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Interactive Law Encoding Diagrams for learning and instruction

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Abstract

Law Encoding Diagrams (LEDs) appear to be effective for learning and instruction, because they make the underlying relations of a domain more readily accessible than do traditional representations. Two systems of interactive computer based LEDs are described. The empirical evaluation of one system is reviewed. The potential of LEDs is analysed in terms of how they support different classes of activities that can be done with notation systems (Kaput, 1992). Implementing LEDs as interactive computer based representations alleviates some of the potential difficulties of using them for learning. Strategies for effective learning with LEDs are discussed. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The roles and potential benefits of graphical representations for problem solving, learning and instruction are considered here. The approach is to investigate what may be considered a special class or extreme case of diagrammatic representations. An effective strategy in pursuing scientific discovery is to study examples of phenomena that are novel, unusual or apparently rare. The study of Law Encoding Diagrams, LEDs (Cheng, 1994, 1995a, 1996c) is an example of the execution of this strategy in the investigation of graphic representations. There are examples of

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other researchers who can be viewed as adopting a similar strategy (e.g., diSessa, 1985; White, 1989), but in the main, the focus of work in the field has involved theoretical and empirical studies of more common graphic representations than LEDs, such as histograms, Cartesian graphs, maps, schematic diagrams, flow diagrams, Euclidean geometry and so forth (e.g., Schnotz, Zink & Pfeiffer, 1995; Gattis & Holyoak, 1996; Tabachneck, Leonardo & Simon, 1994; Koedinger & Anderson, 1990; Bertin, 1983; Larkin & Simon, 1987).

Law Encoding Diagrams are representations that capture the important relations of a law in the structure of a diagram using geometric, topological or spatial constraints, such that each instantiation of a diagram represents an instance of the phenomenon modelled or one case of the law represented. Examples of LEDs can be found in early physics, including Galileo's and Newton's diagrams (Cheng, 1996c). Problem solving and learning with law encoding diagrams have been empirically studied (Cheng, 1994, 1995a, 1996a) and investigated through the construction of computational models (Cheng, 1996c). This paper will analyse LEDs from the perspective of interactive computer based graphic representations for learning and instruction.

To provide a context for this analysis, Kaput's (1992) thorough investigation of the state of the art in the use of modern computers for mathematics education will be considered. The analysis here will range more broadly than mathematics, but most of Kaput's conclusions are as applicable to instruction in science and engineering as they are to mathematics. Given the fundamental new features of this technology, Kaput identifies four potential educational payoffs: (1) The medium of computers allows routine computations to be off-loaded enabling learning to be more compact and enriched. (2) The nature of the traditional representations can be transformed due to the plasticity added by modern computer interfaces. (3) Computational power can be put to use to focus learners attention on the essentials of the domain. (4) Notational systems may capture procedures or abstract structure in perceptually concrete symbols, so making them potential subjects of "didactic discussion".

Three related points will be made in this paper. First, LEDs are forms of graphical representation which may be effective additions to the existing repertoire of representations for learning and instruction because they make the relations governing a domain more readily accessible to the learner. This is different to Kaput's second point, in that traditional representations are not just being transformed, but are being supplemented (or even supplanted) with a new class of representations for instruction.

The second point is that the power of modern interactive computer graphics can overcome some of the practical difficulties in using LEDs. The difficulties are mainly associated with the need to construct and transform LEDs, but such activities can be largely off-loaded onto the computer (Kaput's payoff number 1).

The third point is that new instructional strategies will be needed to support effective learning with LEDs. These strategies will need to focus learners' attention on the essentials of the LEDs, which will require methods that will use computers to help the learner focus on the important structural features of the LEDs (an example of Kaput's payoff number 3).

Much of the exploitation of computers in science and engineering has been to make traditional notation systems for learning and instruction, such as algebra, graphs and numerical tables, more effective. Together the three points constitute a fifth educational pay-off of modern computer technology—making feasible the introduction of powerful new classes of representations. The paper will examine the three points in detail. First, two interactive graphical systems that use LEDs for science and engineering instruction are presented. The systems will provide thoroughgoing examples for the analysis of the three points. The science example also provides empirical evidence for the effectiveness of LEDs for instruction.

2. Two systems for interactive instruction with LEDs

This section has several purposes. Two LEDs are presented to illustrate quite different examples of this class of representations. The forms of problem solving that can be done with them are discussed to show their utility. How they have been implemented in two quite different computer programs is described as this will provide some of the background needed for the analysis in the rest of the paper.

2.1. AVOW Tutor

AVOW Tutor combines two representations for learning about electrical circuits (Cheng, 1995b). The first are conventional circuit diagrams and the second are LEDs called AVOW diagrams. *AVOW* stands for Amps, Volts, Ohms and Watts, which are the units for the electrical properties of current, voltage, resistance and power. Figure 1 shows a screen snap shot of the AVOW-Tutor system. The user can construct conventional circuit diagrams on the left, by dragging resistors from the store at the top of the screen on to the "circuit board" in the middle. The resistors can be positioned and connected together as desired.

