Introduction to Logic 1

- What is Logic?
- Why Study Logic?
- Object-language/Meta-language.
- Propositions, Beliefs and Contradictions.
- Formalisation

What is Logic?

Logic is ...

- the study of the ... principles used to distinguish good (correct) from bad (incorrect) reasoning (Copi);
- the study, by symbolic means, of the exact conditions under which patterns of argument are valid or invalid (Lemmon);
- the study of formal (that is symbolic) systems of reasoning and of methods of attaching meaning to them (Reeves & Clarke).

– Abstract principle or patterns
– Distinguishing valid/invalid patterns
– Formalisation/abstraction

Patterns of Reasoning

e.g.

If Logic is fun, then Bill is happy.

Logic is fun.

(It follows that) Bill is happy.

If mungs are flit, then mizzles are gloggy

Mungs are flit.

(It follows that) mizzles are gloggy.

Question: *Do the conclusions follow in the above?*

Questions: *What is noticeable about these two patterns?*

Abstraction

if A then B	$A \supset B$	Premiss
A	A	Premiss
_____	_____	_____
B	B	Conclusion

Question: *What about the following pattern? Does it fit; is it good?*

> *If Logic is fun, then Bill is happy.*
>
> *Programming is fun.*
>
> *(It follows that) Bill is happy.*

Why Study Logic?

A little history:

Aristotle (384–322 BC): first systematic study of "patterns of reasoning"; development of syllogistic reasoning.

Boole (1815–1864): develops algebraic system of symbol manipulation now regarded as the basis of propositional logic and computer hardware.

Frege (1848–1925): studies the foundations of mathematics with the objective of deriving all mathematics from logical principles; introduces a new notation and language which provides the basis of modern logic (the first order predicate calculus).

But why study logic as computer scientists?

Logic and Computer Science

- Foundational issues:

  – there are intimate links between computation and logic

- Analytic tool:

  – use of logic as a tool for formalizing/studying properties of programs.

- Hardware Design:

  – conventional computer hardware is is based on electronic devices called *logic gates*.

- Automated reasoning:

  – mechanical generation of proofs: automated theorem proving, artificial intelligence

  – logic as a programming language (e.g. Prolog)

Some Terminology

We will, in this course, be looking at *mathematical* (formal/symbolic) logic.

In fact, we are using mathematical techniques to study a branch of mathematics called 'logic'

Question: does this make sense?

- It does, as long as we are careful.
- We use different languages to separate the object of our study (i.e. logic) from the means of our study.
- The former is called the *object language*; the latter is called the *meta-language*.

Example

- "The expression 'Karl ist krank' is a well-formed sentence of German"

  Here, the object language is German, while the meta-language is English.

- "$x = 0;$ is equivalent to $x = 1; x = x - 1;$ in *Java*"

  In this case, the object language is presumably *Java*, while the meta-language is English again.

- "The sentence 'John likes Mary' is true"

  Note that is this case the object language is the same as the meta-language (i.e. English).

Propositions

Logic is is concerned with objects called *propositions* and the relationships between them, but what are propositions?

- Language can be used to *express* propositions:

  Bill teaches Logic
  Logic is taught by Bill
  I teach Logic

  – propositions communicate judgements or beliefs about the world.

- Note that not all sentences express propositions:

  Who teaches Logic?
  Teach Logic!!

  – **declarative sentences** express propositions

Simple test for declarative sentences:

"It is true that SENTENCE "

Beliefs and Contradictions

- Our beliefs provide our 'world view' or picture

  They allow us to reason about the world, to construct hypotheses and to draw conclusions

- Is there any restriction on the beliefs that we may hold or entertain?

  We cannot knowingly entertain, simultaneously, contradictory beliefs:

  "Bill teaches Logic/Bill does not teach Logic"

A fundamental task of logic is to be able to decide whether or not a set of beliefs (propositions) is contradictory.

Formalization

We are interested in a *formal* approach to the study of logic. Actually, there are two different senses of 'formalization' here:

1. The process of constructing an object language and the rules needed to manipulate sentences.

2. The provision of a means of manipulating objects according to their *form* rather than their *content*.

   i.e. we can work without understanding exactly what we are doing (there's no need to know what individual propositions mean or what the manipulations achieve)!

Summary

- Logic is concerned with abstract principles of reasoning and the notion of truth;
- Mathematical logic began at the turn of the last century (Frege);
- Logic is an important area of study for computer scientists;
- Logic deals with propositions, which express beliefs;
- Sets of beliefs may be contradictory (we would like to know when this is so);
- Formalization allows us to 'mechanize' the process of reasoning.