Table 1. The four schemas for LEDs

	Phenomena level	Theoretical level
Intra-component relations	LED schemas (LS)	Meta-LED schema (MLS)
Inter-component interactions	Composite-LED schema (CLS)	Meta-composite-LED schema (MCLS)

Table 2. LED schemas

Slots	Default	Zero-momentum	Total-transfer	Planet-and-pea
Diagram	Figure 1 (left)	E.g., Figure 1 (left)	Figures 3a	Figure 3b
Diagram-features	Diagonal line is vertical; horizontal symmetry.	Diagonal line is vertical.	Diagonal intersects opposite corners of rectangle.	Diagonal line cuts mass line at (near) side of rectangle.
Domain-conditions	$u_2=-u_1=v_1=-v_2$, $m_1=m_2$	$v_1=-u_1$, $v_2=-u_2$, $m_1/m_2=\|u_2/u_1\|$.	$u_1\neq0$, $u_2=0$, $v_1=0$, $v_2=u_1$, $m_1=m_2$.	$u_2=-u_1$, $m_1>>m_2$, $v_2\approx-3u_2$, $v_1\approx u_1$.
Interpretation	Simplest symmetrical case.	Overall momentum is zero.	All energy/momentum transferred.	Limiting case on masses.

Table 3. Composite-LED schema.

Slots	Simple-Newton's-Cradle	General-Newton's-Cradle
Diagram	Figure 4.	Stack of *N*-1 1DP diagrams (Figure 3a)
Composite-features	Column of 4 1DP diagrams.	Column of *N*-1 1DP diagrams.
Domain-conditions	5 equal mass bodies. Only one initially moving.	*N* equal mass bodies. Only one initially moving.
Interpretation	4 Total-transfer pair-wise collisions.	*N*-1 Total-transfer pair-wise collisions.

Table 4 Meta-LED Schema for the 1DP diagram

Slot	1DP diagram	Plastic-1DP diagram
Diagram-features	Arrows: **U1**, **U2**, **V1**, **V2**; lines: **m1**, **m2**.	Arrows: **U1**, **U2**, **V1**, **V2**; lines: **m1**, **m2**.
Diagram-constraints	E.g., local: **U1** and **U2** heads adjacent, **V1** and **V2** tails adjacent, **m1** and **m2** end to end. Global: rectangle rule; diagonal rule.	As 1DP diagram + Global: plastic extension rule — **xy** vertical, $\mathbf{xs_1}$:$\mathbf{xr_1}$=$\mathbf{xs_2}$:$\mathbf{xr_2}$ (Figure 4).
Domain-properties	U and V - initial and final velocities; m - mass; subscripts for each body.	U and V - initial and final velocities; m - mass; subscripts for each body.
Encoded-laws	Momentum conservation law: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ Energy conservation law: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$	Momentum conservation law: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ Energy distribution law: $k\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ energy loss coefficient, k<1
Property mappings	**U** and **V** arrow lengths and orientation give *U* and *V* velocities; lengths of **m** lines give relative mass, *m*.	**U** and **V** arrow lengths and orientation give *U* and *V* velocities; lengths of **m** lines give relative mass, *m*.
Interpretation-rules	One dimensional elastic collision between 2 bodes.	One dimensional in-elastic collisions between 2 bodes.
Cases	E.g., Default, Zero-momentum, Total-transfer, Planet-and-pea. (Table 2)	E.g., Figure 4.

Table 5. Meta-Composite-LED schema.

Slots	1DP series collisions
Component-LEDs	1DP diagrams.
Composition-constraints	In successive 1DP diagrams, **a** and **b**, for a given **m1**, **V1a** and **U1b** may share the same arrow.
Domain-description	Multiple successive collisions in one dimension.
Encoded-interaction-laws	Independent pair-wise collisions.
Mapping-rules	One 1DP diagram for each collision.
Interpretation-rules	Collisions between multiple bodies moving in one dimension. Free (not shared) **U** and **V** arrows are the overall initial and final velocities, respectively.
Cases	Simple-Newton's-cradle, General-Newton's-cradle (Table 3).