The AVOW diagram occupies the large area on the right. The user constructs AVOW diagrams by dragging AVOW boxes (rectangles) from the store into the middle. The location, size and shape (aspect ratio) of the AVOW boxes can be adjusted as required, but the system prevents any boxes overlapping each other or the sides of the diagram. Each AVOW box represents one resistor. The height, width, gradient of the diagonal, and area of the AVOW box are, respectively, the voltage (V) across, current (I) through, resistance of (R), and power (P) dissipated by, the resistor. AVOW boxes encode Ohms law ($V = I \cdot R$) and the power law ($P = V \cdot I$). There are simple rules for drawing AVOW diagrams for different circuit configurations. Series resistors are placed vertically on top of each other (e.g., Fig. 1, R1 and R2). Parallel resistors are placed side by side (e.g., R3 and R4). Sets of resistors follow the same rules (e.g., R1 and R2 are in series with R3 and R4). The overall structure of AVOW diagrams is consistent with Kirchhoff's circuit laws, which are algebraic statements about the overall relations among currents and voltages in a circuit (e.g., 'the algebraic sum of the currents flowing towards a node is zero'). A

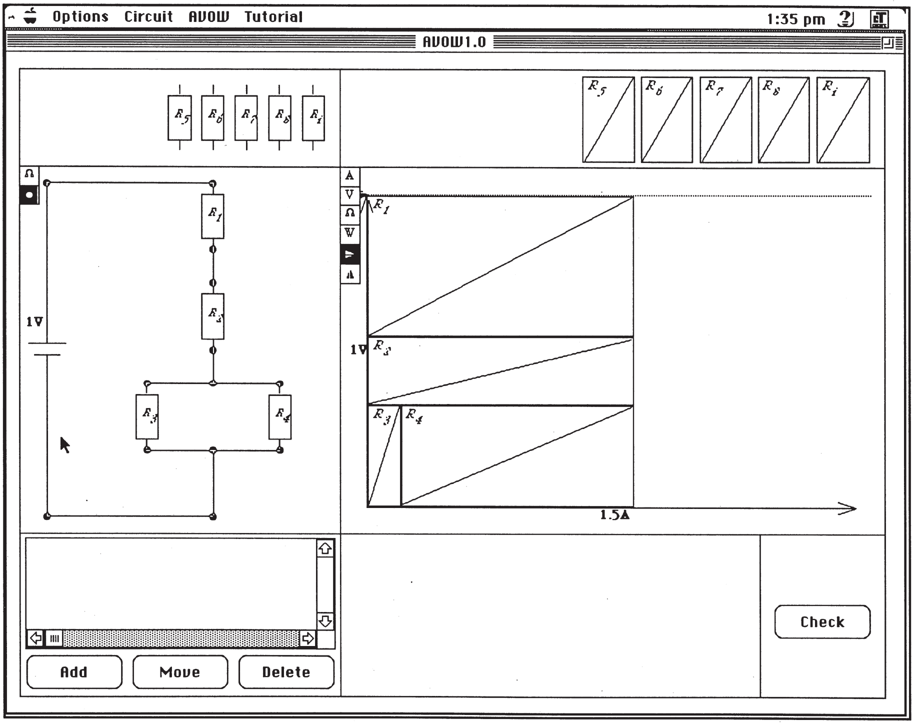

Fig. 1. A screen snap shot of AVOW-Tutor.

well-formed AVOW diagram is a rectangle completely filled with AVOW boxes and has an overall height equalling the voltage of the source in the circuit.

(In Fig. 1, the lower part of the screen under the diagrams contains controls which the instructor uses to write sets of tutorial exercises based on circuit and AVOW diagrams. The tutorial authoring and execution facilities of AVOW tutor are not considered here.)

Various forms of problem solving can be done with the AVOW diagrams. Quantitative reasoning is possible as AVOW boxes directly represent values of circuit properties. For example, given the voltage of the battery in Fig. 1 and resistances R1, R2, and R3, the user can construct a complete AVOW diagram, find the unknown gradient of the diagonal of the fourth AVOW box, which is the value of R4. Qualitative reasoning is based on the relative sizes of the AVOW boxes. For example, from the areas of the boxes it is obvious that R1 is consuming the most power. Similarly, extreme and special cases can be considered. For example, the AVOW box for a component with zero resistance, a pure conductor, is a horizontal line segment; such as the nodes between resistors in Fig. 1. In the same vein, insulators are vertical line segments.

Strategies for solving typical exercise problems on electrical circuits are transformed by AVOW diagrams. For example, a common problem is to determine the overall resistance of a circuit. A typical approach is to find sub-networks of a few resistors, for which it is simple to compute an equivalent single resistance. These new resistances are then themselves considered as parts of other sub-networks, in a recursive fashion, until the whole circuit has been analysed. For example, in Fig. 1, resistors R1 and R2, and resistors R3 and R4, can be reduced to equivalent resistances, say R12 and R34, using standard formulas for resistors in series or in parallel (i.e., $R12 = R1 + R2$, and $1/R34 = 1/R3 + 1/R4$). As R12 and R34 are themselves in series, they can be reduced to a single resistance for the whole circuit ($Rtotal = R12 + R34$).

