SCIENTIFIC DISCOVERY WITH LAW ENCODING DIAGRAMS

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Abstract

This paper introduces the concept of Law Encoding Diagrams, LEDs, and argues that they have had a role in scientific discovery, which has not been previously recognised. A LED is a representation that correctly encodes the underlying relations of a law, or a system of simultaneous laws, in the structure of a diagram by the means of geometric, topological and spatial constraints, such that the instantiation of particular diagram represents a single instance of the phenomena or a particular case of the law(s). Examples of LEDs in the history of science are discussed and the benefits of using LEDs in discovery are considered. LEDs are distinguished from other forms of diagrammatic representation. Previous work on the computational modelling of diagrammatic law induction is reinterpreted in terms of the search for diagrammatic constraints of LEDs. A general characterization of the role of LEDs in discovery is considered and a framework for classifying processes of discovery based on LEDs is proposed.

INTRODUCTION

An approach to the study of scientific creativity is to treat processes of discovery as complex phenomena for scientific study in their own right. An approach to the study of complex phenomena is to assume that the phenomena constitute systems that can be decomposed in to sub-components for separate investigation. In Simon's (1981) terms, the creative processes are assumed to be a *nearly-decomposable* system. When various sub-components have been individually characterized, studies of the interactions between the sub-components can be conducted to understand the phenomena as wholes. One of the fundamental tasks in scientific research is the discovery of components of such systems, which includes the identification of sets of entities that have not previously been recognized as a unique class.

This paper identifies and explicates what seems to be a novel class of entities, a new component, that may have had substantive roles in scientific creativity. The class is a particular type of diagrammatic knowledge representation, called Law Encoding Diagrams (LEDs), which encapsulate laws in the structure of diagrams. The concept of LEDs was first developed in research on science and mathematics instruction with diagrams (Cheng, 1994, and in press). However, once defined, the applicability of the concept of LEDs to understanding some aspects of scientific discovery quickly became apparent. There is a parallel between students attempts to comprehend laws and scientists original struggle to understand phenomena and formulate theories. LEDs can be found in published works and private notes of scientists and have properties that facilitate different forms problem solving. They may, thus, have had a role in the making of some discoveries.

This work is part of the research in cognitive science and Artificial Intelligence that is investigating processes of discovery. This work adopts two fundamental characterizations; (i) problem solving is complex information processing in the form of heuristic search in physical symbol systems (Newell & Simon, 1972); and (ii) scientific discovery is a variety of problem solving (Langley, Simon, Bradshaw, & Zytkow, 1987). Under these conceptions, many empirical investigations of the processes of discovery have been conducted (e.g., Klahr & Dunbar, 1988; Qin and Simon, 1990), and numerous computational models have been constructed (e.g., Langley et al., 1987; Shrager & Langley, 1990; Kulkarni and Simon, 1988; Cheng, 1992a). Significant work on the role of diagrammatic representations and visual imagery in problem solving and creativity has been conducted (e.g., Novak, 1977; Larkin & Simon, 1987; Koedinger & Anderson, 1990; Finke, 1990; Stenning and Oberlander, 1991). An example of work more directly focused on discovery is that of Shepard (1978, 1988), who examined the role of visual imagery in the discoveries of numerous central figures from the history of science. Although the evidence used was mainly anecdotal, Shepard considers various reasons for the effectiveness of mental imagery, including amongst others, their richly concrete and isomorphic structure and their engagement of highly developed, innate mechanisms of spatial intuition. Visual images help the scientist to notice significant details and relations not adequately preserved in a purely verbal formalism. There has also been some computational modelling of the inductive discovery of laws using diagrams (Cheng, 1992b; Cheng & Simon, 1992, and in press).

To introduce and justify a novel class of entities various tasks need to be undertaken. First, there is the identification of examples of the entities that may be members of the class. Second, a definition of the class needs to be formulated, which covers the existing examples and that can be used to identify further examples of the class. Third, the new class has to be distinguished from known classes that are seemingly similar. Forth, explanation or reasons for the existence of the class need to be considered. Fifth, the wider consequences and implications should be addressed. This may take a number possible forms, but just two will be mentioned. (i) Previous work may be reinterpreted in terms of the new class and shown to be subsumed within the new conceptualization. (ii) Analysis of the interactions between the new class and other components of the system can be conducted to show how the new class provides a more complete and potentially more coherent characterization of the system as a whole.

This paper attempts to carry out these tasks for LEDs in discovery, but not strictly in the order given. In the next section the definition and properties of LEDs are considered. Examples from the history of physics and chemistry are considered in the third and fourth sections. The firth section reinterprets previous work, on the computational modelling of diagrammatic law induction, in terms of LEDs. LEDs are distinguished from other diagrammatic representations in the sixth section and reasons for the use in discovery are considered in the seventh section. The penultimate section considers a general conceptualization of LEDs in discovery and presents a framework for classifying processes that employ LEDs in discovery. The conclusion discusses issues and research questions raised by the introduction of the concept.

LAW ENCODING DIAGRAMS

A Law Encoding Diagram, LED, is a representation that encodes the underlying relations of a law, or a system of simultaneous laws, in the structure of a diagram by the means of geometric, topological and spatial constraints, such that the instantiation of particular diagram represents a single instance of the phenomena or a particular case of the law(s). The structure of a LED is defined by its *elementary* and *law-encoding* constraints. This section considers these aspects of LEDs in the context of an example.

A LED from the history of physics is Corollary I of Newton's (1687) *Principia*, which will be called the *Motion-parallelogram* LED. Newton's three laws of motion consider forces and motions in one-dimension. Corollary I combines the first and second laws, by considering the action of two forces, so that motion in two dimension can be examined. Figure 1 shows the diagram that accompanies the corollary. A force *M* acts on the body at *A*, in the direction *AB* and would make it travel distance *AB* in a given time. A force *N* acting independently on the body at *A*, in the direction *AC*, and would make the body travel *AC* in the same time. The combined force of both *M* and *N* would make the body travel in the direction *AD* covering *AD* in the same time. The location of *D* is found by completing the parallelogram.

Figure 1 about here

In the definition of LEDs, the term *representation* means (i) a system of symbols that can be arranged in meaningful configurations to express states of knowledge and (ii) the associated rules, or constraints, for generating new legal states from existing states. For example, a LED for a law conventionally stated as an algebraic formula may use diagrammatic elements, such as lines and shapes, to stand for variables. The relations defined by the formula would be captured by geometric, topological and spatial relations, such as congruence, adjacency and symmetry. In the Motion-parallelogram the symbols are the lines such as those in Figure 1 and the relations are those described in the previous paragraph.

The structure of a particular LED is defined by its diagrammatic constraints, which can be classified as *elementary* and *law-encoding* constraints. The elementary constraints indicate the referents of the diagrammatic elements and also specify how they represent. For example, in the Motion-parallelogram the direction and length of the line *AB* (and *AC* or *AD*) indicates the direction and distance travelled in the given time when the body is acted upon by a force (or combined forces). The elementary constraints may also prescribe some basic structural requirements of a LED, which do not directly encode the laws. The law-encoding constraints capture the underlying relations defined by the laws. They specify how the diagrammatic elements must be related geometrically, topological and spatially, for the LED to encode the law. For example, the relation between the motions *AB*, *AC* and *AD* in the form of a parallelogram, which is essentially vector addition, is an example of a simple geometric law-encoding constraint. The diagrammatic constraints can be used as rules for the legal manipulation of the LED. As long as they are satisfied, the LED will be consistent with the encoded laws. To summarise, the elementary and law-encoding constraints of Motion-parallelogram are:

E1. The length of each line is in proportion to the distance traversed by a body, in a given time, due to an impulse that occurs at the beginning of the line.

E2. The orientation of the line indicates the direction of the impulse and the direction of motion of the body.

L1. The resultant motion, direction and magnitude, of the simultaneously action of the two forces on the body is given by the completion of the parallelogram.

Capturing laws in the form of a diagram permits LEDs to benefit from the advantages that diagrammatic representations confer over verbal or mathematical representations, whilst still grounding the inferences in the laws. Various forms of reasoning and problem solving can be conducted easily with LEDs. Quantitative reasoning can be performed with LEDs that represent magnitudes of variables by the size some diagrammatic element. For example, by drawing the Motion-parallelogram to scale, it is possible to determine the magnitude and direction of the resultant motion, AD, given the angles and lengths of AB and AC. One form of qualitative reasoning involves gradually varying the size of one diagrammatic element and observing how the rest of the LED changes. For example, in the Motion-parallelogram, the direction or magnitude AC can be gradually changed to observe the effect on AD, with AB fixed. Extreme case reasoning considers what happens when one or more of the variables tends to some limiting value, such as infinity or zero. For example, Figure 2 shows the limiting magnitudes of resultant motion, AD, when the directions of AB and AC are (almost) in the same or in opposite directions. LEDs permits constraint based reasoning, in which the value of any variable can be determined when the others are known, irrespective of the which variables were independent or dependent in the physical situation. For instance, given the direction and magnitudes of resultant AD and AB, the direction and magnitude of component AC can be inferred knowing that it must be one side of the parallelogram of which AD is the diagonal.

Figure 2 about here

The next section considers how LEDs can also be the basis for more complex and higher level forms of reasoning and problem solving.

DEMONSTRATION OF NEWTON'S FIRST PROPOSITION

The Motion-parallelogram was used by Newton to derive the first proposition in his *Principia* (Proposition I, Section II). This example will demonstration the utility of the concept of LEDs for understanding the role diagrams in discovery and will illustrate some of the more complex forms of problem solving that can be done with LEDs.

