THE UNIVERSITY OF SUSSEX

5059

BA and BSc EXAMINATIONS 1997

INTRODUCTION TO LOGIC

Saturday 21 June, 1997 — 2.00 p.m. – 3.30 p.m.

Introduction to Logic — 3 Questions

Candidates should answer TWO questions. The time allowed is ONE AND A HALF hours.

Q.1 **a** Draw up truth tables for each of the following statements of the Propositional Calculus and indicate their status (inconsistent or consistent):

1. $\neg p \to p$
2. $p \land \neg(q \land p)$
3. $p \land \neg(\neg q \to p)$
4. $(p \land q) \leftrightarrow (p \lor r)$
5. $(p \to (q \lor r)) \land (p \land \neg(\neg q \to r))$

[10 marks]

b Use the method of truth tables to determine which of the following pairs of statements are logical equivalences:

1. $(p \to q)$ and $(\neg p \lor q)$
2. $\neg(p \land q)$ and $(\neg p \lor \neg q)$
3. $p \to (q \to r)$ and $(p \to q) \to r$

[10 marks]

c Formalize the following arguments as statements of the Propositional Calculus and determine whether or not they are sound.

1. Either x is negative or y is positive. If z is negative, then y is positive. Therefore, if x is positive or y is negative, then z is positive.
2. If Tweety is a migrant, then Tweety can fly. If Tweety can fly then Tweety is not an Ostrich. So, if Tweety is an Ostrich, then Tweety is not a migrant.

[15 marks]

d Identify the premises and the conclusion in the following (sound) argument. Can this argument be formalized appropriately as a statement of the Propositional Calculus? (Give reasons for your answer.)

> If Fido is a dog, then Fido is a mammal. All mammals are warm-bloodied. Therefore, if Fido is not warm-bloodied, then Fido is not a dog.

[5 marks]

e Explain why the method of truth tables cannot generally be used to determine whether a sentence of the First Order Predicate Calculus is tautologous.

[10 marks]

Q.2 **a** Use the method of semantic tableaux to determine the status (consistent or inconsistent) of the following sets of sentences:

1. $\{p \wedge q, \neg p, q\}$
2. $\{p \vee q, \neg p, \neg q\}$
3. $\{p \rightarrow r, q \rightarrow r, (p \wedge q) \rightarrow r\}$
4. $\{(p \vee q) \rightarrow q, p \vee (r \rightarrow q), p, q\}$

[10 marks]

b Display the following derivation rules of the system of natural deduction. Using the method of semantic tableaux, demonstrate that they are sound:

1. $\wedge$ – introduction
2. $\rightarrow$ – elimination
3. $\vee$ – elimination

[10 marks]

c Let $\circ$ be a logical connective such that for any pair of statements A and B, the statement $(A \circ B)$ is logically equivalent to $\neg(B \rightarrow A)$. Display the truth table for the connective $\circ$ and devise suitable tableaux derivation rules for sentences of the form $(A \circ B)$ and $\neg(A \circ B)$.

[10 marks]

d What does it mean to say that a system of formal logic is

1. sound
2. complete

[10 marks]

e Sketch an argument to show that the relation of semantic entailment (symbolized as $\models$) satisfies the following rule (where A and B are arbitrary statements):

$$\text{if } G \cup \{A\} \models B \text{ then } G \models (A \rightarrow B)$$

[10 marks]

Q.3 **a** Express the following English sentences as formulas of the First Order Predicate Calculus, providing informal interpretations for any symbols that your introduce.

1. If $x > y$ and $y > z$, then $x > z$
2. Every natural number is less than its successor.
3. There is a prime number that is even.
4. Every natural number is the sum of two natural numbers
5. There is no number that is larger than all other numbers

[10 marks]

b Explain how the method of semantic tableaux for the Propositional Calculus is extended to the case of the First Order Predicate Calculus.

[10 marks]

c Let a and b be constants and f, m, s and t function symbols with the interpretation:

$I(a)$ is Ann	$I(b)$ is Bob		
$I(f)$ is father	$I(m)$ is mother	$I(s)$ is sister	$I(t)$ is brother

Express the following concepts as first order terms:

1. Ann's sister
2. Bob's paternal grandmother
3. The father of Ann's mother's sister.
4. The brother of the mother of Bob's paternal grandfather.

[5 marks]

d Let a, b, c and d be individual constants and M, W, L and H predicate symbols with the interpretation:

$I(a)$ is Ann	$I(b)$ is Bob	$I(c)$ is Cath	$I(d)$ is Dave
$I(M)$ is man	$I(W)$ is woman	$I(L)$ is likes	$I(H)$ is hates

Assuming the domain of all people, provide paraphrases in English for the following formulas:

1. $\forall x.H(x,b) \to \neg L(b,x)$
2. $\forall x.L(x,d) \to (L(x,c) \vee H(x,b))$
3. $\forall x.\exists y.M(x) \to (W(y) \wedge L(x,y))$
4. $\exists x.W(x) \wedge \neg(H(x,d) \vee L(x,d))$
5. $\neg\exists x.\exists y.W(x) \wedge W(y) \wedge H(x,y) \wedge L(y,x)$

[10 marks]

e For each of the following formulas of the First Order Predicate Calculus, specify (a) an interpretation which makes the formula **true**, and (b) an interpretation which makes the formula **false**.

1. $\forall x.\forall y.A(x,y) \vee \neg A(y,x)$
2. $\exists x.\forall y.B(x,y)$
3. $\forall x.\forall y.\forall z.(C(x,y) \wedge C(y,z)) \to C(x,z)$

[15 marks]