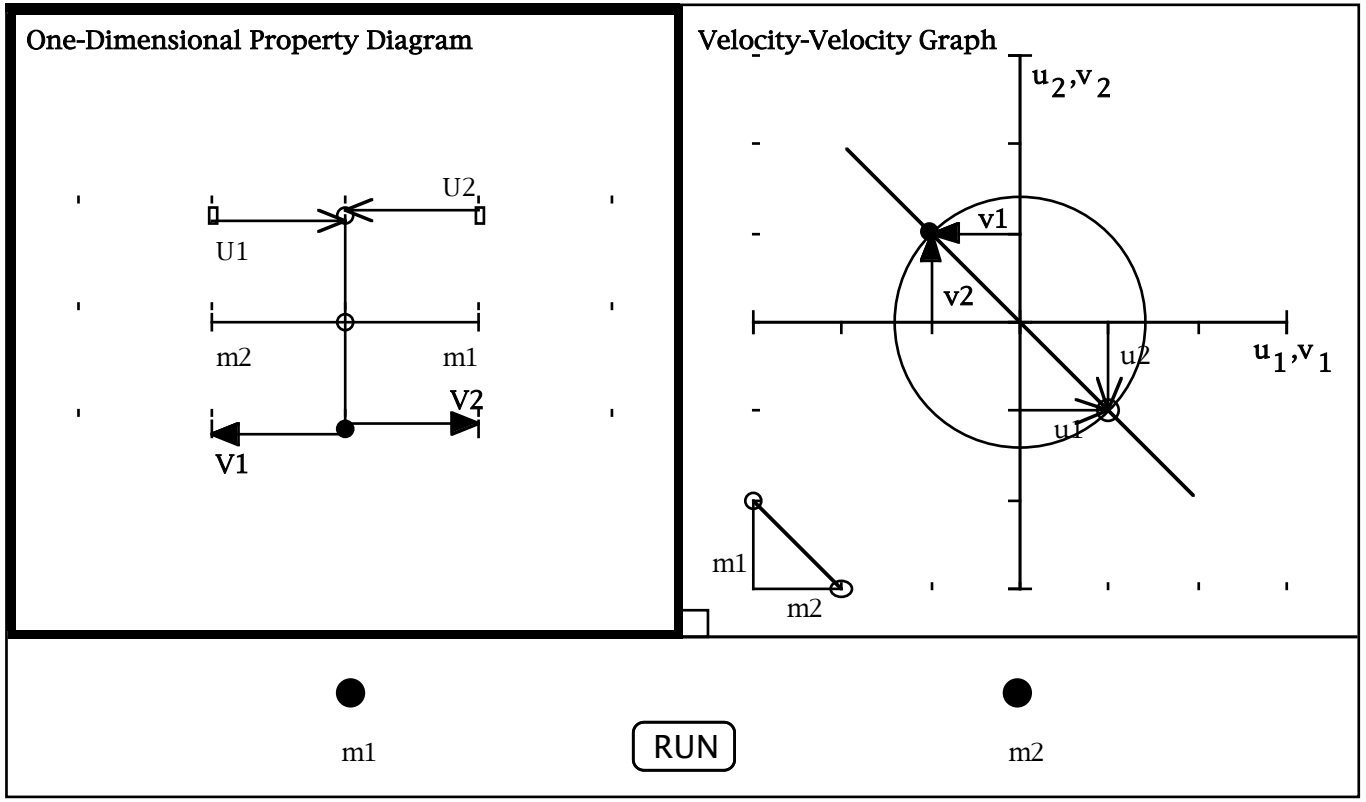

Figure 1 ReMIS-CL Learning Environment with LEDs in their Default Configurations

