A Framework for Scientific Reasoning with Law Encoding Diagrams: Analysing Protocols to Assess Its Utility

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Abstract

Law Encoding Diagrams (LEDs) are classes of representations for problem solving and learning in science. A framework consisting of four schemas has been proposed to account for problem solving and learning with LEDs. This paper assesses the utility of this framework by using it to analyse verbal and behavioural protocols of a subject involved in problem solving with a class of LEDs for electricity.

Introduction

Understanding the nature of representations and the role they have in cognition is fundamental to Cognitive Science. Different representations of formally isomorphic problems can dramatically change the difficulty of the problems (e.g., Kotovsky, Hayes and Simon, 1985). Experts and novices may have different problem representations for the same domain that substantially affect their approaches to problem solving (Egan and Schwartz, 1979; Koedinger and Anderson, 1990). There are computational benefits of diagrammatic representations over sentential representations that are informationally equivalent (Larkin and Simon, 1987). The focus here is on the internal mental and external aspects of representations that must be analysed and related to adequately understand cognition (Tabachneck-Schijf, Leanordo and Simon, 1997; Zhang 1996).

This paper is part of a research programme that is studying the role of representations in cognition by examining a particular class of representations with some interesting properties — Law Encoding Diagrams, LEDs. LEDs, are representational systems that capture the laws of a domain in the structure of a diagram using geometric, topological or spatial constraints, such that each instantiation of a diagram is a single case of the laws or an example of the phenomena of the domain. The role of LEDs in scientific discovery has been investigated (Cheng, 1996a, Cheng and Simon, 1995) and their potential for learning science is being pursued (Cheng, 1996b, 1996c).

The particular class of LEDs considered here was invented by the author and has been called *AVOW* diagrams (Amps, Volts, Ohms and Watts — the units of the basic electrical properties). The elementary diagrams are rectangles, *AVOW boxes*, that represent the properties of a single resistor or load, as shown in Figure 1. The mappings of properties to diagram features are: voltage (V) to height; current (I) to width; resistance (r) to gradient of the diagonal; power (P) to area. As V=I*r (Ohm's law) and P=V*I (power law), the

geometry of the AVOW-boxes encodes these laws (i.e., height=width*gradient, area=height*width).

To model networks of resistors, AVOW boxes can be composed, as in the diagrams in Tables 4 and 5. The composition constraints on such *composite AVOW diagrams* encode Kirchoff's Laws that govern the behaviour of electrical networks. A well formed composite AVOW diagram must be a rectangle that is completely filled with AVOW boxes, with no overlap or gaps.

Cheng (1997) proposes a framework to characterize the nature of information processing with LEDs. This paper describes how the framework has been applied to the problem solving protocols of a subject using AVOW diagrams. The completeness, coherence, consistency and parsimony of the analysis argues for the adequacy of the framework.

Here, the framework is first described, with electricity providing an example domain. Second, the framework is used to analyse protocols of problem solving, to assess the utility of the framework.

Figure 1 An AVOW Box for a resistor

Finally, the discussion section considers the adequacy of the framework and related work.

Schemas for LEDs

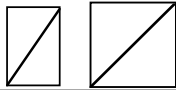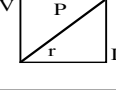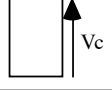The four schemas named in Table 1 were proposed as the basis of a framework for understanding problem solving and learning with LEDs in scientific domains by Cheng (1997, in press). The schemas are distinguished on two dimensions. One pair of schemas (1, 2) holds information about the basic units or components of a domain. The other pair (3, 4) deals with the interactions among those components. The phenomena level schemas (1, 3) are for particular classes of phenomena or cases, and the schemas (2, 4) at the theoretical, or meta, level hold information about the laws or principles governing the domain. The four classes of schemas are considered in turn with examples from the electricity domain.