With AVOW diagrams the approach is quite different. Once constructed, the total resistance of an AVOW diagram is simply the gradient of the diagonal line from the top right to the bottom left of the complete diagram. For some circuit problems, AVOW diagrams are a more effective approach (i.e., require fewer inference steps) than traditional techniques of circuit decomposition and algebra. The circuit in Fig. 2 illustrates this point, as it is impossible to reduce the circuit to pairs of simple series or parallel resistors. R3 is simultaneously in parallel and in series with all the other resistors, so the simple formulae for pairs of resistors are inapplicable. The problem solver will have to set up and solve simultaneous equations for combinations of currents and resistors in different paths through the circuit. With AVOW diagrams the same diagrammatic strategy remains appropriate. The AVOW diagram in Fig. 2 shows the solution (imagine the box for R2 covers the shaded region; the shaded region is explained below). The diagram shows that R3 is really in parallel with R2 and R4, and in series with R1 and R5.

Trouble shooting is also an important skill in this domain and AVOW diagrams also transform the approach with this kind of problem solving. For example, trouble shooting may take the form of comparing the AVOW diagram for the ideal circuit with an AVOW diagram constructed from measurements taken from the malfunctioning circuit. Visual comparison for AVOW boxes of the wrong shape or size will localize the fault. A “short circuit”, for instance, would appear as an unexpectedly squat AVOW box.

Extensions of the basic form of AVOW diagrams are being developed to deal with more complex circuits containing multiple voltage sources, other components such as diodes, capacitors and transistors. Evaluations of AVOW diagrams and the AVOW-Tutor system have just begun. The next two subsections will briefly describe another computer based system that exploits LEDs for learning which has been evaluated.

2.2. ReMIS-CL

This is a system for learning about the quite different domain of elastic collisions in physics. ReMIS-CL deals only with impacts between two bodies (balls) travelling in a straight line. Figs. 3 and 4 are screen snap shots of the system. At the bottom there is an animated simulation of the collision that the user runs at will. In the two areas above there are two interactive LEDs: *the one-dimensional property diagram* (1DP diagram) and *the velocity-velocity graph* (V-V graph). Both LEDs encode the

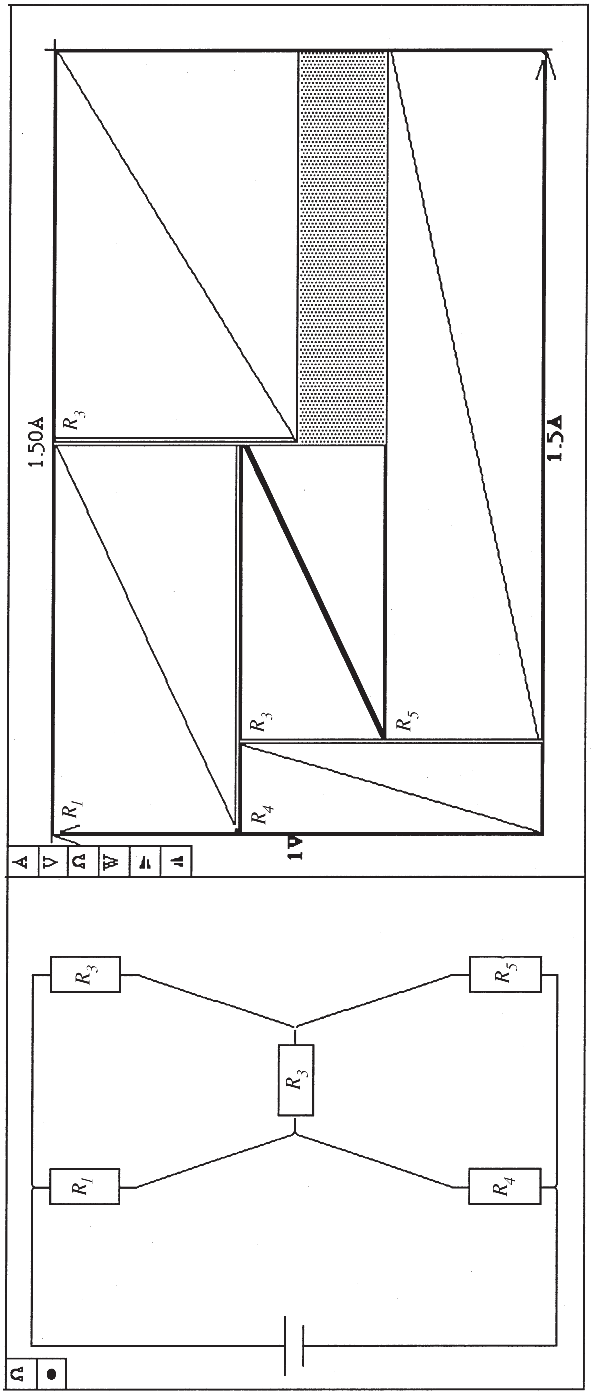

Fig. 2. Five resistor problem in AVOW Tutor. The AVOW diagram is not quite complete, the shaded area shows that box R2 should extend down to the top of box R5.

