THE UNIVERSITY OF SUSSEX 5059

BA and BSc EXAMINATIONS 2001

INTRODUCTION TO LOGIC

June, 2001 –

Introduction to Logic 3 Questions

Candidates should answer TWO questions. The time allowed is ONE AND A HALF hours. Each question is worth 20 marks.

1. (a) For each of the following statements of the Propositional Calculus, use the method of truth tables to determine whether it is **contingent**, **tautologous** or **inconsistent**:

   i. $p \to \neg p$
   ii. $(p \vee q) \to q$
   iii. $(p \vee \neg q) \leftrightarrow (q \to p)$
   iv. $r \to \neg((p \wedge q) \vee r)$

   [4 marks]

   (b) Using the Propositional Calculus, provide a formal representation of the following argument. Using the method of truth tables, determine whether or not the argument is sound.

   Either the solution is acid or it is alkaline. If the solution is acid, then the litmus paper will be red. It follows that if the litmus paper is not red, then the solution is alkaline.

   [4 marks]

   (c) Using the formal laws of logical equivalence, simplify the following statements of the Propositional Calculus as far as possible:

   i. $p \to (p \wedge q)$
   ii. $(p \to q) \wedge (p \to \neg q)$
   iii. $((p \to r) \to ((r \wedge p) \wedge q))$

   [4 marks]

(d) Justify the following proposition about the semantic entailment relation (where G is a set of statements and A and B are statements).

$$\text{if } G \models A \text{ and } G \cup \{A\} \models B, \text{ then } G \models B$$

[4 marks]

(e) What does it mean to say that a system of formal logic is;

i. sound
ii. complete
iii. decidable

[4 marks]

2. (a) Use the method of semantic tableaux to determine whether the following sets of sentences are **consistent** or **inconsistent**:

i. $\{(p \to q), \neg q, (r \wedge \neg p)\}$
ii. $\{(p \leftrightarrow (q \vee r)), (\neg p \vee \neg q)\}$
iii. $\{((p \vee q) \wedge (p \vee \neg q)), \neg p\}$

[4 marks]

(b) Let $\circ$ be a logical operator with the following truth function:

$\circ$	t	f
t	t	t
f	f	t

Devise suitable tableaux derivation rules for sentences of the form $(A \circ B)$ and $\neg(A \circ B)$

[4 marks]

(c) Using the method of natural deduction, show that the following statements are **theorems**

i. $(p \wedge q) \to (q \wedge p)$
ii. $((p \to q) \wedge p) \to q$
iii. $((\neg p \to q) \wedge \neg q) \to p$

[4 marks]

(d) Present an argument to show that the natural deduction proof rule of **reductio ad absurdum** (proof by contradiction) is sound.

[4 marks]

(e) With reference to the First Order Predicate Calculus, explain what is meant by the following terms:

i. a first order theory
ii. an interpretation
iii. a model

3. (a) Is the language of the Propositional Calculus appropriate as a means of representing the following argument? Give reasons for your answer.

   All men are mortal. Socrates is a man. So, Socrates is mortal.

   [4 marks]

   (b) Express the following English sentences as statements of the First Order Predicate Calculus, providing informal interpretations for any symbols that you introduce.

   i. Every integer is either even or odd.
   ii. No integer is both even and odd.
   iii. The sum of two odd integers is an even integer.
   iv. Every integer is smaller than some other integer.
   v. Negative integers are smaller than positive integers.

   [4 marks]

   (c) Explain why the method of truth tables cannot generally be used to determine the status (i.e. **contingent**, **tautologous**, **inconsistent**) of statements of the First Order Predicate Calculus.

   [4 marks]

   (d) Let C_1 and C_2 be two clauses, as shown below. Show that C_1 and C_2 **resolve** and display their **resolvent**.

$$
\begin{aligned}
C_1 &= \{\neg P(a, x), \neg Q(x, a)\} \\
C_2 &= \{Q(b, y), R(y)\}
\end{aligned}
$$

   [4 marks]

   (e) In resolution theorem proving what, is meant by

   i. a literal
   ii. a definite clause
   iii. a goal clause

   [4 marks]