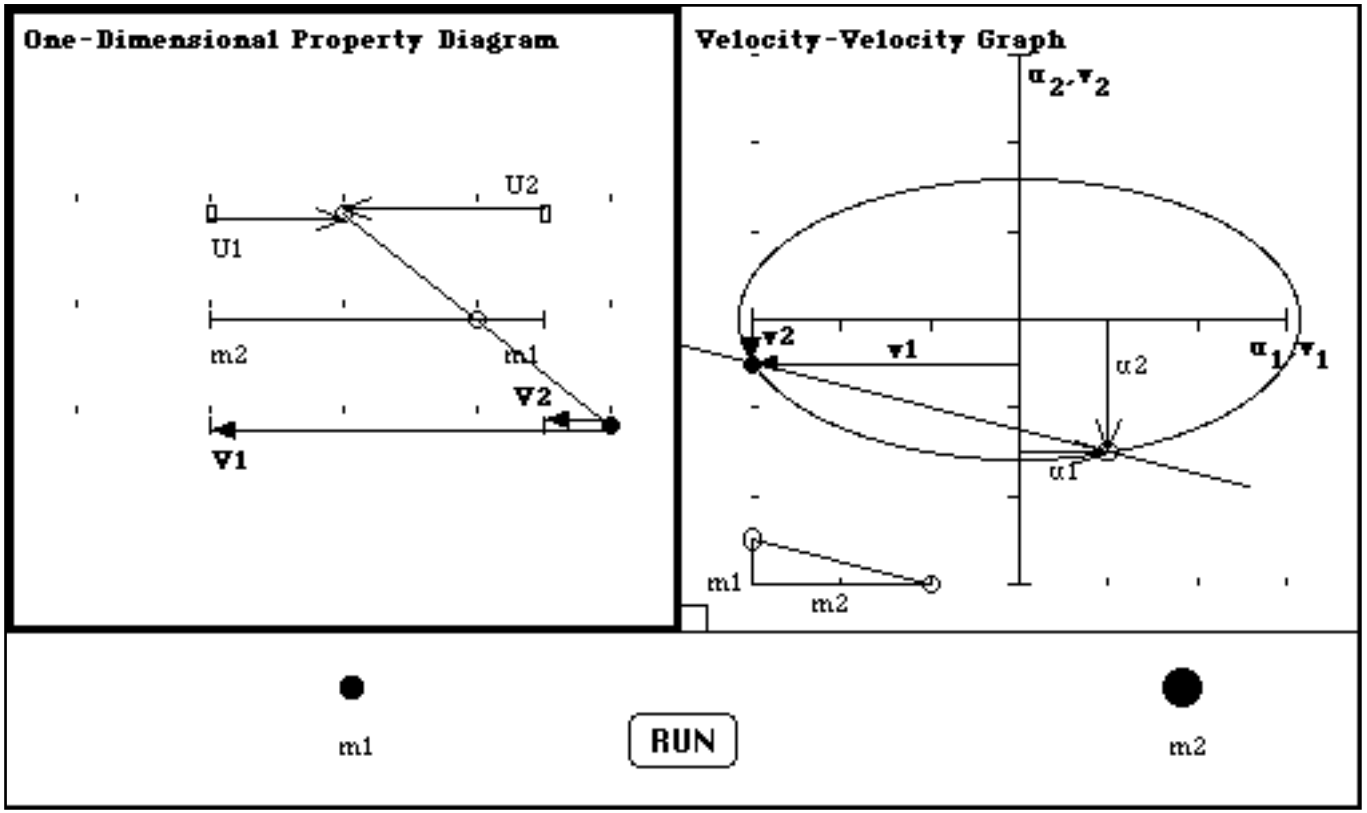

Figure 2 ReMIS-CL Showing a Collision Between Unequal Masses and Speeds.