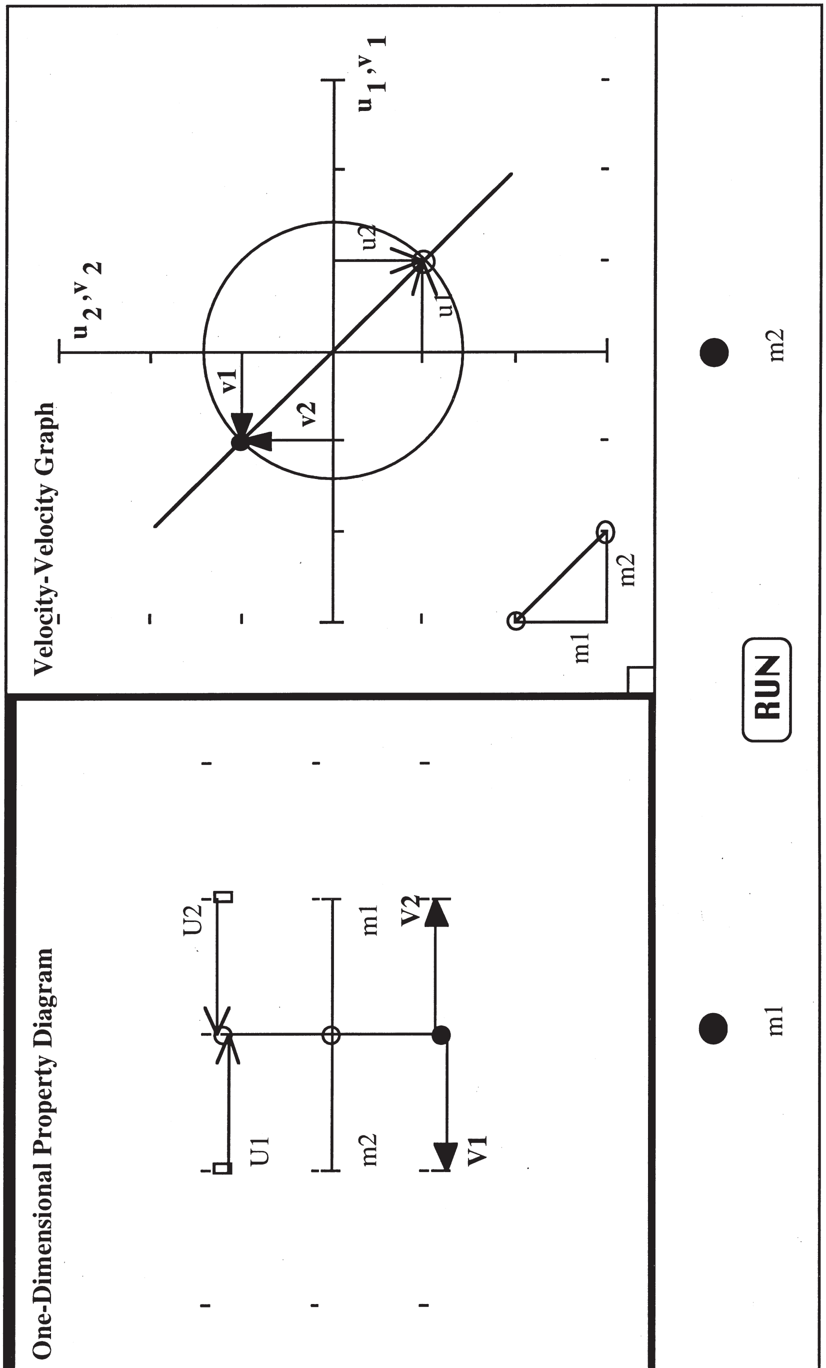

Fig. 3. ReMIS–CL showing a simple equal mass collision.