LED schemas (LS). Table 2 shows examples of this class of schemas for selected circuit components. They have slots containing information about (i) particular aspects of a component and (ii) its AVOW diagram representation. For example, consider the Typical-load LS. The **interpretation** slot indicates that this schema deals with typical loads with

Table 1 Schemas for LEDs

	Phenomena level	Theoretical level
Intra-component relations	(1) LED schemas (LS)	(2) meta-LED schema (MLS)
Inter-component interactions	(3) Composite-LED schema (CLS)	(4) Meta-composite-LED schema (MCLS)

Table 2 LED schemas (LS)

Slots	{1} Typical-load	{2} Known-load	{3} Insulator	{4} Conductor	{5} Voltage-source
Diagrams					
Diagram-configuration	Rectangle (not especially thin or squat)	Rectangle with known dimensions.	Vertical line (narrow, gradient huge)	Horizontal line (flat, gradient tiny)	Rectangle with constant height
Domain-conditions	"Normal values" (not tiny or huge)	Values of some of I, V, r, P given.	I=0 or r=∞ or (P=0 and V≠0).	V=0 or r=0 or (P=0 and I≠0)	Vc=constant, P and I variable
Interpretation	Typical load	Particular resistor	Insulator, "broken circuit"	Conductor, "short circuit"	Idealized battery

properties (domain-conditions) that are not unusual. The loads can be represented by a rectangle (diagram-constraints) that is similar to one in the diagrams slot. The Known-load, Insulator and Conductor LSs are specializations of the Typical-load LS. The Voltage-source LS is for idealised batteries, which supply a constant voltage. One use of LSs is to complete the information about a component when only partial information is given. For example, if one considers a resistor of unknown but not extreme value, then matches with the interpretation and diagram-conditions slots of the Typical-load LS will be made and a rectangle (diagram-configuration) to represent it may be drawn (diagrams).

Meta-LED schema (MLS). These schemas hold information about (i) the laws governing a class of components and (ii) how they can be represented as LEDs. In Table 3 the AVOW-box MLS is for resistors/loads and the Source-box MLS is for power supplies such as batteries. MLSs store information that in effect define the natures of particular sets of LSs. For example, how should a component with (near) infinite resistance be represented?

First, there will be one diagram box (interpretation-rules) which must be a rectangle of some sort (diagram-constraints). Ohm's law (encoded-laws) tells us that current will be nearly zero (domain-properties) as resistance is almost infinite. As current is represented (property-mappings) by the width of the box (diagram-features), so the rectangle will be very narrow. This is the Insulator LS {3} (Numbers in curly brackets identify schemas in Table 2 to 6).

Composite-LED schema (CLS). These share a common structure to LSs but they deal with networks rather than individual components. The diagrams (slot) contains configurations of more than one AVOW box. The diagram-features specify with relative positions of whole boxes rather than their individual elements. The domain-conditions relate properties of different boxes, and the interpretation of CLSs are topologies of components. Table 4 gives some examples. A CLS may be used in a similar fashion to a LS. For instance, given two loads in series (interpretation), the Typical-series CLS can be used to draw the given diagram, or to complete a diagram if one

Table 3 Meta-LED schema (MLS)

Slot	{6} AVOW-box	{7} Source-box.
Diagram-features	**A** box width, **V** box height; **Ω** box diagonal gradient, **P** box area	**As** box width, **Vs** box height; **Ps** box area
Diagram-constraints	Rectangle (box) with bottom-left to top-right diagonal.	Rectangle.
Domain-properties	current, I (A); voltage, V (V); resistance, R (Ω); power, P (W)	Source voltage, Vs. Source current, Is. Power supplied by source, Ps
Encoded-laws	Ohms law: $V=IR$. Power law: $P=VI$	Power law: $Ps=VsIs$
Property mappings	I — **A**; V — **V**; R — **Ω**; P — **P**; Horizontal lines are equi-potentials.	$I n$ — **A**; Vn — **V**; Pn — **P**; Top of box is at Vs and bottom is at 0.
Interpretation-rules	Single component, or an isolated network (with a one input and one output connection).	An ideal power source.
Cases	Table 2 {1}, {2}, {3}, {4}	Table 2 {5}.