Proposition I concerns the motion of bodies in space around fixed points to which they are attracted, such as a moon around a planet. Using geometric reasoning, the relation between the speed of the body and its distance from the fixed point are derived from the laws of motion.

Newton is showing that Kepler's laws are a consequence of his own, more fundamental, laws of motion.

The various steps in Newton's demonstration of Proposition can be analysed, or reinterpreted, in terms of the application of two LEDs. One is the Motion-parallelogram. The other will be called the *Time-triangle* LED, which encodes the definition of speed, distance divided by time, but in an unexpected manner. (This LED does not correspond to a particular corollary or proposition in the *Principia*, but is implicit in Proposition I.)

Figure 3 about here

In the Time-triangle time is represented by the area of a triangle and the distance by the length of its base, *AB*, Figure 3. Given that the area of a triangle is proportional (1/2) to its base times its height, then the speed is inversely proportional to the height. For example, a fixed distance, *AB*, is covered at twice the speed (half the height) in half the time (half the area). These relations can be restated as elementary and law-encoding constraints.

E1. The direction of the body is given by the orientation of the base of the triangle.

E2. The distance covered is in proportion to the length of the base.

E3. The time is in proportion to the area of the triangle.

E4. The speed is inversely proportional to the perpendicular height of the triangle.

L1. The relations between the distance, time and reciprocal of speed are given by the geometric relations between the base, area and height of the triangle.

Figure 4

The reason for adopting this seemingly awkward LED, for what is really a simple relation, is that it greatly simplifies the demonstration of Proposition 1. The Time-triangle can be directly applied to considerations of motions of a body about a fixed point. Figure 4 shows three Time-triangles, with *S* as a fixed point and *AB*, *BC* and *Bc* as motions of bodies. As each triangle shares a common side *SB*, and *SB* is parallel to and equi-distant from the two dashed lines, the areas of the triangles are equal, so the times are equal. Thus, comparison of different motions in equal times, with respect to some common fixed point, is feasible and simple.

Figure 4 is a portion of the full diagram accompanying Proposition I, which is shown in Figure 5. A body moving relative to *S* from *A* to *B* will continue in the same amount of time from *B* to *c*. However, the body is being attracted to *S*, so assume that the centripetal force occurs as a single impulse at *B*, such that if B were stationary it would travel in the direction *BS*, reaching *V* in the given time. Now, the Motion-parallelogram is immediately applicable to *BV* and *Bc*, with a resultant of *BC*. The centripetal force in the direction *BS* turns the body so that its new path is *BC*. Further, as *Cc* is parallel to *SB*, the times for motions BC and Bc are equal. The body would continue in motion in a straight line covering *Cd*, but another impulse occurs at *C*. Hence, by similar repeated application of the two LEDs, the path *ABCDEF* is generated. All the areas of triangles *SAB*, *SBC*, etc., are equal, thus the relative velocities at *A*, *B*, etc., are given by the reciprocals of the heights of these respective triangles. By considering what happens as the area of the Time-triangles tends to zero, Newton shows that the path of the object around *S* will be a curve.

Figure 5

The successive applications of the Motion-parallelogram and the Time-triangles demonstrate the compositional property of LEDs, in which outcomes generated by one LED are used as the basis for the construction of further LEDs to gradually build up an analysis of a complex situation[1]. Proposition I is a good example of how the diagrammatic instantiation of abstract laws, encoded in the form of LEDs, makes it possible to apply the law to a particular situation.

LEDs that encode laws of motion occurs elsewhere in the history of physics. For example, one of the several ways that Galileo represented the relation between the distances and times of a body in free fall motion was using the diagram shown in Figure 6. *DC* is the vertical diameter of a circle and *DGF* is a right angled triangle with *F* on the circumference of the circle. The ratio of the times of two bodies falling from rest along *DG* and *DC*, is given by the ratio of the lengths of *DG*

[1] See Cheng, in press, for compositional analyses of Newton's cradle.

to *DF*. This LED forms the basis for the derivation of several of Galileo's proposition in the *Two New Sciences*. (See Cheng, 1992b, and Cheng & Simon, in press, for more on Galilean diagrams.)

Figure 6

LEDs have had a significant role in some discoveries of physics. The next section considers some LEDs in chemistry.

LEDS IN CHEMISTRY

Processes of discovery in chemistry are different from those in physics, because the nature of the phenomena and laws in the two disciplines. Whereas discovery in physics can be (grossly) characterised as finding mathematical laws that govern sets of continuous phenomenon, discovery in chemistry can be (just as grossly) characterized as finding classification schemes for groups of substances according to their properties or compositional structure. Thus, it seems reasonable to expect that the function of diagrams, including LEDs, will be different in chemistry. The LEDs in this section are further examples of this class of representations that contrast with the physics LEDs already considered.

A central part of the history of chemistry has been concerned with structure of substances. This section will consider: (i) a system of LEDs used for the explication of the structure of compounds in terms of the electronic structure of the elements, Lewis structures; and (ii) Summerfeld's diagram for the electronic structure of atomic elements in terms of quantum numbers.

Lewis Structures and Octet Theory

Figure 7

At the beginning of the Twentieth century G. N. Lewis discovered a simple way to account for chemical bonding by assuming electrons surround atoms in concentric cubes (Russell, 1971). The outer cube has between one and eight electrons, which are located at the corners of the cube. The inner cubes are "kernels" with complete compliments of eight electrons. Figure 7a show the structure for oxygen atoms. Bonding is represented by the alignment cubes so each atom has a complete outer cube of eight electrons. In *single* bond two cubes share a common edge, and in a *double* bond two cubes share a face; Figure 7b shows the double bond structure for an oxygen molecule composed of two atoms (O_2).

Lewis further refined his diagrammatic representations electronic structure, abandoning the image of concentric cubes, because the assumed cubic structure was too strong a constraint; for example, triple bonds could not be represented. The ultimate form of Lewis's structures was in the form of dot, or colon, diagrams. These diagrams are LEDs and will be called Lewis Structure, LS, diagrams. The main elementary constraints of the LS diagrams are:

E1. An atom of a particular element is represented by its chemical symbol.

E2. Each electron in the outer shell of an atom is represented by a dot around the symbol.

Figure 8a shows the six dots (electrons) of a neutral oxygen atom. Lewis's idea about complete cubes of electrons now takes the form of a rule, the Octet rule, in the dot diagrams. The rule states that in all compounds, atoms achieve a complete octet of electrons in the outer shell. The main law-encoding constraint of LS diagrams captures this idea:

L1. Legal dot diagrams for molecules (compounds) are those which have eight dots surrounding the symbol for each atom.

The LS diagram for an oxygen molecule, O_2, is shown in Figure 8b. LS diagrams were important in the history of chemistry (Russell, 1971), for example: (i) they showed for the first time the fundamental similarity between oxyacids, borofluoric and hydrochloric acids; (ii) they explained the relationship between ammonia and the ammonium ion; and (iii) they explained the existence of dimeric molecules and dative covalent bonds. From the law encoding constraint we can infer that an molecule comprised of three oxygen atoms is possible, that is Ozone, Figure 8c. By encapsulating the octet rule in the form of a LED, Lewis was able to demonstrate how the structure of complex compounds was a direct consequence of the simple arithmetic octet rule, exploiting the compositionality of the LS diagrams.

Figure 8

The definition of LEDs states that a particular diagram should correctly encode the given laws. The LS diagram does capture the octet rule, but it is possible to generate structures that are impossible compounds. For example, Figure 8d shows an oxygen molecule composed of four atoms, which satisfies the law-encoding constraint that each atom is surrounded by eight electrons. The explanation for this state of affairs, is not that the LED is an imperfect representation of the given rule, but that the rule itself is only an approximation to the correct underlying laws of chemical bonding. The octet rule does not take into account the individual nature of the bonds between particular orbitals of the electrons, nor does it capture considerations about the energetic stability of compounds. In other words, a LED is only as good as the laws which it encodes. The laws underlying the electronic structure of atoms are considered in the next subsection.

The Electronic Structure of the Atom

The complexity of the laws underlying the electronic structure of the atom is apparent from a summary of the laws[2]. The electronic structure of the atom and therefore the form of the period table is governed by four quantum numbers. The *principle* quantum numbers are integers; n=1 to 4. The principle quantum numbers correspond to electron shells, designated by the capital letters, K, L, M, and N. For each n certain integer *angular* or *orbital* quantum numbers are possible, l=0 to (n-1). These angular quantum numbers are also called electron types and are designated by the letters, *s*, *p*, *d* and *f*. For each l there are certain integer *magnetic* quantum numbers; m=-l to l. Finally, for each m there are two *spin* quantum numbers, s=+1/2,-1/2. Each configuration of the first three quantum number corresponds to a unique electron *orbital*, which can accommodate two electrons with different spin numbers. In "building up" of the structure of an element two rules are followed. First, Pauli's exclusion principle which states that no two electrons can have exactly the same quantum numbers. Second, the *aufbau* principle states that the order of filling of the orbitals is in order of increasing energy; 1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4d, etc. This is not the same as the order of the designation of orbitals. The, none to obvious, consequences of this are that the outer electron configuration of the groups of elements, and thus the form of the period table, can be derived: Groups I and II (s-Block) — ns^{1or2}; Groups II-VIII (p-Block) — ns^2np^{1to6}; transitional group (d-block) — $(n-1)d^{1to10}ns^2$; and, inner transitional group $(n-2)f^{1to14}(n-1)d^1ns^2$.