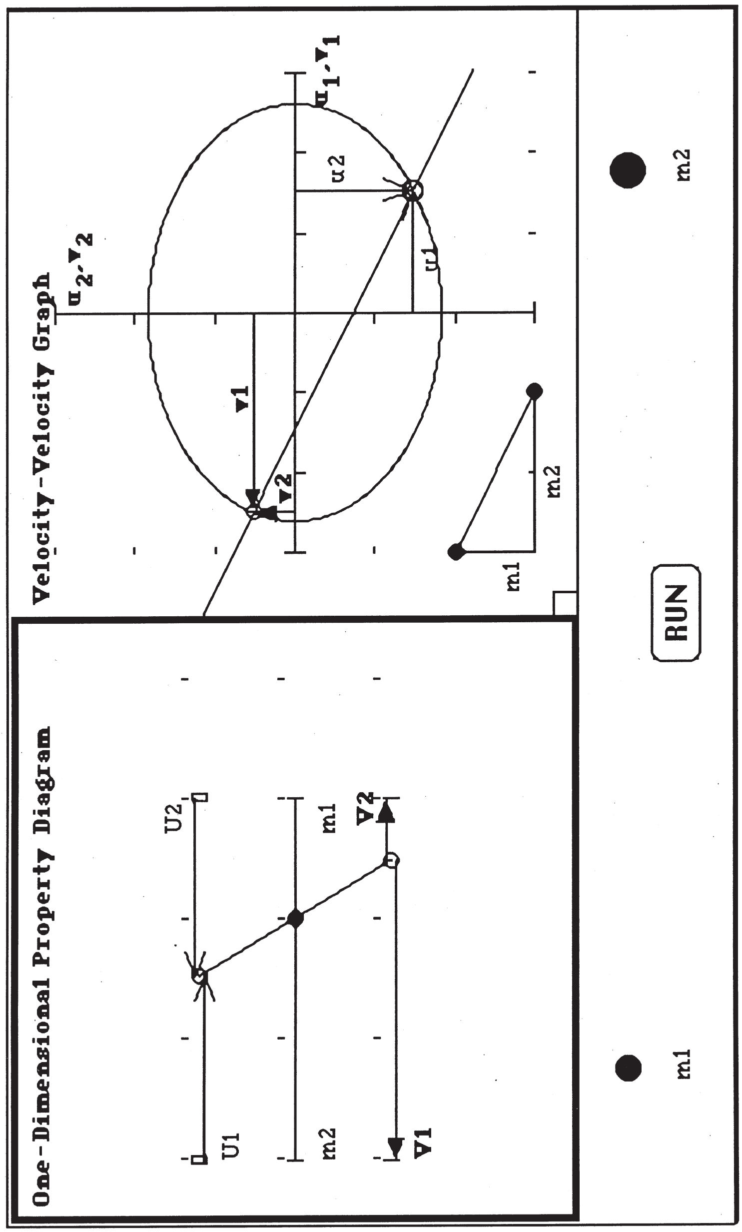

Fig. 4. Unequal masses colliding in ReMIS-CL.

laws of momentum conservation and energy conservation simultaneously, which are both true in elastic collisions.

The lines in the diagrams represent magnitudes of velocities and masses: *U*1 and *U*2 are the velocities before impact; and *V*1 and *V*2 are the velocities after collision. In Fig. 1 the bodies approach and depart in different directions but with equal speeds. In the 1DP diagram, masses lines, *m*1 and *m*2, are drawn equidistant between the *U*1–*U*2 and *V*1–*V*2 lines. In the V-V graph the masses are represented by the sides of the small triangle. The ratio of the lengths of the mass lines in both LEDs equals the ratio of the masses of the two balls. The large circle in the V-V graphs is a constant energy contour and the long diagonal is a constant momentum contour. The LEDs within each figure represent the same collision, but between figures they show different collisions.

LEDs can be directly manipulated to change the values of the variables. Figure 4 shows the effect of increasing the speeds of *U*1 and *U*2 and making *m*2 twice the size of *m*1. The rest of the 1DP diagram has been automatically updated. The LEDs are inter-linked so that the structure of V-V graph is also updated automatically. ReMIS-CL ensures that the LEDs are always consistent with their diagrammatic constraints and thus satisfy both conservation laws. Cheng (1995a) describes the LEDs and their constraints in more detail, but comparison of the two figures gives a good impression of the rules which govern the diagrams.

Like the AVOW diagrams, both LEDs for this domain can support a variety of forms of problem solving. This includes quantitative and qualitative reasoning, extreme and special case problem solving (Cheng, 1995a). More complex collisions (e.g., Newton's cradle) and non-elastic collisions can also be handled.

2.3. Evaluations of ReMIS-CL

Two empirical studies have been conducted to evaluate the ReMIS-CL. The first was a small scale one, in which the problem solving behaviour and learning on the system was examined in detail (Cheng, 1994, 1995a). The subjects were six physics students. They were initially given pre-test qualitative and quantitative problems on collisions which they solved using conventional approaches, such as algebraic manipulation of the equations of the conservation laws. In the trial on ReMIS-CL they were given instruction how to use the interface, with a brief description of the LEDs. A short period (up to 15 minutes) of free investigation followed, during which they were allowed to explore the system for themselves. This was followed by a series of ordered questions that asked them to try to discover qualitative and quantitative relations among the variables. The session on the computer typically lasted 90 minutes. A post-test with a similar format to the pre-test was administered two weeks later. The subjects fell into two groups defined as LED-*users* and the *conventional methods group*, which happened to have equal numbers. The conventional methods group did not change their approach to the problems in the post-test, but the LED-users solved the problems using LEDs in the post-test. It appears that the LED-users had a better grasp of the underlying constraints of the LEDs, as shown by their more complete reproductions of the diagrams prior to the post-test. This is, in turn, related to their more complete exploration of the range of possible structural forms of the LEDs during the free investigation phase of the trial. LED-users used novel diagrammatic problem solving strategies in the post-test. For example, some quantitative problems were solved by drawing LEDs to scale and measuring the diagram to obtain the required values, which were then cross checked by direct substitution into the equations to show that momentum and energy were conserved, without any algebraic manipulations. The conventional methods group, meanwhile, persisted unsuccessfully with algebraic representations in the post-test.

The second study showed that LEDs can facilitate thorough exploration of the range of possible configurations of the collisions (Cheng, 1996a). Three groups of subjects were compared: (i) *LED* group, which used LEDs in the ReMIS-CL system; (ii) *Num* group, which used an alternative version of the system with no LEDs, but had an interface that presented the equations and a table of numerical values; (iii) a control group. The pre-test and post-test mainly consisted of qualitative questions about particular configurations of collisions. The post-test was given two weeks after the pre-test and the trial on the system. During the trial the LED and Num group were instructed to find out as much as they could about elastic collisions. Only the LED group showed a significant improvement in their qualitative reasoning scores from pre- to post-test. During the trial, the LED group considered a greater variety and more configurations of collisions than the Num group, whilst the Num group ran a greater proportion of simulations. Thus, it appears that the improvements in LED group stemmed from their more complete examination of the space of collisions, whilst the Num group expended their effort attempting to relate observation of specific cases to the equations. The LEDs seem to make it easier to comprehend the relations amongst the variables in each case.