Table 4 Series Composite-LED schemas (CLS)

Slots	{8} Typical-series	{9} Equal-series-pair	{10} Series-load+conductor	{11} General-series
Diagrams				
Diagram-configuration	stack, same width	stack, same width, boxes same height	stack, same width, 1 box squat	stack, not of uniform width.
Domain-conditions	$VT=V1+V2$, $I1=I2$, $RT=R1+R2$	$VT=V1+V2$, $I1=I2$, $RT=R1+R2$, $R1=R2$	$VT\approx V1$, $V2\approx 0$, $I1=I2$, $RT\approx R1$, $R2\approx 0$.	$VT=V1+V2$, $I1\neq I2$, $RT=$n.a.
Interpretation	resistors in pure series	equal resistors in pure series	resistor in pure series with conductor	two resistors partly in series with each other.

AVOW box has already been drawn. Particular relations between the loads can also be asserted (domain-conditions); such as the current is the same through both loads.

The Typical-series CLS may be specialized to give Equal-pair-series, Series-load+conductor and Series-load+insulator (not shown) CLSs. Further, it is itself a specialization of the General-series CLS. There are equivalent CLSs for parallel loads (i.e., Typical-parallel, Equal-parallel-pair, etc.). Table 5 shows some other interesting examples of CLSs, which are useful for the analysis below.

Meta-composite-LED schemas (MCLS) These schemas capture (i) the laws governing the interactions among components and (ii) how they are to be represented as configurations LEDs. Table 6 gives three examples, two of which, Series and Parallel MCLSs, are specializations of the third, General-Network MCLS. An interesting example to consider is the generation of the Series-and-parallel CLS {13} from MCLSs. From a circuit diagram with three resistors, R1 in parallel with a sub-network comprising R2 and R3 in series, the Series MCLS can first be applied to the sub-network of R2 and R3 to draw a stack of two AVOW boxes. Then taking this sub-network and R1 together, the application of the Parallel MCLS puts another box for R1 next to the stack for R2 and R3, like the drawing in Table 5.

Although the schemas are similar to each other, these particular pairs of schemas appear necessary on logical and psychological grounds (Cheng, in press). Much of the explanatory power of the framework for scientific reasoning with LEDs resides in the dimensions identified in Table 1.

Protocol Analysis

In other applications of the framework, it was used to characterize Galileo's kinematics discoveries (Cheng, 1997) and to analyse the nature of scientific understanding using particle collisions as an example domain (Cheng, in press). Here, it is applied to protocol data from experiments to continue to assess its utility. If coherent, parsimonious, complete and consistent accounts can be given of how subjects learn and do problem solving with AVOW diagrams, this will argue for the adequacy of the framework. However, the complexity of scientific domains and the normal use of multiple representations in problem solving makes this a significant challenge. For instance, a subject saying 'resistance' may be referring to (i) a variable in an equation, (ii) a particular circuit component, (iii) the magnitude of a property, or (iv) the gradient of the diagonal in an AVOW box. Here, an opportunity provided by particularly good quality verbal and behavioural protocols of a single subject was taken up. The subject, SL, participated in a pilot experiment on the effectiveness of AVOW diagrams for learning about electricity. Not only were his verbalizations detailed but he did a lot of meaningful pointing, which provided the means to disambiguate the referents of verbal expressions. SL is a graduate electrical engineer. In pre-test problem solving with the conventional algebraic approach to electricity, it was found that he had a reasonable, but not an expert, understanding of the domain.

SL received brief instructions on AVOW diagrams (see below) and attempted three problems using AVOW diagrams. The processing of SL's verbal protocols mainly followed the guidelines of Ericsson and Simon (1993), with transcripts of the verbalizations segmented into expressions and a simple coding scheme devised to catergorize drawing activity, computations and pointing. To identify when the subject appeared to be using a particular schema various coding rules based on the definition of the different schemas were devised. Basic schemas for components, LS or MLS, are implicated when: an expression refers to single resistor

Table 5 More Composite-LED schemas (CLS)

Slots	{12} N-equal-parallel	{13} Series-and-parallel
Diagrams		
Diagram-configurations	side-by-side only, same height	stack of 2 boxes side by side with one box, equal height
Domain-conditions	$V1=V2=..Vn$, $R1=R2= ... =Rn$ $1/RT=1/R1+1/R2+ ... +1/Rn$,	$V1=V2+V3$; $I2=I3$ $1/RT=1/R1+1/(R1+R2)$
Interpretation	N equal components in parallel	A pair in series in parallel with a single

