THE UNIVERSITY OF SUSSEX 5059

BA and BSc EXAMINATIONS 2000

INTRODUCTION TO LOGIC

June, 2000 –

Introduction to Logic 3 Questions

Candidates should answer TWO questions. The time allowed is ONE AND A HALF hours. Each question is worth 20 marks.

Q.1 **a** Draw up truth tables for each of the following statements of propositional logic and indicate their status (i.e. inconsistent, contingent or tautologous):

1. $p \rightarrow \neg p$
2. $p \wedge \neg(p \vee q)$
3. $p \wedge (p \rightarrow \neg q)$
4. $\neg(p \wedge q) \leftrightarrow (p \rightarrow \neg q)$
5. $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow r)$

[4 marks]

b Use the method of truth tables to determine which of the following pairs of statements are logical equivalences:

1. $(p \wedge q)$ and $\neg(p \rightarrow \neg q)$
2. $\neg(p \wedge q)$ and $p \rightarrow q$
3. $p \rightarrow (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$

[4 marks]

c Formalise the following argument as a single statement of propositional logic and determine whether or not it is sound:

> *If Spock is a Vulcan, then Spock is logical. If Spock is logical, then Kirk is nervous. So, either Spock is not a Vulcan or else Kirk is nervous.*

[4 marks]

d Let $\downarrow$ be a logical connective such that for any sentences A and B, $(A \downarrow B)$ is equivalent to $\neg(A \wedge B)$. Show that the following statements can be expressed using $\downarrow$ as the sole connective:

1. $\neg A$
2. $(A \rightarrow B)$
3. $(A \vee B)$

[4 marks]

e Construct an example, in English, of a simple argument that cannot be adequately formalized using propositional logic.

[4 marks]

Q.2 **a** Use the method of semantic tableaux to determine whether the following sets of statements of propositional logic are consistent or inconsistent:

1. $\{p, (p \rightarrow q), \neg q\}$
2. $\{(p \vee q), (\neg q \rightarrow r), (p \rightarrow \neg r)\}$
3. $\{(\neg p \vee q), (q \rightarrow r), (p \rightarrow (q \rightarrow r)\}$

[4 marks]

b Use the method of semantic tableaux to determine whether the the following statements of semantic entailment hold:

1. $\{(\neg p \vee q), \neg q\} \models \neg p$
2. $\{(p \vee q) \rightarrow p, q\} \models p$

[4 marks]

c Consider the following putative rule of inference:

> For any statements of propositional logic A, B and C, given $A \rightarrow B$ and $B \rightarrow C$ as premises, obtain $A \rightarrow C$ as a direct consequence.

Show that this rule is sound.

[4 marks]

d Use the method of natural deduction to show that the following statements are theorems:

1. $(p \wedge q) \rightarrow (p \vee q)$
2. $p \rightarrow (q \rightarrow (p \wedge q))$

[4 marks]

e Sketch an argument to show that the relation of semantic entailment (symbolized as $\models$) satisfies the following rule (where G is a set of statements and A and B are arbitrary statements):

$$\text{if } G \models B \text{ then } G \cup \{A\} \models B$$

[4 marks]

Q.3 **a** Provide paraphrases in English for the following sentences of the predicate calculus, assuming that d and m are individual constants "Dumbo" and "Micky", E, F and M are one-place predicate symbols "is an elephant", "is flying" and "is a mouse", and S is a two-place predicate symbol such that $S(x, y)$ means "x is smaller than y".

1. $E(d) \vee \neg E(m)$
2. $\exists x.(E(x) \wedge F(x))$
3. $\forall x.(M(x) \rightarrow S(x, d))$
4. $\forall x.(M(x) \wedge \exists y.(E(y) \rightarrow S(x, y)))$

[4 marks]

b Express the following English sentences in the predicate calculus. (You may wish to make use of the individual constants and predicate symbols introduced in part **a** above.)

1. Dumbo is a flying elephant
2. Micky is smaller than any elephant
3. No elephant is smaller than a mouse.

[4 marks]

c Assuming the domain of natural numbers, for each of the following formulas of the predicate calculus, find a satisfying interpretation.

1. $\forall x.A(x, x)$
2. $\forall x.\exists y.B(x, y)$
3. $\forall x.\forall y.\forall z.(C(x, y) \wedge C(y, z)) \rightarrow C(x, z)$

[4 marks]

d In general, can the method of truth tables be used to determine whether a sentence of the first order predicate calculus is a tautology? (Give reasons for your answer).

[4 marks]

e Explain how the method of semantic tableau for propositional logic is adapted to the predicate calculus.

[4 marks]