To summarise the two experiments: There is evidence that LEDs transform problem solving strategies. To use a LED effectively, problem solvers must have an understanding of the constraints governing the structure of the LED. Exploring a wide variety of the cases of the phenomenon is one approach to learning about a LED. LEDs can in some circumstances be more effective than traditional representations, because they reveal the relations implicit in their laws whilst visually depicting particular instances of the laws.

3. Representations to make relations accessible

LEDs attempt to make relations in a law explicit by encoding them in the structure of a diagram using relatively simple diagrammatic rules, such that each instantiation of the diagram is one case of the laws. Making the relations accessible in this way seems to confer some benefits to problem solvers and learners. Evidence for this comes from the two evaluations of ReMIS-CL described above. In this section the potential benefits of LEDs are analysed in more detail by considering how they support the four general classes of activities that Kaput (1989, 1992) has identified for notational systems. These classes are: (1) syntactic manipulation within a notational system; (2) translations between notational systems; (3) construction and testing of mathematical models; (4) consolidation or crystallization of relationships and/or processes into conceptual objects. How LEDs can enhance each of the four activities will be addressed in turn.

Syntactic manipulations of a representation are a central part of problem solving in many areas of science and engineering. Students can gain some understanding of the relations and constraints imposed by a law when they manipulate and examine the mathematical expressions in which the law is expressed. Kaput argues that traditional instruction in mathematics has, in the past, focused too much on such syntactic manipulations. The same is true in mathematically oriented engineering and science subjects. Nevertheless, this class of activity is important for learning. LEDs may be viewed as specialized computational devices for particular laws. Adopting LEDs and replacing syntactic manipulation in traditional notational systems with the manipulation of diagrams may confer various benefits on students. (1) The range of manipulations possible with LEDs is typically more restricted than the equivalent traditional representations. All legal changes to an AVOW diagram will produce a diagram that correctly represents a circuit, but not all mathematically correct manipulations of an equation will yield meaningful expressions (e.g., squaring both sides of the equation for the total resistance of parallel resistors). (2) In a similar vein, it is often easy to identify the boundary conditions or special cases of the laws with LEDs. For example, finding the maximum power dissipation for a component in a circuit is a matter of maximising the area of its AVOW box, within the constraints given. The equivalent problem with an algebraic representation requires: (i) the derivation of an equation for the power; (ii) differentiating it with respect to one of the other variables to find the turning points; and, (iii) solving the new equation. This is a more difficult and laborious task. (3) As a LED represents one instance of a phenomenon and also shows the relations among the variables as a coherent structure, it is often easier to spot mistakes in the manipulations of LEDs than it is to find algebraic errors. In the detailed small scale ReMIS-CL study, described above, none of the undergraduate science students successfully completed algebraic solutions to the quantitative problems in the pre-test, because of algebraic mistakes or inappropriate approaches. In the post-test, subjects using the 1DP diagram on equivalent quantitative problems found the correct solution quickly and then cross-checked their answers with the equations.

Kaput's second class of activities is translations between notational systems, encompassing the co-ordination of actions across systems. For example, adding a positive constant to the right hand side of the equation '$y = x^2$', corresponds to an upward shift of a parabola in a Cartesian graph. In general terms, a LED may benefit learning and instruction in a scientific domain by increasing the number and diversity of representations, so that there is more choice in the selection of appropriate representations for students with different levels of knowledge. For example, AVOW diagrams could be used for students who do not have a firm grasp of algebra. In more specific terms, LEDs have some inherent advantages over traditional representations with respect to translation activities. LEDs often have easy mappings into other notations, at the level of individual variables and their magnitudes and at the level of abstract relations. For example, it is relatively simple to relate the lengths of the sides of an AVOW box to a point in a graph of current versus voltage. It is possible to translate from the area of the box to the power law ‘P = V·I’ knowing the formula for the area of a rectangle. Similarly, the vectors in the V-V graph are velocities and the circle can be mapped onto the energy conservation law, because its algebraic form is the same as the expression for circles and ellipses.

The third form of activities with notational systems is the construction and testing of models, which involves the translation between aspects of phenomena (or simulations of them) and the notations. An example is the application of a quadratic function to the relation between distance and time of falling objects. LEDs are models in this sense and possess some advantages over models in other traditional representations. The most obvious is that each instantiation of a LED represents one case of the law. Thus, each 1DP diagram represents one collision and each AVOW diagram represents an electrical circuit. Mappings between a LED and a phenomenon occurs at the same level of description rather than across different levels of abstraction. Further, for some LEDs this mapping is itself facilitated by the resemblance of the LED to the topology of the phenomenon. For example, the left to right positioning and the direction of the vectors in the 1DP diagram matches the location and motions of the colliding balls. Similarly, the positioning of AVOW boxes reflects the connectivity of the components in a circuit.