Table 6. Meta-composite-LED schemas (MCLS)

Slots	{15} General-Network	{16}Series	{17} Parallel
Component-LEDs	AVOW boxes	AVOW boxes	AVOW boxes
Composition-constraints	Completely filled rectangle (no overlaps or gaps)	Stacked	Side by side
Domain-description	Any (2D) network	series components	parallel components
Encode-laws/relations	Kirchoff's laws: general	Kirchoff's laws: current conserved , voltage distributed	Kirchoff's laws: voltage conserved, current distributed
Mapping-rules	1 box for each component or sub-network. Topologies of circuit and diagram match.	1 box for each component or sub-network. Vertical order of boxes matches loads.	1 box for each component or sub-network. Horizontal order of boxes matches loads.
Interpretation-rules	current, voltage (and power) shared among components	Current shared through loads. Voltage split across loads. Power distributed among loads.	Current split over loads. Voltage shared across loads. Power distributed among loads.
Configurations	All (non-crossing) networks.	E.g., {8} {9} {10} {11}.	Parallel configurations.

or one AVOW box; or a group of boxes are treated as a unitary entity; or a network is taken as a whole with no consideration of its internal topology. Composite schemas, CLS or MCLS, are implicated when relations among boxes or the structure of a network are mentioned. Phenomena level schemas, LS and CLS, are indicated when there is a inference consisting of the recognition of a configuration/case followed immediately by the recall of associated information or the drawing of a diagram, with no intervening inference steps. Theoretical level schemas, MLS and MCLS, are deemed to operate when there are explicit inferences to obtain new information or the incremental piece-wise consideration/construction of a diagram.

To enable the identification of new schemas being generated/learned, four conditions were specified: (i) the subject comes across a novel domain configuration not previously solved; (ii) solves this sub-goal with a series of inferences that invoke existing schemas; (iii) there is no single schema that could have been used to make that inference; and (iv) the subject later solves another sub-goal for an isomorphic domain configuration without making a series of inferences. These conditions restrict the hypothesizing of new schemas to a minimum, so that difficulties in the explanation of the protocol cannot simply be circumvented by proposing new conceptual entities. This in turn makes the overall test of the framework as stringent as possible.

Working through the protocols, schemas to explain particular inferences were proposed based on the information available to the subject at each point in the protocol and the new information he inferred. All three problems have been analysed, but only the instructions and the first problem will be considered (given the limited space), to illustrate the nature of the analysis.

Instructions

These must be considered as they provided SL with his initial schemas. They consisted of two brief pages of written text and diagrams. From their content and SL's protocols it is clear that SL learnt the following schemas: Known-load {2} and Voltage-source {5} LSs; AVOW-box MLS {6}; Typical-parallel and Typical-series {8} CLSs;

Parallel {17} and Series {16} MCLSs. Consider, for instance, how the instructions supplied the information for all the slots of the Voltage-source LS {5}. On the second page of the instructions was a circuit diagram showing a network connected to a battery and a corresponding AVOW diagram. Both the battery and the overall height of the AVOW diagram shared a common symbol to indicate the voltage of the battery across the whole network. Thus, SL had a **diagram** representing a battery, which he knew was an idealised voltage source (**interpretation**), and the symbol told him that the fixed height of the rectangle (**diagram-configurations**) represents that constant voltage of the battery (**domain-conditions**).

Problem 1

In this problem the overall resistance of the network in Figure 2 had to be calculated, using the given values of the resistors. (Resistors will be identified in the text by a 'R' plus their value; e.g., R1/12.) Figure 3 shows SL's solution to the problem, with labels [A] to [I] for the AVOW boxes that were drawn. SL's general approach to this problem was first to incrementally construct the AVOW diagram from the circuit, starting with box [A] and working through to box [I]. Then SL assigned values to boxes in turn starting with [A] and propagating the known values through the diagram to help constrain the further assignment of values, which included a little algebra to find the values for [H] and [I]. Then from the calculated total height and width, SL found the overall gradient and thus the resistance of the network.