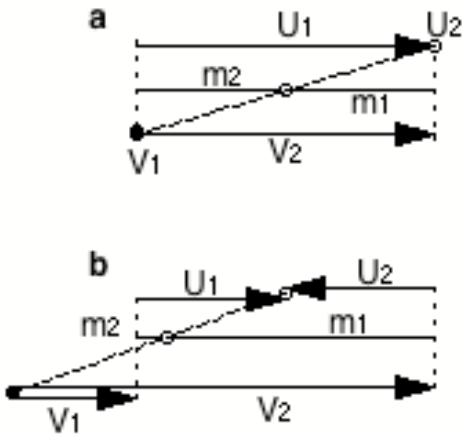

Figure 3 Special Case 1DP Diagrams

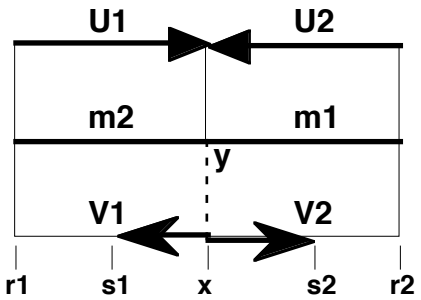

Figure 4 A Plastic-1DP Diagram.

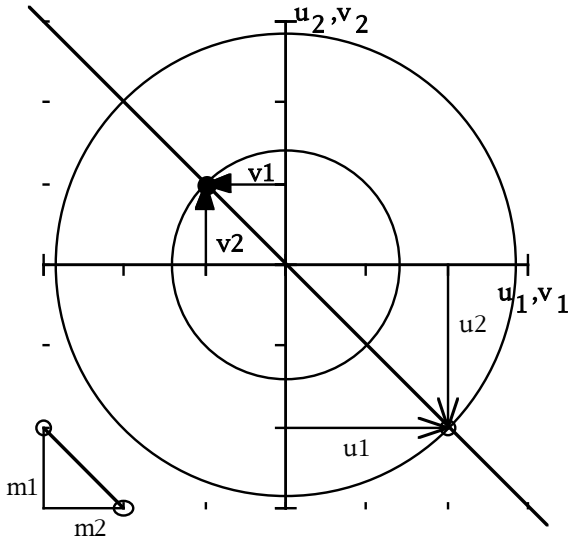

Figure 5 A Plastic-VV graph.

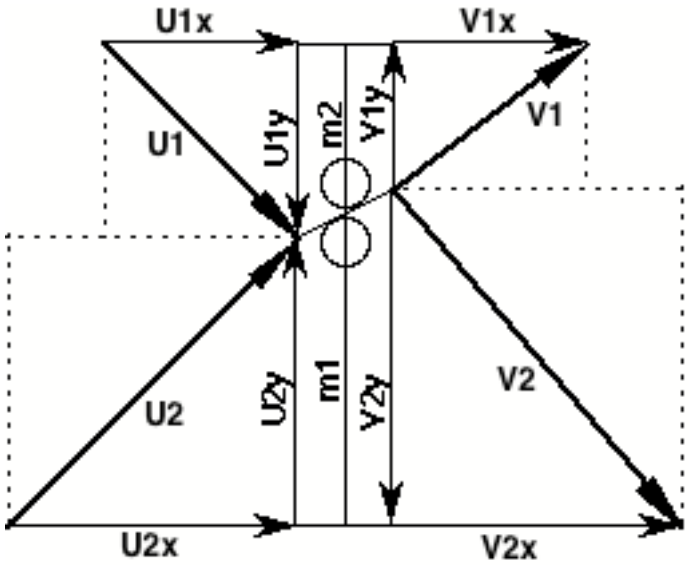

Figure 6 A 2DP Diagram.