[2]Full accounts are given in most general chemisty texts; e.g., Buttle, Daniels, and Beckett 1981.

Figure 9

Diagrams of various kinds have been used to show the regularities that are present in the (seemingly) complex structures just summarized. Mendeleev's Periodic table shows the consequences of the relations by displaying the elements in groups and periods. One particularly interesting and useful example was a scheme devised by Sommerfeld in 1925 (Mazurs, 1974). The diagram is based on the quantum numbers underlying the electronic structures and can be considered as a LED, Figure 9. Here, it will be called the Quantum Number Electronic Structure (QNES) diagram. The elementary constrains are:

E1. The two dimensions of the diagram are the principle and angular quantum numbers.

E2. The "pyramid" of squares are the possible configurations of the two quantum numbers that can be "filled" with electrons.

E3. The number in a square is the number of electrons for a particular pair of principle and angular quantum numbers.

The law encoding constraints are:

L1. The maximum number in a square on a particular tier is indicated by the numbers on the right. These numbers are the number of electrons permitted by the combination of the magnetic quantum numbers (i.e., 1, 3, 5, 7) multiplied by two for the spin numbers.

L2. The filling of the squares begins at the bottom left and continues in the order shown by the diagonal arrows in Figure 10. This constraint encodes the *aufbau* principle, because the order of the squares given by the sets of diagonals is that of increasing energy[3].

L3. The dashed lines in Figure 11 indicate sets of squares that relate to particular periods in the periodic table. The lines will be called *period lines*. From left to right these are periods 1 to 7, respectively. The outer electron configuration of an element is obtained by reading-off the numbers of electrons on the period line furthest to the right, which does not cut squares with all zero entries.

[3]The structure of the first few inner transition elements are a slight exception to this.

Figure 10

For example, in Figure 9 the structure of Bromine is given. Of the eight squares possessing electrons, the first seven in order of filling (L2) are complete, with their maximum numbers of electrons (L1). The outer electron structure of Bromine has a configuration of $3d^{10}4p^{5}4s^{2}$ (L3).

Figure 11

Like other LEDs, the QNES diagram encodes the underlying law and makes explicit relations between the properties of the phenomenon that are obscure in a verbal or mathematical representation. The maximum numbers of electrons that are permitted in each electron shell, K to N, can be found by summing the maximum number of electrons in each column (i.e., K=2, L=8, M=18, etc.). Written as a linear series, the pattern underlying the order of filling of the orbitals is not obvious, but in the LED it follows a simple diagonal pattern (L2). The maximum number of elements in each period, which is given by the number of outer electrons, can be found from the QNES diagram by summing the maximum number of electrons for each square along each period line. Figure 11 shows the number of members of each period shown under the diagram. The jump from 8 to 18 members in period 4 corresponds to the transition elements, and the jump from 18 to 32 in period 6 corresponds to the inner transition elements. The basis for Lewis's theory of electrons in concentric cubes can also be seen in the QNES diagram. The sum of the maximum number of outer *p* and *s* orbital electrons is eight, and the left to right order of the period lines in Figure 11 is analogous to Lewis's succession of concentric cubes. The similarity of the outer electronic structures of members of the same group is responsible for their similar chemical properties. The interaction of the three law-encoding constraints results in the similar pattern on the outer period lines of QNES diagrams for members of same group. Figure 12 shows the configurations for the first four members of group VII.

Figure 12

The reasoning conducted with the LEDs in this and the previous section has been based on existing LEDs, but nothing has been said about the discovery of LEDs in the first place. Lewis's notion about the electronic structure of atoms being composed of concentric cubes occurred whilst he was trying to explain the ideas involved in the periodic table to an elementary class, after he had become interested in the (then) new theory of the electron (Russell, 1971). The combination of the ideas seems to have been important to the discovery, but the underlying processes involved are far from apparent. The next section considers certain processes of discovery of a LED in detail.

DIAGRAMMATIC LAW INDUCTION WITH LEDS

Examples of the use of LEDs in deductive forms of discovery can be found in published works, but reports of cases of discoveries with an inductive character are rarer, because of the hypothetico-deductive slant of formal scientific writing. Details of the processes of data driven discovery can, for example, be retrieved from the note-books of scientists and by retrospective interviews. However, such methods do not usually provide adequate information for complete characterizations, although they may yield sufficient details to enable computational models to be built (e.g., Kulkarni and Simon, 1988).

The general point is also the case with the role of LEDs in discovery. Thus, in this section the consideration of inductive discovery with LEDs will take the form of the examination of a computational model. Cheng and Simon (1992) argued that it would have been easier for the early physicists to have discovered the conservation of momentum using a diagrammatic representation than conventional algebraic formulas. Huygens and Wren presented their findings on elastic collisions using LEDs (Hutton, Shaw & Pearson, 1804), but whether the LEDs were instrumental in the discovery or merely used for the purposes of exposition is a question yet to be answered, and requiring further historical evidence. However, the possibility of law induction, more generally, using diagrams has been considered (Cheng and Simon, in press), with the construction of a computational model, called HUYGENS, that finds laws in the form of one-dimensional diagrams. HUYGENS follows in a long line of computational models of discovery in Artificial Intelligence and this research was originally conceived in that context. However, HUYGENS can also be considered as a model of discovery of LEDs from data and thus demonstrates the possibility of the inductive discovery of LEDs. This section will first introduce a historical example that HUYGENS models and then considered how HUYGENS makes the discovery in terms of the search for diagrammatic constraints.

Huygens and Wren's LEDs

Figure 13

In papers to the Royal Society of London, in 1669, on the investigations of the collisions of elastic bodies, both Huygens and Wren presented similar diagrams (Hutton, Shaw & Pearson, 1804). The collisions were head-on impacts between two elastic bodies travelling in a straight line. The diagrams were central to their expositions on the collision and fit the definition of LEDs. Figure 13 shows three examples of Huygens's and Wren's diagrams, on the top and bottom rows, respectively. In Huygens's diagrams *A* and *B* are the two bodies, their velocities before impact are denoted by the lines *AD* and *BD*, the velocities after impact, by *EA* and *EB*; and the masses of *A* and *B* by *BC* and *AC* [*sic*], respectively. In the diagrams, the lengths of *DC* and *CE* are always equal. Wren's diagrams, second row in Figure 13, are essentially the same, except for differences in notation and that Wren explicitly states that the diagrams are reversible; that is, either *Ro* and *So* or *Re* and *Se* can indicate the initial velocities, with *eR* and *eS* or *oR* and *oS* as the final velocities, respectively.

Figure 14

The same diagrams may be re-drawn to show their various elements and their structure more clearly, Figure 14. These diagrams are called one-dimensional property diagrams (1DP diagrams for short), because the properties are represented by lines in one-dimension. The labels indicate the properties for which each line stands: m_1 and m_2 are the masses of the two bodies; U_1 and U_2 are their velocities before collision, and V_1 and V_2 their velocities after collision. The left diagram shows that body-1 comes in from the left and impacts body-2, which is initially stationary. Body 1 is bigger than body 2, as shown by the middle line. The bottom line shows that both bodies travel off to the right. The speeds of the bodies are in proportion to the lengths of their respective lines. The centre diagram shows a collision where the bodies approach from opposite directions with equal speeds, but depart with different speeds in opposite directions, because the masses have different magnitudes. The right diagram shows that when the ratio of the initial speeds, U_1/U_2 , is equal to the inverse of the ratio of their masses, m_2/m_1, then the final speeds for each body is the same as it was before collision, but the bodies' directions are reversed.

Table 1

The elementary and law-encoding constraints for 1DP diagrams are given in Table 1. The velocity difference constraint (L1) directly encodes the relation:

$$U_1 - U_2 = V_2 - V_1 \qquad \ldots (1)$$

which will be called the *velocity difference* law. The mass-line scaling constraint (L2) and the diagonal rule (L3) together capture the momentum conservation law, which in conventional algebraic form is,

$$m_1.U_1 + m_2.U_2 = m_1.V_1 + m_2.V_2 \ . \qquad \ldots (2)$$

Further, by combining Equations 1 and 2, an additional law can be derived,

$$^1/_2 m_1.U_1^2 + {}^1/_2 m_2.U_2^2 = {}^1/_2 m_1.V_1^2 + {}^1/_2 m_2.V_2^2 \ . \qquad \ldots (3)$$

This is the energy conservation equation for collisions between bodies, which holds because the collisions are perfectly elastic and no energy is lost.

Given the elementary and law encoding constraints it is possible to efficiently solve a wide variety of problems (see, Cheng, in press). The question now is: how can the 1DP diagrams, like those in Figure 14, be found from sets of values obtained from collision experiments? Table 2 shows some idealised data which HUYGENS has used to successfully find the 1DP diagram.

Table 2

How HUYGENS Discovers LEDs

The HUYGENS discovery system (Cheng and Simon, in press) has modelled the inductive discovery of the 1DP diagram and a version of Black's law (Cheng, 1993). Like other discovery programs that do law induction, HUYGENS takes sets of data and recursively searches for regularities by exploring a space of terms. In previous systems, such as BACON (Langley et al., 1987), the terms have been algebraic combinations of variables. In HUYGENS the terms are combinations of diagrammatic elements that are potential components of LEDs.