This problem solution can be characterized in terms of the schemas learned from the instructions, plus the generation of 3 new schemas. Table 7 summarizes the main steps in the solution with the particular schemas that were invoked or created at each step. The information read from or placed into particular slots are not given, but some examples will give the flavor of the types of inferences made and how they were coded.

(Step 1) The initial drawing of [A] is an application of the Typical-load LS {1}, with SL quickly drawing the AVOW box **diagram** for R1/12, assuming an

interpretation of it as a typical load. This is the first AVOW box SL has drawn. (Step 2) He checks what the diagram's features represent by comparing the given values to the example on the first page of the instruction sheet, which is an application of the AVOW-box MLS {6} to match particular diagram-features to domain-properties using the property-mappings. (Step 3) By examining the values SL may be considered to have specialized the Typical-load {1} LS into a Known-load {2} LS for R1/12, associating particular values with domain-properties, rather tha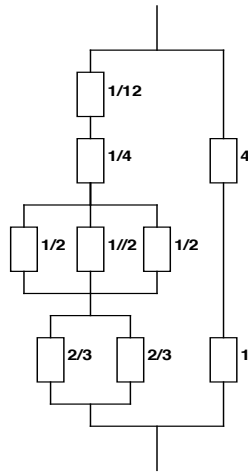n assuming they are just "typical". (Step 4) SL sees that R1/4 is in series with R1/12 and states that 'series are on top of each other', implying the Series MCLS {16} was used to decide where to place box [B]. (Step 5) The actual drawing of [B] is just an application of the Typical-load LS {1}.

Skipping to Step 11, we find an interesting example of the generation of the Series-and-parallel CLS {13}. SL is trying to decide how to draw [H] and [I] given [A-G] using the Typical-series {8} and Typical-parallel CLSs. The interpretation of R4 and R1 from the circuit diagram means that
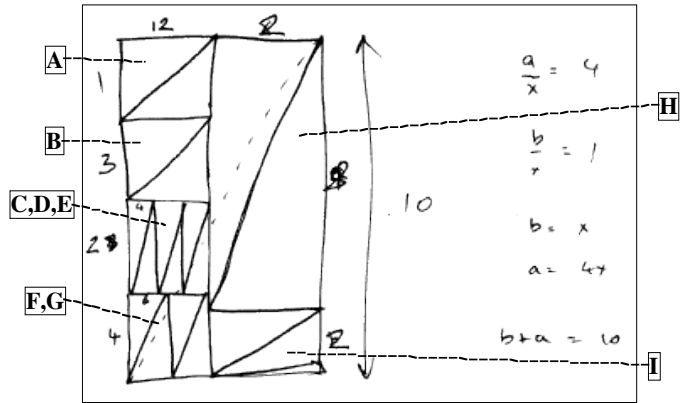

Figure 2. Circuit for problem 1.

Figure 3. Solution to Problem 1

both the schema are simultaneously applicable, so the diagram has [H] stacked on [I] and both next to [A-G]. Now, given this new configuration SL can reasonably be assumed to have generated a new schema, CLS Series-and-parallel CLS {13}, which happens to be used in the next problem.

The analyses of the second and third problems are largely similar, but other new schemas are generated.

Discussion

The framework has been used to analyse SL's protocols for the three problems. As the same conditions for identifying the different schemas were used throughout the instructions and all three problems, the analysis appears to give a fairly consistent interpretation. The analysis can be considered to

