# What is Logic?

Logic is ...

- the study of the ... principles used to distinguish good (correct) from bad (incorrect) reasoning (Copi);
- the study, by symbolic means, of the exact conditions under which patterns of argument are valid or invalid (Lemmon);
- the study of formal (that is symbolic) systems of reasoning and of methods of attaching meaning to them (Reeves & Clarke).

– Abstract principle or patterns
– Distinguishing valid/invalid patterns
– Formalisation/abstraction

# Patterns of Reasoning

e.g.

*If Logic is fun, then Bill is happy.*

*Logic is fun.*

*(It follows that) Bill is happy.*

*If mungs are flit, then mizzles are gloggy*

*Mungs are flit.*

*(It follows that) mizzles are gloggy.*

**Question:** *Do the conclusions follow in the above?*

**Questions:** *What is noticeable about these two patterns?*

# Abstraction

| | | |
|---|---|---|
| if A then B | $A \supset B$ | Premiss |
| A | A | Premiss |
| ________ | ______ | ________ |
| B | B | Conclusion |

**Question:** *What about the following pattern? Does it fit; is it good?*

*If Logic is fun, then Bill is happy.*

*Programming is fun.*

*(It follows that) Bill is happy.*

# Why Study Logic?

A little history:

**Aristotle (384–322 BC):** first systematic study of "patterns of reasoning"; development of syllogistic reasoning.

**Boole (1815–1864):** develops algebraic system of symbol manipulation now regarded as the basis of propositional logic and computer hardware.

**Frege (1848–1925):** studies the foundations of mathematics with the objective of deriving all mathematics from logical principles; introduces a new notation and language which provides the basis of modern logic (the first order predicate calculus).

But why study logic as computer scientists?

## Logic and Computer Science

- Foundational issues:

  – there are intimate links between computation and logic

- Analytic tool:

  – use of logic as a tool for formalizing/studying properties of programs.

- Hardware Design:

  – conventional computer hardware is is based on electronic devices called *logic gates*.

- Automated reasoning:

  – mechanical generation of proofs: automated theorem proving, artificial intelligence

  – logic as a programming language (e.g. Prolog)

# Some Terminology

We will, in this course, be looking at *mathematical* (formal/symbolic) logic.

In fact, we are using mathematical techniques to study a branch of mathematics called ‘logic’

Question: does this make sense?

- It does, as long as we are careful.
- We use different languages to separate the object of our study (i.e. logic) from the means of our study.
- The former is called the *object language*; the latter is called the *meta-language*.

# Example

- "The expression 'Karl ist krank' is a well-formed sentence of German"

  Here, the object language is German, while the meta-language is English.

- "$x = 0;$ is equivalent to $x = 1; x = x - 1;$ in *Java*"

  In this case, the object language is presumably *Java*, while the meta-language is English again.

- "The sentence 'John likes Mary' is true"

  Note that is this case the object language is the same as the meta-language (i.e. English).

# Propositions

Logic is is concerned with objects called *propositions* and the relationships between them, but what are propositions?

- Language can be used to *express* propositions:

  *Bill teaches Logic*
  *Logic is taught by Bill*
  *I teach Logic*

  – propositions communicate judgements or beliefs about the world.

- Note that not all sentences express propositions:

  *Who teaches Logic?*
  *Teach Logic!!*

  – **declarative sentences** express propositions

Simple test for declarative sentences:

"It is true that SENTENCE "

## Beliefs and Contradictions

- Our beliefs provide our 'world view' or picture

  They allow us to reason about the world, to construct hypotheses and to draw conclusions

- Is there any restriction on the beliefs that we may hold or entertain?

  We cannot knowingly entertain, simultaneously, contradictory beliefs:

  "Bill teaches Logic/Bill does not teach Logic"

A fundamental task of logic is to be able to decide whether or not a set of beliefs (propositions) is contradictory.

# Formalization

We are interested in a *formal* approach to the study of logic. Actually, there are two different senses of 'formalization' here:

1. The process of constructing an object language and the rules needed to manipulate sentences.

2. The provision of a means of manipulating objects according to their *form* rather than their *content*.

   i.e. we can work without understanding exactly what we are doing (there's no need to know what individual propositions mean or what the manipulations achieve)!

# Summary

- Logic is concerned with abstract principles of reasoning and the notion of truth;
- Mathematical logic began at the turn of the last century (Frege);
- Logic is an important area of study for computer scientists;
- Logic deals with propositions, which express beliefs;
- Sets of beliefs may be contradictory (we would like to know when this is so);
- Formalization allows us to 'mechanize' the process of reasoning.