Discovery systems have operators, regularity spotters and heuristics. The operators generate new terms by combining variables or existing terms; for example multiplying two variables together or finding the difference between them. The regularity spotters look for particular relations among the terms that may be of interest; for example, that two terms are equal, or that one term increases, monotonically, as another decreases. The heuristics have the job of guiding the direction of the search and of managing the overall size of the search space. The heuristics may use the information about particular relations found by the regularity spotters to suggest the application of particular operators to generate new terms that may be closer to a complete law.

HUYGENS's operators, regularity spotters and heuristics work on a diagrammatic representation comprised of one-dimensional lines. The system works in recursive cycles of operator and spotter application, taking into consideration in successive cycles new terms found in the previous cycles. Over a number of cycles *groups* of diagrams are generated. A group is a set of diagrams obtained by applying the same sequence of operators to all the sets of data. Different groups are generated by different sequences of operators. With judicious guidance of the heuristics, HUYGENS aims to find a group of diagrams that has the same consistent pattern across all its members, covering every variable in each diagram. From the sequence of operators and the final pattern, it is possible to determine the algebraic form of the relation.

Figure 15

Figure 15 shows some of the groups of diagrams, in the rectangles, that HUYGENS considers in making this discovery. Of the three sets of data in Table 2, only diagrams for cases 1 and 3 are included. Various symbols are used to indicate significant parts of the lines. The origin or beginning of a line is indicated by an 'o' and its point of interest (interest point) by a 'x'. The position of the interest point is determined by the operations performed on the line. When the interest point is to the right of the origin (o—x), then the magnitude is positive; otherwise, it is negative (x—o). Where appropriate, construction points are also marked by a '|'; a construction point being an intermediate point identified and used by operators in the construction of a new line. The lines are labelled with symbols for their variables.

The first step in HUYGENS simulation of the discovery is to plot the data in Table 2 as a group of diagrams; the top rectangle in Figure 15. The origins of the lines are at the same location and the lengths of the line are in proportion to the magnitudes of the variables, which gives the position of their interest points. The sign of each variable determines the "direction" of the line. Plotting the lines in this fashion embodies the elementary constraints concerning the direction and lengths of the diagrammatic elements; E1, E5, and E8, Table 1. As the variables for different properties have different units, the scale of the lines for different properties is arbitrary, but consistent across variables for the same property. The lines are shown with vertical separation in Figure 15, but in the program they are considered as line segments on the number line. This, in effect, satisfies the elementary constraint, E9, relating the vertical relation between the mass and velocity lines.

In the second cycle, HUYGENS's heuristics select the two pairs of velocities for consideration, because *v1* and *v2* are dependent variables and because *u1* and *u2* stand for the same property as them (see Cheng and Simon, in press, for descriptions of the heuristics). When the regularity spotters were applied to these variables no patterns applicable to the whole group was found. Thus, by default, ADD and SUBTRACT operators were applied to the group, each generating a single new group; shown as the second row in Figure 15. The ADD operator finds the sum of each pair of velocities, by moving one line in relation to the other, so that its origin is in the same position as the interest point of the other line. The new line is the total length of the combined line. The SUBTRACT operator is similar, but places both interest points together to find the difference between two lines. The new line connects the two origins, and the shared point of interest is remembered as a construction point. The elementary constraints, E3 and E4, which specific that the tips of the U_1 and U_2 arrows, and the tails of the V_1 and V_2 arrows, are adjacent, is introduced by the SUBTRACT operation.

In the third cycle, the regularity spotters find no common patterns in the group generated by the ADD operator, so it is abandoned. However, the spotters find that the interest points of the $U_1 — U_2$ and $V_1 — V_2$ lines are on opposite sides of their common origins (in every diagram of the group). Thus, the NEGATE operator is applied to the pairs of lines and the direction of the $V_1 — V_2$ line is reversed, so that the interest points are all on the same side of the origins. What this has done is to make the pairs of velocity lines consistent with the elementary constraint E2, which in effect states that the U_1 and V_1 (or the U_2 and V_2) arrows are on the same sides in the 1DP diagram.

In the fourth cycle, the regularity spotters find that the lengths of the $U_1 — U_2$ and $V_2 — V_1$ lines are equal; the interest points are in the same position. Thus, HUYGENS has found the velocity difference law-encoding constraint, L1. The algebraic form of this law can be inferred from the operators that have been used. The SUBTRACT operator gave u_1-u_2 and v_1-v_2 and the NEGATE operator changed v_1-v_2 into v_2-v_1, which equals u_1-u_2; Equation 1. As there are still mass lines to be considered, the ends of the $U_1 — U_2$ (and $V_2 — V_1$) line are taken as construction points, and the construction point made by the SUBTRACT operator are changed into new points of interest; fourth row of Figure 15.

In the final cycle, HUYGENS combines the lines for mass with the relation already found using a set of five *normalization* operators. These operators take two lines standing for one property, in this case mass, and use a line standing for another property, velocity, as the basis against which to scale, or standardize, the mass lines. In this case the common $U_1 — U_2$ and $V_2 — V_1$ line is the standard. Consider the NORMALIZE-ADD-2 operator as an example. The operator re-draws the lines for the masses end to end and scales them so that their total length is equal to that of the standard. The operator is applied twice, because the order of m_1 and m_2 may be reversed; Figure 15 shows (bottom right) the two groups of diagrams generated. The other normalization operators have different ways to relate the mass lines to the standard and the five operators produce ten (5 pairs) groups of diagrams in total, of which four are shown in Figure 15. These operators consider various combinations of relations between the masses and the velocity lines, so are exploring possible diagrammatic constraints that specify the scaling of the mass lines and their relation to the velocity lines. NORMALIZE-ADD-2 happens to be one that encapsulates the relevant elementary constraints E6 and E7, and the two law-encoding constraints L2 and L3. It is the only operator that produces a group of diagrams with a common pattern; shown by the thicker rectangle in Figure 15. The pattern which the spotters identify is the equal distance from the interest point on the mass line to the interest points on the lines for U_1 and $-V_1$. Given the way NORMALIZE-ADD-2 manipulates the mass lines, the following a relation between the velocities and masses has been found:

$$\frac{m_2}{m_1+m_2}(U_1-U_2) = \frac{U_1 - V_1}{2} \quad . \qquad \text{. . . (4)}$$

With a little algebraic manipulation and given Equation 1, previously found by HUYGENS, this equation can be easily shown to be equivalent to the momentum conservation law, Equation 2[4].

HUYGENS demonstrates that it is possible to find LEDs from sets of data, by searching a space of diagrams that are consistent with different sets of diagrammatic constraints. When a set of diagrams with a common pattern is found, the set of constraints encoding the laws underlying the data have been discovered. An obvious limitation of HUYGENS is that the system only considers one-dimensional structures. Nevertheless, this section has shown that LEDs can be inductively discovered and that the concept of LEDs has utility in that the operation of HUYGENS was successfully interpreted in terms of heuristic search for LEDs.

LEDS, GRAPHS AND NOMOGRAMS

One of the tasks to be performed when a new class of entities is discovered is to consider how the class differs from, or subsumes, other existing classes, in order to show that the new class is unique and novel. Two recognized classes of diagrammatic representation need to be distinguished from LEDs. They are co-ordinate graphs and nomograms and are dealt with in turn.

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[4]I.e., rearranging Equation 4 gives $2m_2u_1-2m_2u_2=(u_1-v_1)(m_1+m_2)$ then $m_2(u_1+v_1)=m_1u_1-m_1v_1+2m_2u_2$. Substituting for u_1+v_1 on the left hand side using Equation 1, gives $m_2(u_2+v_2)=m_1u_1-m_1v_1+2m_2u_2$, which simplifies directly to Equation 2.

Figure 16

Graphs typically are not LEDs because they do not encode laws in the manner specified in the definition of LEDs. In a typical graph a curve, or family of curves, shows the relations among the variables in a continuous fashion over a range of values. The shape of the curve captures the relation among the variables. The relations in a LED are captured by the law encoding constraints that specify the structure of the diagram in geometric, topological or spatial terms. Some LEDs employ graph-like co-ordinate systems, but each instantiation of a LEDs shows one set of values for the underlying law, one case of the phenomenon rather than a range of behaviours. Thus, given a graph and a LED for the same laws, each point on a curve in the graph corresponds to one instantiation of a LED. For example, Figure 16 is a distance time graph with two lines for different speeds. Time-triangles are shown for four points on the lines. The time-triangles 1 and 2 have twice the height of triangles 3 and 4, because the speed on the lower line is one half that of the upper. The areas of triangles 1 and 3 are half those of triangles 2 and 4, because the times are doubled. The lengths of the based of triangles 2 and 3 are equal as the distances are equal.

Figure 17

Nomograms are a class of diagrammatic representations that were used for computing quantitative solutions to equations before the advent of electronic calculators. A nomogram still sometimes found in road atlases is one for calculating the distance that a car, with a known fuel consumption (m.p.g.), can travel on a given amount of fuel. Figure 17 shows a schematic example of a nomogram for a formula relating three variables, *u*, *v*, and *w* (e.g., quantity of fuel, m.p.g., and distance). The lines *U* and *V*, *W* are, respectively, scaled axes for three variables and *I* is the *index* line. The choice of scales and the design of the shape and position of the lines is such that the index line can be used to compute values for the variables that are consistent with the formula. Given two values, say for *u* and *v*, the index line is extrapolated through their points on *U* and *V* until it intersects *W*. That point on the *W* scale gives the value of *w*. Johnson (1952) and Allcock, Reginald-Jones, & Michel (1950) are books on the design of nomograms.