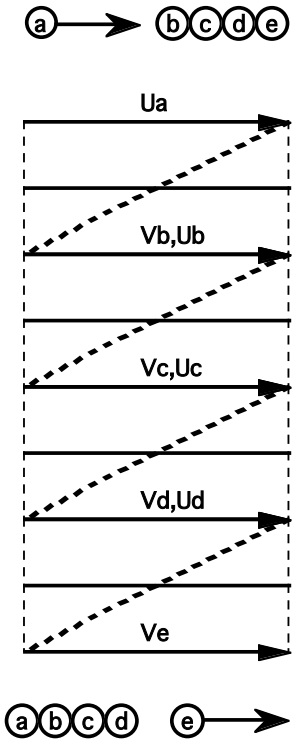

Figure 7 A Composite LED for Newton's Cradle

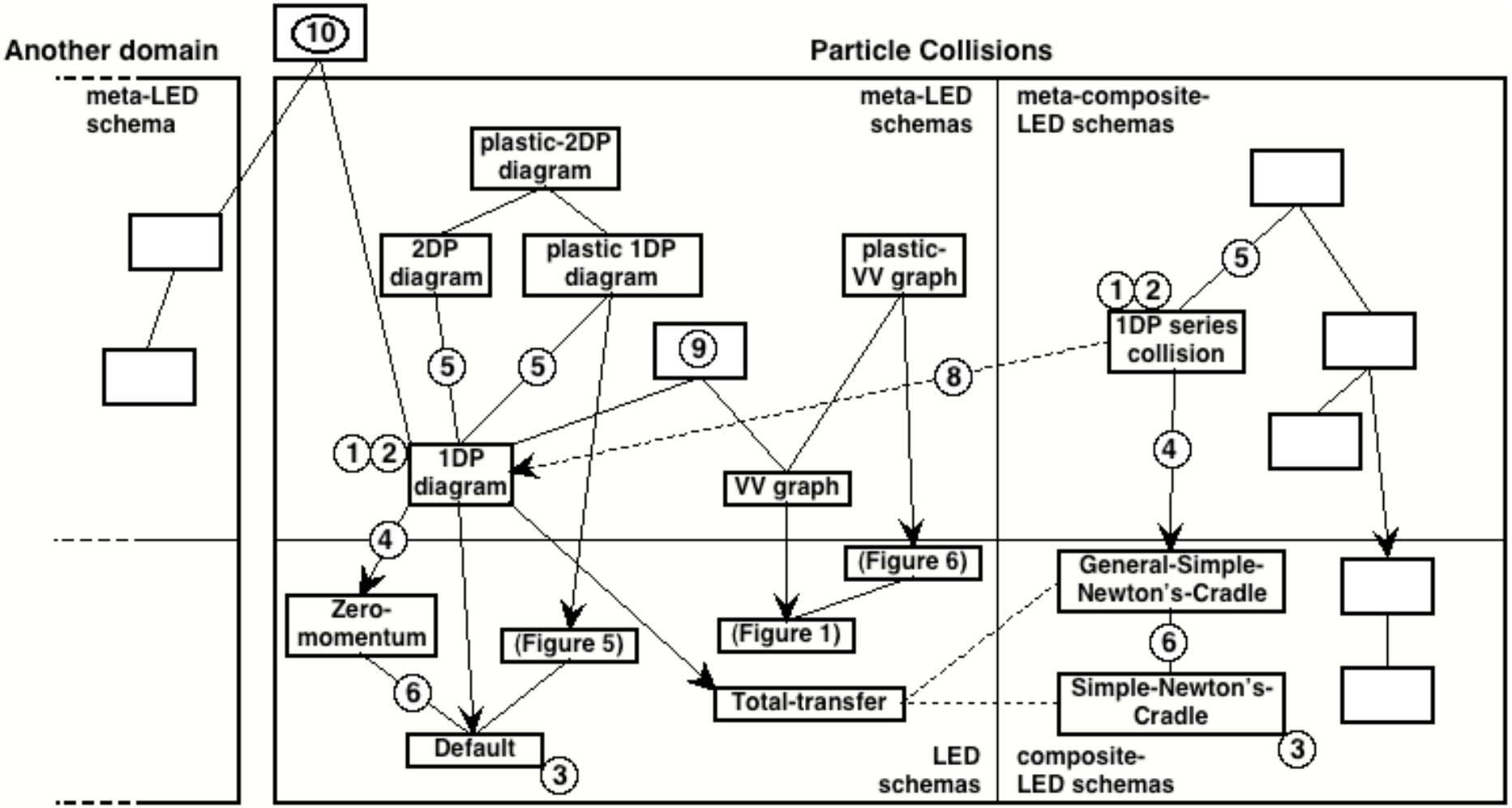

Figure 8 The framework applied to understanding the particle collisions domain.