The issue of the selection of appropriate analogical models for learning is relevant here, both in terms of the activities of translation between representations and the construction and testing of models. Gentner and Gentner (1983) note that there is good evidence that peoples’ mental representations of physical phenomena often contain “profound errors”, such as being fragmentary, inaccurate, internally inconsistent. As the representations strongly effect a person’s construal of new information in a domain, appropriate representations are crucial to scientific reasoning and learning. Gentner and Gentner studied problem solving on electrical circuits with analogies based on both water flowing in pipes driven by pumps or reservoirs, and crowds teeming around a race track. Their predictions about the performance of subjects naive in physics were only partially supported, because the subjects intuitive models of the base domains differed from the experimenters’ own expectations about peoples’ understanding. Gentner and Gentner (1983) conclude: ‘Our investigations bring home the point that an analogy is only useful to the extent that the desired relational structure is present in the person’s representation of the domain’ (p.124). The approach with LEDs is quite different, as it does not rely upon prior analogies, but aims to provide learners with correct, complete and comprehensible models from scratch. It is hypothesized that learners will be able to cope with both parallel and series circuit problems using AVOW diagrams, because both types of circuit are equally supported by this LED system.

The final class of Kaput’s activities concerns the consolidation, or crystallization, of relations and/or processes into conceptual objects that can then be used in relations or processes at a higher level of organization. For example, finding the roots of a quadratic equation is initially a task in itself for students, but later it may be just one step in the solution to a larger problem. To consolidate relations or crystallize procedures the learner must be able to identify the significant aspects of the representation from those that are irrelevant. LEDs may be effective in this respect because they attempt to capture co-ordinated variables as compact patterns in diagrams using relatively simple rules. A well understood LED can become a conceptual object for problem solving at a higher level of organisation. For example, each AVOW box gets incorporated into a larger AVOW diagram and 1DP diagrams can be used in a compositional fashion to model series of multiple collisions (e.g., Newton's cradle, see Cheng, 1995a).

The potential benefits of LEDs can be summarised in the idea that they make relations and constraints more accessible to learners. Accessibility is meant both in terms of facilities to help visualize the form of the relations and to examine the implications of the relations in particular cases. An alternative and complementary perspective is to consider LEDs as representations at an intermediate level of abstraction (White, 1989). White advocates such representations for science education, as a means to bridge the conceptual gulf, that exists for most students between abstract laws and concrete cases. White (1993) has demonstrated that dramatic improvements in students understanding of Newtonian mechanics can be achieved with *intermediate causal models*, ICMs, which are representations that emphasise temporality and causation. LEDs differ in that they focus on relations and constraints. Clearly, selecting representations at the right level of abstraction is important, but whether they should be constraint-based or reflect causality is a question open to empirical investigation. LEDs may have an advantage, in that they can be the basis for mental models like ICMs, but they may also provide a more direct link to the formal abstract representations of laws.

The next section considers some of the potential problems with LEDs and considers how they can be overcome.

4. Making LEDs interactive graphical representations

There are at least four potential difficulties with LEDs for problem solving and instruction. (i) Some forms of reasoning with LEDs involve significant amounts of drawing and re-drawing of the diagrams. (ii) Constructing LEDs requires some ability in diagrammatic and geometric reasoning. Learners lacking such knowledge could be disadvantaged. (iii) For complex phenomena, LEDs may become unwieldy more quickly than the equivalent algebraic expressions. Problem solvers may find it hard to keep track of all the interrelated constraints of large composite LEDs. (iv) A problem for instructors is that it is harder to design effective LEDs for new domains than it is to generate appropriate graphs or mathematical models. Fortunately, the first three difficulties can be ameliorated by making LEDs interactive graphical representations. Four potential educational payoffs of computers to support notations (mentioned above) provide a convenient basis for this analysis (Kaput, 1992).

The first educational pay-off that Kaput has for computers is that they allow routine computations to be off-loaded, enabling learning to be more "compact and enriched". Most of the effort required in the drawing and redrawing of LEDs (difficulty i) can be placed onto the computer by designing systems that automatically maintain the constraints of the LEDs. In ReMIS-CL the system automatically updates LEDs whenever a part of it is manipulated. As the representations are linked, the other LEDs on screen are also re-drawn. In some circuit problems AVOW diagrams must be iteratively re-drawn to satisfy all the constraints on each AVOW box and amongst them. AVOW Tutor facilitates such problem solving as AVOW boxes can be relocated or re-sized by simple "dragging" actions. The system maintains the rectangular shape of the box and automatically prevents them from overlapping.

Kaput's second educational pay-off is the plasticity added to representations when implemented on computers. For instance, static notations that are traditionally used to display information (e.g., graphs) can be changed into action notations that are interactively manipulated on screen. By implementing LEDs on computers, the extent to which students must actively maintain the constraints can be reduced, and consequently lessen the need for a good understanding of geometry (solution to difficulty ii). Computers allow LEDs to become information display notations for students with little geometry knowledge. For example, to reason with the V-V graph one often has to construct an ellipse, given a centre and two points on its circumference, a nontrivial task. Knowing how to do this is unnecessary with ReMIS-CL, as the system draws the ellipse, so learners can benefit from the visualization of the energy conservation law as an ellipse. Further, LEDs for students with little geometry knowledge does not mean that they have to be passive learners, because computer based LEDs can be interactively manipulated, whilst the computer maintains the correct structure of the diagram, as in ReMIS-CL.