Nomograms are like LEDs in that each use of a nomogram with the index line at a particular angle and position represents one set of values of the encoded law, equivalent to an instantiation of a particular LED. A nomogram's index line in different locations and orientations corresponds to different sets of values for the same law. The relations between the variables are determined by the scaling and spatial arrangement of the axes, which are arranged to facilitate calculations with the underlying laws. This, however, typically means that the structure of nomograms obscures the form of the relations. For example, different formulas can be represented in a nomogram with identical spatial arrangement of axes but using different scales; a nomogram for the calculation of sums using linear scales can be changed into one for computing products by adopting log scales. The relations are implicit in the nature and arrangement of the scales. LEDs, on the other hand, aim to reveal the relations implicit in a law using the structure of diagrams defined by explicit law encoding constraints that specify legal configurations of diagrammatic elements. Thus the spatial and geometric arrangements of diagrammatic elements in LEDs change for different sets of values, for example Figures 2 and 14, whereas only the index line of nomograms changes and not their overall form.

LEDs are a class of diagrammatic representations that are distinct from co-ordinate graphs and nomograms.

REASONS FOR USING OF LEDS IN DISCOVERY

To justify the introduction of a new class of entities it is necessary to explain the reason for their existence. In chemistry the concept of atomic elements is needed for the explanation of reactions and compounds. For LEDs in processes of discovery, this means explaining why LEDs are useful for discovery and considering the benefits they confer over other forms of representation; in other words, why scientists would have devised and used them.

Discovery with LEDs benefits from the advantages that diagrammatic representations, in general, often confer over other forms of representation in problem solving. These include: reductions in the amount of computation required in search and recognition processes (Larkin & Simon, 1987); the replacement of laborious sequences of logical or mathematical inferences by quick perceptual inferences that exploit the power of the visual system; and, the reduction of working memory loads. Cheng & Simon (1992) argue that the discovery of the conservation of momentum would have been easier under a diagrammatic representation than using algebra. LEDs may also be considered as encoding knowledge in the form of perceptual chunks, like Koedinger and Anderson's (1990) *diagrammatic configuration schemas*, which are part of the foundation of expertise. However, the two forms of representation are not equivalent. The diagrams in diagrammatic configuration schemas are representations of the physical situation or phenomenon, whereas the diagrams of LEDs encode the structure of laws governing a phenomenon (see Cheng in press, for a more detailed discussion). LEDs may also be considered as *abstraction representational systems* (Stenning and Oberlander, 1991), which benefit computationally by being a small subset of all possible models, rather than the complete and large set of all conceivable models.

In contrast to other forms diagrammatic representations, such as graphs, LEDs exploit the variety and richness of geometric, topological and spatial forms to express relations in the laws, whilst using constraints so ensure that only lawful configurations of LEDs are permitted. Some important consequences follow from this.

Encoding of laws in the structure of diagrams makes explicit many relations that are implicit in the laws. In conventional representations and graphs such relations often require deliberate sequences of inferences to be found. For example, the velocity difference relation, Equation 1, can be derived from the two conservation laws, but it is not immediately obvious from the algebraic form of the two equations. In the 1DP diagram, the same relation is captured by the first law-encoding constraint, Table 1, and is quite apparent from the structure of the diagram; the ends of the velocity lines are in line vertically, Figure 14. The idea underlying the HUYGENS discovery program is to find such relations by constructing diagrams using sequences of diagrammatic operators and spotting simple patterns or structures.

Given a set of laws, different LEDs can be devised that facilitate different forms of reasoning and problem solving, either by not being computationally equivalent, or by not being informationally equivalent. Two representations are informationally equivalent when the information in one can be directly transformed into the other. Two informationally equivalent representations are computationally equivalent when the amount of computation needed to make inferences in one is equal to that in the other (Larkin and Simon, 1987). The informational and computational differences between LEDs for the same law can be exploited by choosing a LED best suited to particular forms of reasoning. For example, the 1DP diagram, Figure 14, is useful for qualitative and quantitative reasoning about different configurations of elastic collisions. The mapping from the properties of the phenomenon to the diagram is straightforward and the law-encoding constraints are simple to use; see Table 1. Other LEDs exist that possess diagrammatic elements that explicitly represent momentum and energy in elastic collisions, which makes them suitable for high level conceptual reasoning (Cheng, in press). Such reasoning is difficult in the 1DP diagram, because there is no explicit information about these higher level theoretical terms in the diagram. One would have to visualizing sums of the products and squares of the velocity and mass lines to be able to reason about momentum and energy.

Using the flexibility of different diagrammatic structures, LEDs may be invented to conduct particular steps in a line of reasoning. Newton's use of the Time-triangle to encode the definition of speed is a case in point, Figure 3. By making the area of the triangle represent time, the base of the triangle can stand for the motion of the body. This enabled Newton to use the Time-triangle in conjunction with the Motion-parallelogram; the single straight line for motion was shared by the two LEDs. Thus the large composite diagram explaining the motion of astronomical bodies could be constructed, Figure 5. LEDs other than the Time-triangle can be devised to encode the definition of speed, but they lack this special feature that Newton needed for the derivation of proposition I.

A general explanation for the existence of LEDs in science, is that they confer a range of benefits over other forms of diagrammatic and non-diagrammatic representations in the processes of problem solving and discovery.

LEDS AS INTERMEDIATE REPRESENTATIONS

When a new component of a system is identified, the operation of the system as a whole may need to be reconsidered, as the new component may interact with the other existing components. The sum of the interactions constitute the system as a whole, so new interactions may result in a change of the overall conception of the system. This section first considers a general conceptualization of the role that LEDs have in discovery. Then it will be shown how disparate processes involving LEDs, as considered in the examples, can be brought together under a single framework based on the conceptualization.

The central idea of the conceptualization is that LEDs are intermediate representations that bridge the conceptual gulf between the laws and the phenomena. This view of LEDs comes from the work on LEDs for instruction in mathematics, science and engineering (Cheng, in press). In turn, the general idea of using of intermediate representations or models for instruction originates with White (1989, 1993). On the one hand, a particular instantiation of a LED, a single diagram, represents a single instance of the phenomenon. Each 1DP diagram in Figure 14 is one collision, and each configuration of QNES diagrams is a particular element, Figure 12. On the other hand, the elementary and law-encoding constraints of a LED capture the underlying laws of the phenomenon. The 1DP diagram's constraints capture the two conservation laws, and the QNES's constraints capture the quantum number relations and laws governing the filling of electron orbitals. LEDs are more concrete than abstract general laws, but also more abstract and general than descriptions of phenomena. Thus, LEDs have an intermediate position between phenomena and laws. Being intermediate between laws and phenomena, LEDs may be considered to be models. However, they are special in the way that they represent with the phenomena below and the laws above.

Given that LEDs are intermediate between phenomena and laws, a framework for classifying processes of discovery with LEDs can be defined in terms of two criteria. The first is whether the process acts (I) between the LED and the laws or (II) between the phenomena and the LEDs. The second criterion consider whether: (a) just the phenomena or laws exist; (b) just the

LED exists; and (c) both LED and the phenomena, or both LED and the laws, exist. There are six combinations of these criteria, which are now described in turn, with examples.

Ia. Invention of new LEDs from a law, or laws, stated in some conventional representation; for example, Newton's formulation of the Motion-parallelogram LED from his first and second laws of motion.

Ib. The generalization of a LED into an abstract law stated in the conventional representation for a particular field, such as mathematical formulas. This is the transformation of the elementary and law-encoding constraints into the conventional form. An example is the conversion of the constraints of the 1DP diagram into the algebraic form of the momentum conservation law.

Ic. Relating an existing law and an existing LED. One case of this is the demonstration that a simple law or relation is directly subsumed by a LED. For example, Lewis's cubic ideas about electronic structure of the atom follows from the interpretation of the constraints of the QNES diagram.

IIa. The making of novel prediction from a LED. The prediction of the existence of the four atom oxygen molecule, O_4, using Lewis diagrams is an example, albeit a false one.

IIb The process modelled by HUYGENS is the inductive discovery of a new LED from a set of data or observations, including the specification of the appropriate elementary and law-encoding constraints.

IIc. The explanation of some phenomenon using LEDs; such as Newton's derivation of the curved path of astronomical bodies from the Motion-parallelogram and Time-triangle.

Table 3

Table 3 summaries the six types of processes (ticks indicate the thing exists, question marks indicating that it is to be found, and the arrows show the direction of inference).

Episodes of discovery involving LEDs can be decomposed into to these processes. For example, Newton's demonstration of Proposition I was a case involving process Ia followed by process IIc; inventing the Motion-parallelogram and Time-triangle, followed by the construction of Figure 5 given that a curved path was required.

The general conceptualization of LEDs as intermediate representations makes clearer the overall role that LEDs have in discovery. The framework of processes shows the utility of the conceptualization. Each of the six types processes could be a research topic in its own right.

CONCLUSION

This paper has introduced Law Encoding Diagrams as a class of diagrammatic knowledge representations, which seem to have had a significant role in some discoveries. A LED is diagrammatic representation that encodes the underlying relations of a law, or a system of simultaneous laws, in the structure of a diagram by the means of geometric, topological and spatial constraints, such that the instantiation of particular diagram represents a single instance of the phenomena or a particular case of the law(s). The structure of particular LEDs are defined by their elementary and law-encoding constraints. Examples of discoveries involving LEDs have been described and LEDs have been distinguished from other diagrammatic representations. The reasons for the use of LEDs in discovery have been considered. LEDs are intermediate representations that may bridge the conceptual gulf between phenomena and laws. A framework for the classification of different forms of discovery with LEDs has been presented. There are of course many research questions and general issues raised by LEDs. To conclude, a few are briefly addressed.