Table 7 SL's Solution Path to Problem 1

Step	Actions	Schemas Invoked and New Schemas Created (*)
1	Draw [A]	Typical-load LS{1}
2	Diagram features?	AVOW-box MLS{6}
3	(i) Compare [A] gradient with (ii) instruction example box	(i) Typical-load LS {1} =>Known-load LS {2} (ii) Known-load LS
4	Where to draw [B]?	Series MCLS {16}
5	Draw [B]	Typical-load LS {1}
6	Series sub-network, draw [CDE] perimeter	Typical-series CLS{8}
7	Split [CDE] into three equally	Parallel MCLS{17}, *N-Equal-parallel CLS {12}
8	Add diagonals	3X Typical-load LS {1}
9	Series sub-network, draw perimeter [FG]	Typical-series CLS {8}
10	(i) Split into two and (ii) add diagonals	(i) Typical-series CLS{8}, (ii) 2X Typical-load LS {1}, *Equal-parallel CLS
11	[H] and [I] in series and in parallel [A-G], draw perimeter [HI].	Typical-series CLS {8}, Typical-parallel CLS, *Series-and-parallel CLS {13}
12	Divide [HI] into two using given resistance	Typical-series CLS {8} and 2X Known-load LS {1}.
13	Total resistance is overall diagonal	AVOW-box MLS {6}.
14	(i) Compute values for [A]; (ii) check with Ohms law.	(i) Known-load LS {2}; (ii) AVOW-box MLS {6}.
15	Compute values for [B] given it is in series with [A].	Known-load LS{2} and Typical-series {8}.
16	Compute values for (i) [C] given (ii) it is in parallel with [DE] and (iii) [CDE] in series with [B]	(i) Known-load LS {2}; (ii) N-equal-parallel CLS {12}, (iii) Known-load LS {2}
17	Check with (i) Ohms law and (ii) instruction example.	(i) AVOW-box MLS {6}, (ii) Known-load LS {2}.
18	Compute values for (i) [F] given (ii) it is part of [FG].	(i) Known-load LS {2}, (ii) Equal-parallel CLS.
19	(i) Total height by summing (ii) individual heights	(i) Known-load {2}; (ii) 4X Known-loads {2}
20	Equations to compute (i) heights of [H] and [I] given (ii) total height.	(i) Series MLS {16}, (ii) 2X Known-load {2}.
21	Sum (i) widths, thus (ii) overall gradient	(i) 2X Known-loads {2}, (ii) Known-loads LS {2}.

be complete as there were no substantive expressions or drawings that could not be reasonably accommodated by the schemas. The analysis was fairly parsimonious as only a relatively small number of schemas of each type needed to be hypothesized to explain SL's behaviour and they were typically invoked more than once. The analysis has some coherence as it provides a fairly uniform characterization of the different phases of SL's reasoning and on quite different problems types. There is good reason to think that the number and particular definition of slots is necessary and sufficient for explaining the various inferences observed. At least for AVOW diagrams, there do not seem to be any redundant slots containing unnecessary information and none of the expressions had substantive information that could not be placed in one of the proposed slots.

Further, the framework appears to capture the complex reasoning that relates the values of properties of individual components/AVOW boxes to the global structure of the AVOW diagrams/circuit topology — an important aspect of problem solving with LEDs. The framework also provides an integrated account of problem solving and learning with AVOW diagrams. Problem solving is largely characterized by the application of schemas by matching contents of certain slots to available facts and then asserting the facts from the other slots of the schemas. One important aspect of learning is the generation of novel domain level schemas, LSs and CLSs, using the meta-level knowledge of the domain contained in MLSs and MCLSs.

The various slots of the schemas have, quite deliberately, not been defined in precise terms with the specification of particular data formats, because the exact nature of the information may vary from domain to domain, or even within a domain. For example, domain-conditions may be specified as magnitudes of properties for one aspect of a domain but as topological relations in another. The framework aims to be a general characterization of the classes of information that are necessary for problem solving and learning with LEDs in diverse problem domains, but in doing so trades some precision in the definition of slots for a great scope of applicability.

The proposed framework extends previous work on problem solving with diagrammatic representations based on schemas. In contrast to Egan and Schwartz (1979), who studied the role of perceptual chunks in the recognition of functional units in circuit diagrams, the framework provides a basis for studying problem solving that integrates circuit diagrams with the novel AVOW diagrams, which incorporate information about the underlying laws governing the domain. Koedinger and Anderson (1990) show how expert geometry problem solving can be characterized as the search through a space of *diagrammatic configuration schemas*, DCSs. The LS and CLS schemas are similar to DCSs, but the matching of slots contents is not only to the diagram configuration, as in DCSs, but also to the conceptual information about the domain. Further, the MLS and MCLS schemas provide a basis for beginning to explore the "knowledge-based" processes by which LS and CLS can be learned in the first place.

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