Kaput's third and forth educational payoffs can address the third difficulty of LEDs; i.e., that complex LEDs may be unwieldy and difficult to interpret. With complex LEDs learners attention may be distracted by features of the diagrams that are not essential to understanding the important relations of the domain. However, computational power can be put to use to focus learners attention on the essentials of the domain; Kaput's third pay-off. AVOW tutor provides a good example of how the learner can be encouraged to focus on the unsatisfied constraints of the AVOW diagram. The system checks the structure of the AVOW diagram for consistency with the circuit diagram and highlights the missing AVOW boxes or gaps in the complete rectangle. In Fig. 2, the shaded region in the AVOW diagram shows that it is incomplete; R2 should be deeper.

Keeping track of the constraints of complex LEDs can be facilitated by adding appropriate features during the design of the system. This is an example of Kaput's fourth educational payoff, that notational systems may capture procedures or abstract structure in perceptually concrete symbols. In ReMIS-CL and AVOW tutor, the format of the diagrams is changed when different constraints are placed on the diagrams by the student. In ReMIS-CL the user may make the masses the dependent variables, for instance, so manipulations of either the initial or final velocity vectors will yield different ratios of masses. When this is the case the small circles in the middle of the lines change colour to show their status in this respect. In Fig. 3 V1 and V2 are the dependent variables and in Fig. 4 it is the masses. In Fig. 2 the AVOW box for R3 has a highlighted (thicker) diagonal line to indicate that the box has a fixed aspect ratio, because it represents a resistor with a given fixed value. Any change to the voltage (height) will result in an automatic change to its current (width), and vice versa. AVOW Tutor uses similar highlighting for boxes with set widths (constant current) and heights (fixed voltage). Such features are simple to build into computer based LEDs and can make complex LEDs easier to understand.

Some of the potential problems with LEDs may be alleviated by making them computer based interactive representations. The problem of designing LEDs for new domains is, however, a more fundamental difficulty, the discussion of which is beyond the scope of this paper. The combination of the study of diverse forms of LEDs and the functional roles of diagrammatic representations (Cheng, 1996b) will attempt to address this issue. Studying the history of science and technology has been a profitable way to find new LEDs (Cheng, 1996c).

5. Approaches to instruction with LEDs

Given that computer-based LEDs exist and that LEDs may be useful for learning science and engineering, because they make relations more accessible, the question arises regarding the effective ways to learn LEDs. This section begins to provide some answers.

Introducing LEDs in instruction provides opportunities for new problem solving strategies, which will in turn require different approaches to instruction. A good understanding of the underlying constraints of LEDs is essential to their use. From the small scale ReMIS-CL study it seems that examining a wide variety of configurations of the LEDs is beneficial, so the problem is to find ways to efficiently allow such exploration of diagrammatic forms. In the empirical evaluations conducted so far, the instructions combined brief descriptions of LEDs with free investigation and exploratory problem solving. Three alternative approaches will be briefly considered.

First, the computer systems may be used in their existing form, with learners manipulating LEDs to be consistent with given cases, whilst the computer maintains the correct constraints of the diagrams. But by presenting a wider range of cases, the learners will be exposed to a greater variety of correct diagrammatic configurations. In particular, special and extreme cases that correspond to unusual configurations of the LEDs should be included, as a means to help the learner differentiate real from merely apparent constraints. For example, the case of a collision between a perfectly elastic planet and pea translates into unique forms of the 1DP diagram and the V-V graph (Cheng, 1995a).

The second approach allows the learner to investigate freely, but has the computer temporarily place additional constraints on the diagrams. These extra constraints limit the manipulability of the LED to focus the learner on particular structural components. In the evaluations of ReMIS-CL the learners manipulated initial velocities and masses to find final velocities. In the evaluations the subjects were not allowed to use the dependent variable selection option in ReMIS-CL (mentioned above). However, forcing the learners to have the masses as the dependent variables, may encourage them to examine how different combinations of final and initial velocities affect the masses, and possibly explore further the structural forms of the LEDs.

The final alternative is to have learners attempt to construct LEDs for themselves under carefully scaffolded conditions. For example, learners may be given selected examples of collisions and told to draw pairs of vectors for the velocities and pairs of lines for masses. Then by comparing the sets of lines for each case they may be able to spot constraints such as those encoded in the 1DP diagram. From a constructivist view, having the learners find the rules for themselves might lead to better understanding of the LEDs.

Further work is required to determine which, if any, of these will be the most effective approach to learning LEDs.

6. Conclusion

It has been argued that LEDs may be an effective class of representations to augment existing traditional representations in science and engineering instruction. LEDs make important relations accessible and help the visualization of particular cases. By implementing LEDs as interactive graphics on computers some of the difficulties of using LEDs are circumvented. New instructional strategies may be devised to exploit the benefits of computer based LEDs. The possibility of introducing new representations, such as Law Encoding Diagrams, may be considered as an additional educational payoff to add to Kaput's (1992) list of beneficial features of computers for learning and instruction.

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