Examples of LEDs in early physics and chemistry have been discussed, but are there LEDs in other scientific domain or disciplines? Feynman diagrams are one intriguing candidate that are currently being considered for addition to the list. One approach to the discovery of other LEDs will be to examine major scientific discoveries that apparently involved the use of visual imagery, such as those collected by Shepard (1978, 1988).

The framework for discovery processes involving LEDs identifies six classes of processes, Table 3. Can all forms of discovery with LEDs be placed in one of these categories and can whole episodes of discovery be interpreted in terms of the classes? For example, an analysis of Galileo's use of LEDs, as his science of kinematics matured, could be made in terms of the classes, to determine whether there is shift from processes relating LEDs and phenomena (types IIa-c, Table 3), to processes relating LEDs and laws (types Ia-c).

Above it was argued that LEDs can make processes of discovery easier by being informationally and computationally superior to other forms of representation. Some formal support for this hypothesis comes from Cheng and Simon's (1992) analysis of the processes involved in the discovery of the conservation of momentum. The more general issue is whether the hypothesis is true for other forms of discovery and across different scientific domains.

New discoveries typically raise more questions than they answer, so these questions and issues concerning LEDs in discovery are, perhaps, an indicator of the value of the concept of Law Encoding Diagrams.

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Table 1 Elementary and Law-encoding constraints for the 1DP diagram

	Description of constraint
	Elementary
E1	The velocity arrows are drawn in the same direction as the motion of the bodies.
E2	The relative positions of the arrows in the diagram are the same as the relative physical positions of the bodies to each other. When body-1 is to the left of body-2, then U_1 and V_1 are to the left in the diagram.
E3	The tips of the U_1 and U_2 arrows are adjacent.
E4	The tails of the V_1 and V_2 arrows are adjacent.
E5	The length of the arrows are in proportion to their speeds.
E6	The mass lines are drawn end to end.
E7	The left to right order of the mass lines is the reverse of the velocity arrows.
E8	The lengths of the mass lines are in proportion to their respective masses.
E9	The mass line is drawn equi-distant between the two velocity lines.
	Law-encoding
L1	Velocity difference: the tails of the initial velocities arrows and the tips of the corresponding final velocity arrows must be in line vertically, making the total lengths of the $U_1—U_2$ and $V_1—V_2$ lines equal.
L2	Mass-line scaling: the total length of the mass line, $m_1—m_2$, equals the length of the velocity lines.
L3	Diagonal Rule: the small circles indicate the ends of the lines that are not fixed by L1 and L2; these points must lie on a straight vertical or diagonal line.

Table 2 Sample collision data

Symbol	Property	Type	Case 1	Case 2	Case 3
m_1	mass	independent	2	1	3
m_2	mass	independent	1	2	1
U_1	velocity	independent	3	3	3
U_2	velocity	independent	0	-3	1
V_1	velocity	dependent	1	-5	2
V_2	velocity	dependent	4	1	4

Table 3 Framework of Types of LED Discovery Processes

Process type	Ia	Ib	Ic	IIa	IIb	IIc
Laws	√	?	√			
	⇩	⇧	⇳			
LEDs	?	√	√	√	?	√
				⇩	⇧	⇳
Phenomena				?	√	√

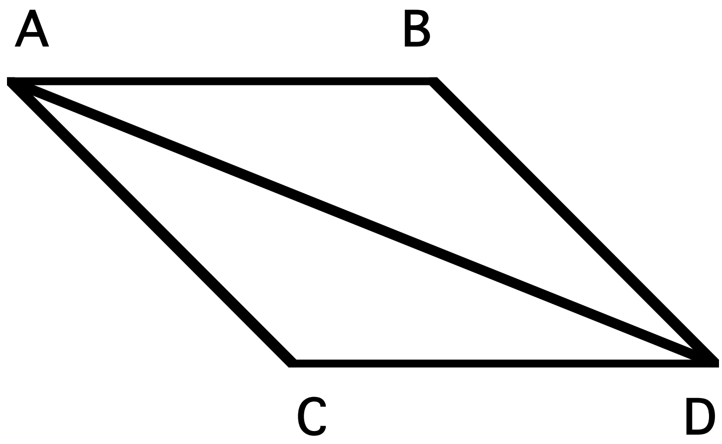

Figure 1 Motion-parallelogram

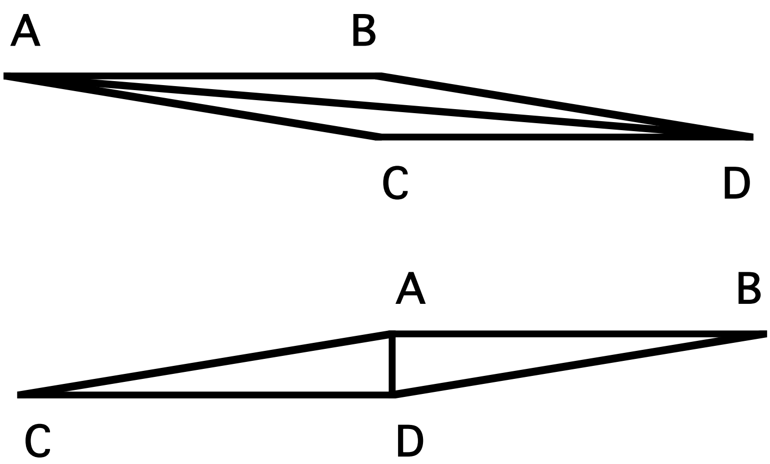

Figure 2 Extreme Case Motion-parallelograms

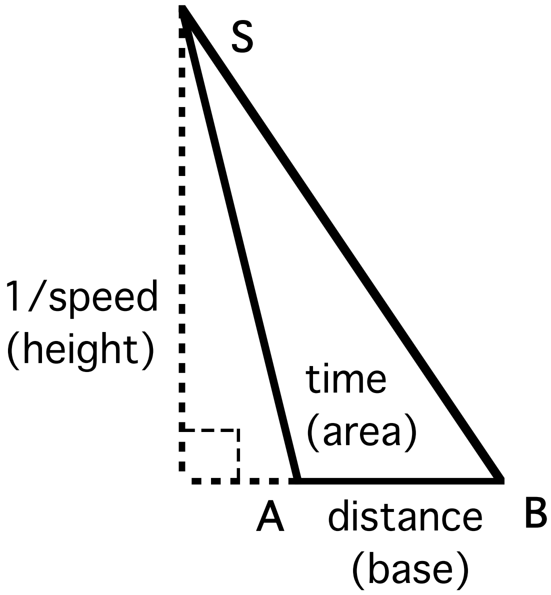

Figure 3 Time-triangle

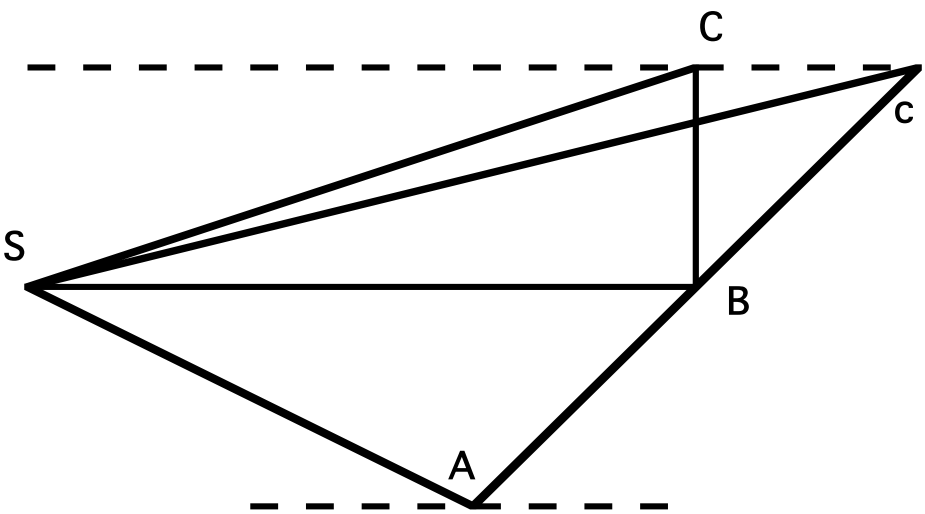

Figure 4 Time-triangle LEDs

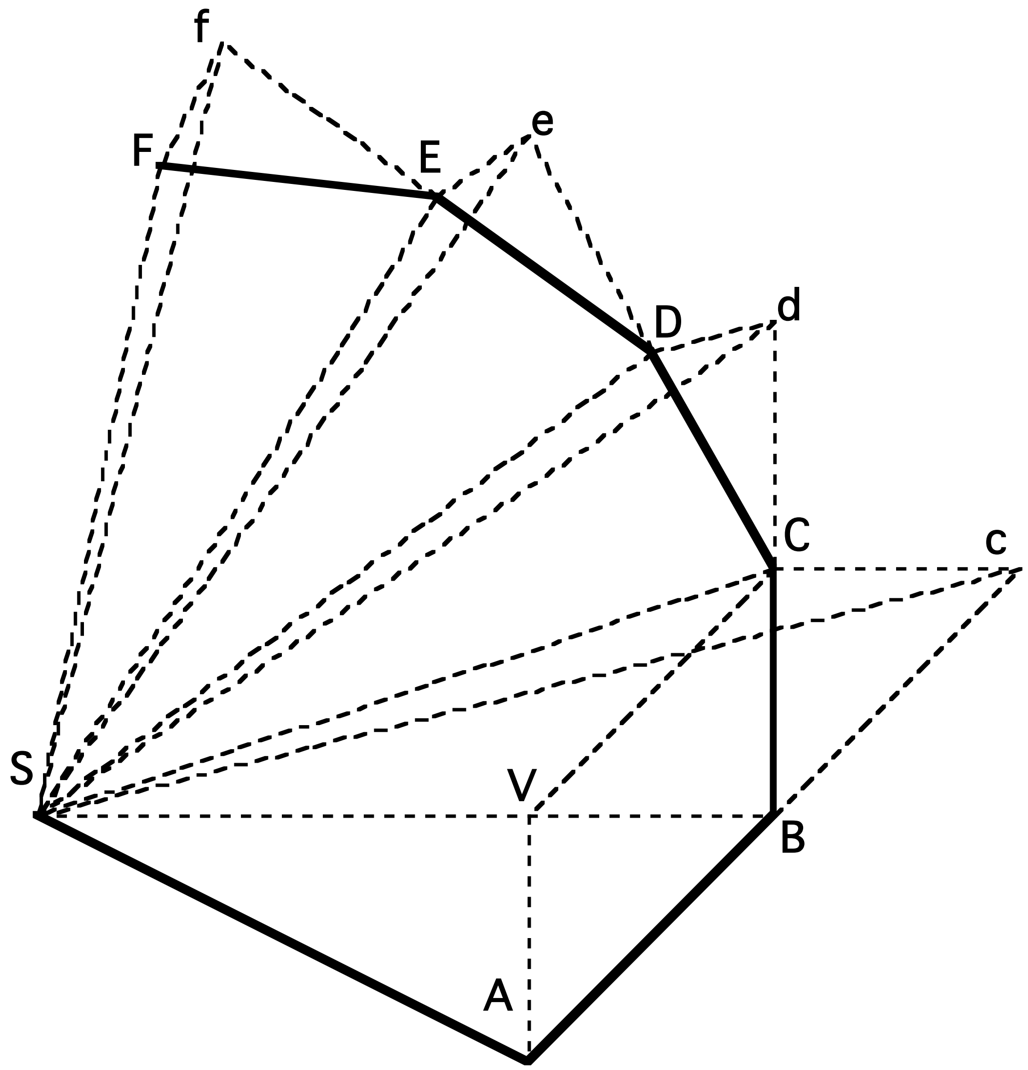

Figure 5 Proposition I

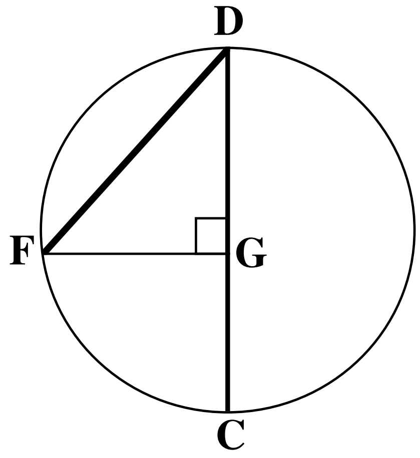

Figure 6 Galileo's Law of Free Fall

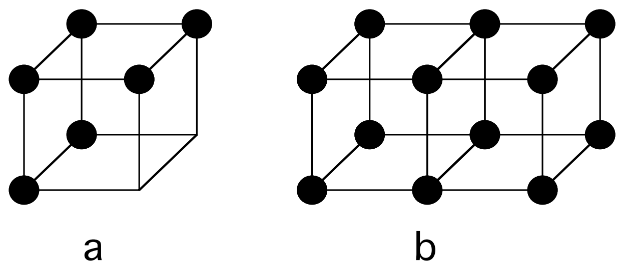

Figure 7 Lewis Cubes (a) Oxygen atom (b) Oxygen Molecule

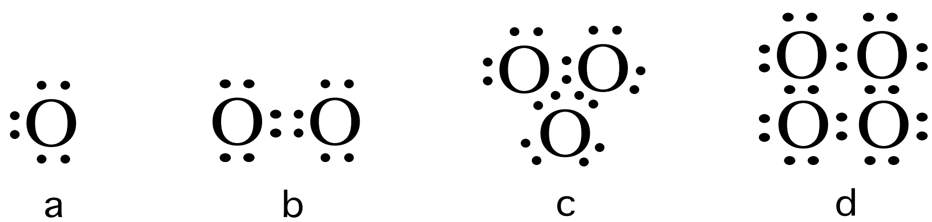

Figure 8 Lewis structures (a) Oxygen atom, (b) O_2, (c) Ozone O_3, (d) O_4

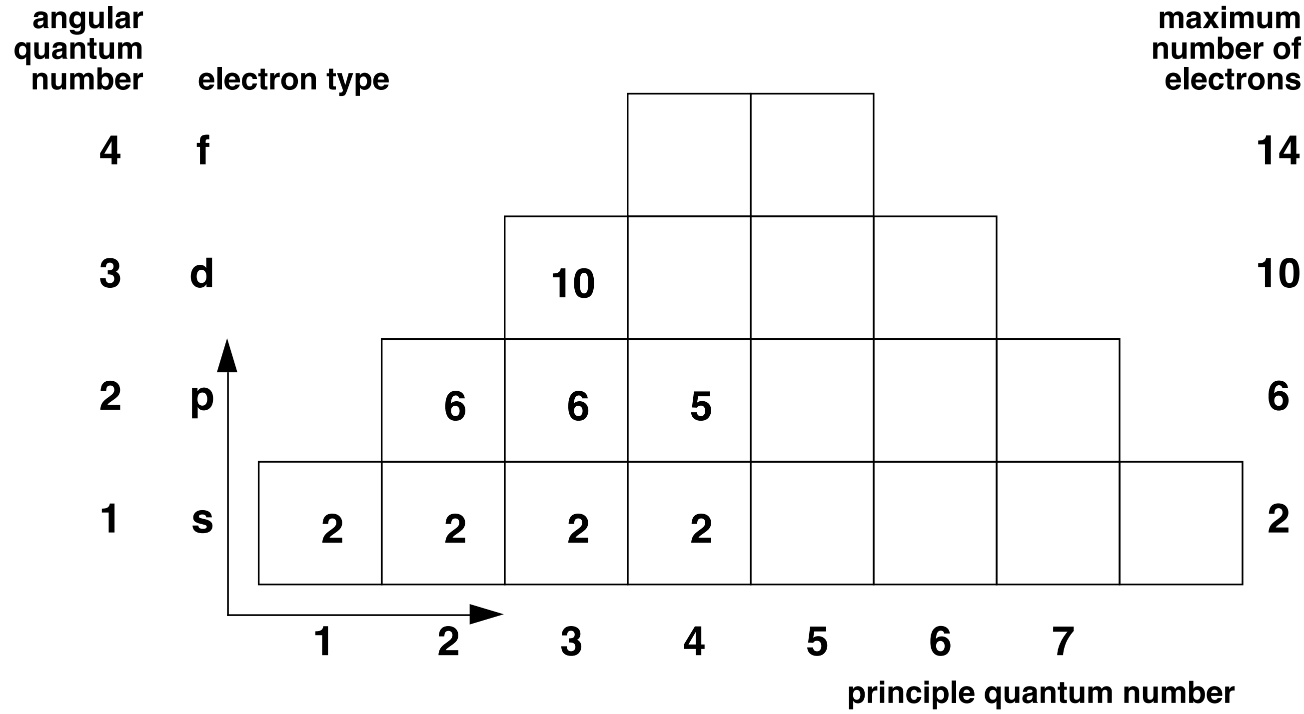

Figure 9 Quantum Number Electronic Structure Diagram

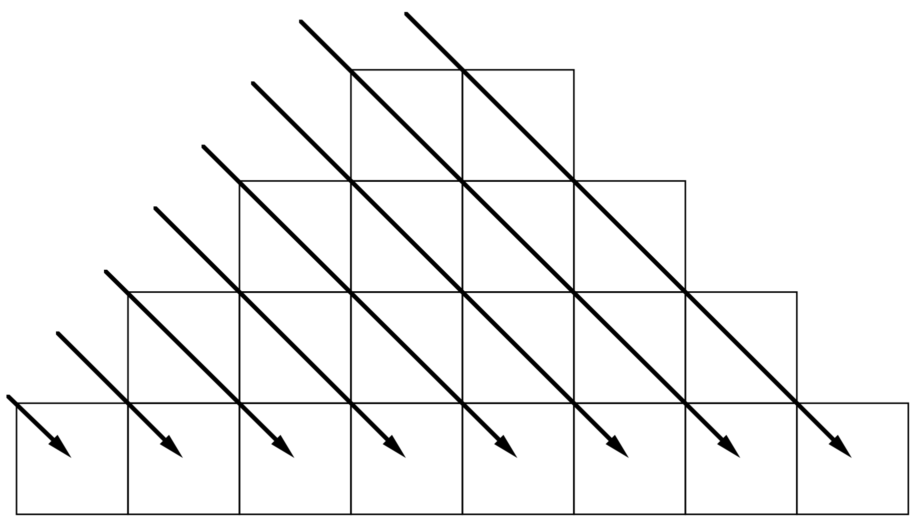

Figure 10 Constraint L3

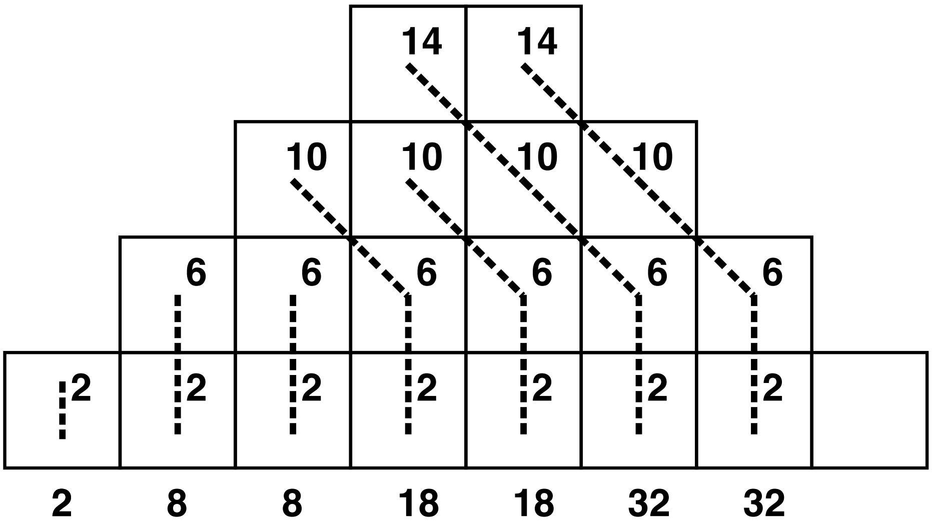

Figure 11 Outer Electron Configurations

```
F          Cl            Br    10        I       10 10
   5          6  5           6  6  5          6  6  6  5
2  2       2  2  2        2  2  2  2       2  2  2  2  2
```

Figure 12 QNES Diagrams for some Group VII Elements

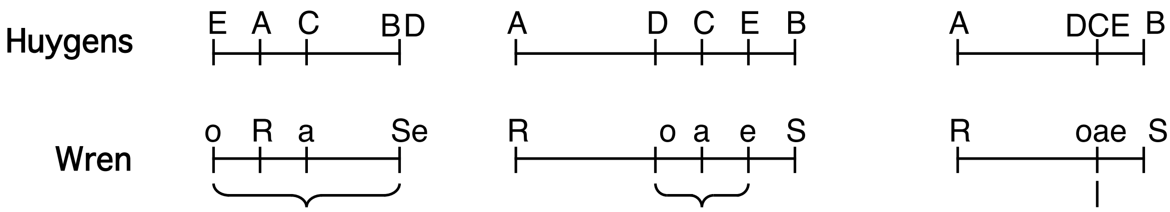

Figure 13 Huygens's and Wren's Elastic Collision Diagrams

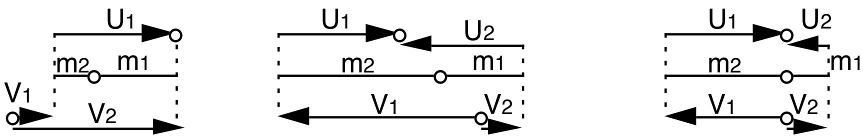

Figure 14 One-dimensional Property Diagrams

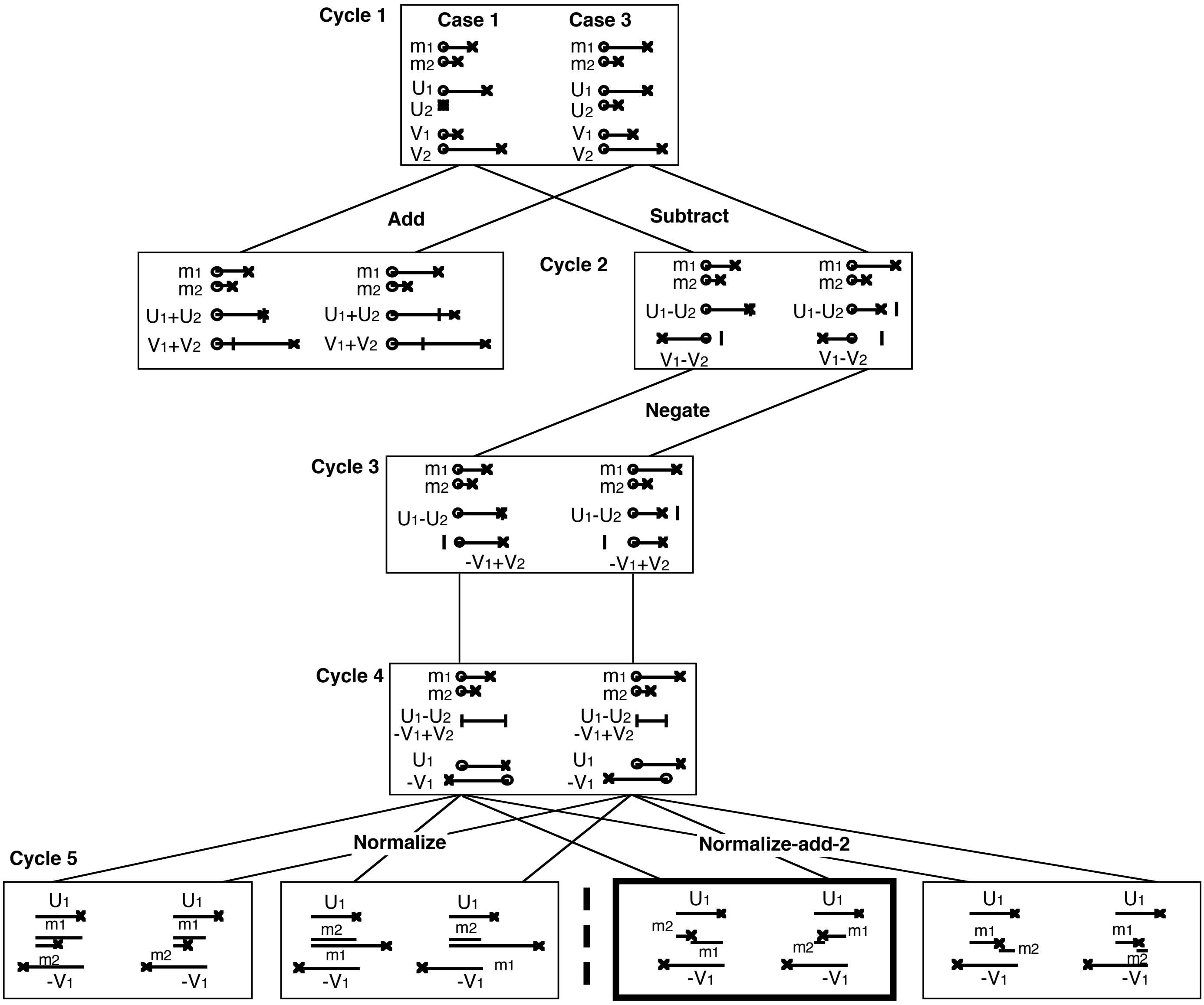

Figure 15 Space of groups of diagrams

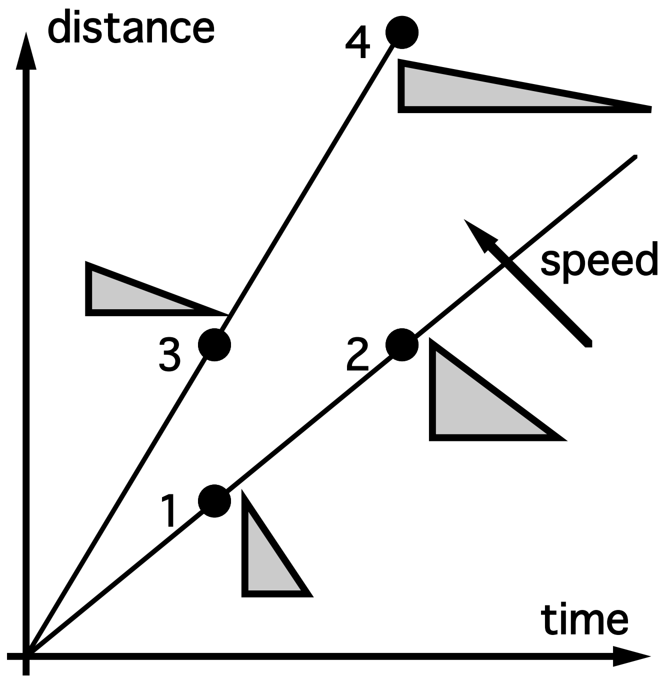

Figure 16 Distance-time graph with time-triangles

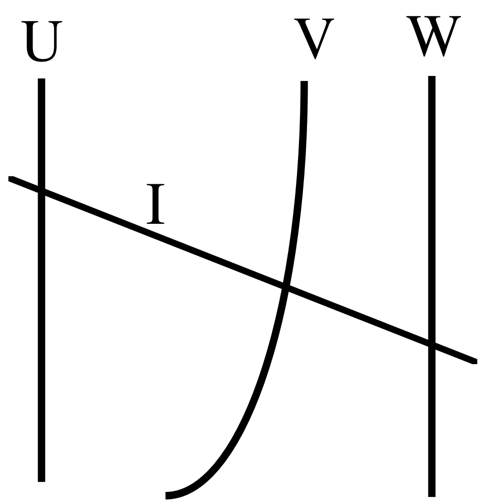

Figure 17 A Nomogram